Spike-Timing Dependent Competitive Learning of Integrate-And-Fire Neurons with Active Dendrites

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Abstract. Presented is a model of an integrate-and-fire neuron with active dendrites and a spike-timing dependent Hebbian learning rule. The learning algorithm effectively trains the neuron when responding to several types of temporal encoding schemes: temporal code with single spikes, spike bursts and phase coding. The neuron model and learning algorithm are tested on a neural network with a self-organizing map of competitive neurons. The goal of the presented work is to develop computationally efficient models rather than approximating the real neurons. The approach described in this paper demonstrates the potential advantages of using the processing functionalities of active dendrites as a novel paradigm of computing with networks of artificial spiking neurons.

1 Introduction

For a long time, dendrites have been thought to be the structures where complex neuronal computation takes place, but only recently we have begun to understand how they operate. Dendrites do not simply collect and pass synaptic inputs to the soma, but in most cases they shape and integrate these signals in complex ways [1]. With our growing knowledge of such processing, there is a stronger argument for taking advantage of the processing power and active properties of the dendrites, and integrating their functionality into artificial neuro-computing models [2]. The features of the models presented here are a computationally optimized interpretation of processing functionalities observed in real neurons.

2 Spike Processing with Active Dendrites

Real neurons show a passive response only under very limited conditions. In many brain areas, such as the cerebellar cortex and neocortex, a reduction of ongoing synaptic activity has been shown to increase the membrane time constant and input resistance, suggesting that synaptic activity can reduce both parameters [3, 4]. The model of a neuron with active dendrites presented in this paper is based on such observations. It builds upon the leaky integrate-and-fire neuron. The developed model of an artificial neuron has a set of new active dendrites. In the equations describing the model, the s, d and m indices indicate that the variable or parameter belongs to a synapse, dendrite or

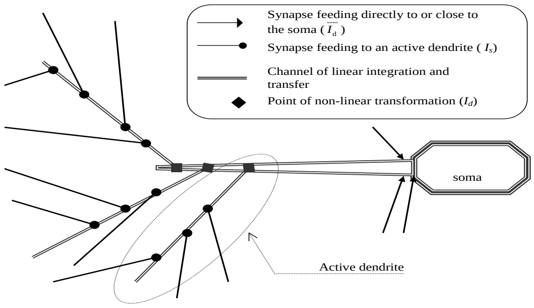

Fig. 1. Model of a neuron with active dendrites.

the membrane respectively. For the active dendrite i with a set of $\mathcal{S}^i$ synapses, the total post-synaptic current I_s^i is described by:

$$\tau_s \frac{d}{dt} I_s^i(t) = -I_s^i + \sum_{j \in \mathcal{S}^i} c^{ij} \sum_{t^{(f)} \in \mathcal{F}^j} \delta(t - t^{(f)})$$

where synaptic connection j at dendrite i has weight c^{ij}, $\mathcal{F}^j$ is the set of pre-synaptic spike times filtered as Dirac δ-pulses, and τ_s is the synaptic time constant. In addition, the neuron has a number of synapses feeding close to or directly to the soma. The same as the above equation holds for the total current $\overline{I_s}$ from these synapses.

Further, the current passing through the dendrite into the soma is described by:

$$\tau_d^i \frac{d}{dt} I_d^i(t) = -I_d^i + R_d^i I_s^i(t)$$

Here, the time constant τ_d^i and resistance R_d^i define the active properties of the artificial dendrite as they depend on the incoming post-synaptic current. They are defined as functions of I_s^{i*} which is the maximum of $I_s^i(t)$ since the last pre-synaptic spike:

$$\tau_d^i \triangleq \tau_d^i(I_s^{i*}) = \tau_m - \frac{\tau_m - \tau_s}{1 + e^{-\frac{10}{\tau_s}(\tau_s^2 I_s^{i*} + 1)}} \qquad \text{and} \qquad R_d^i \triangleq R_d^i(\tau_d^i) = \frac{\theta}{BE}$$

$$\text{with } A = \frac{1}{\tau_s - \tau_d^i},\ B = A\frac{R_m}{\tau_m},\ C = \frac{\tau_m \tau_s}{\tau_m - \tau_s},\ D = \frac{\tau_m \tau_d^i}{\tau_m - \tau_d^i} \text{ and}$$

$$E = min\left(-Ce^{-\frac{t}{\tau_s}} + De^{-\frac{t}{\tau_d^i}} + (C - D)e^{-\frac{t}{\tau_m}}\right),\ t > 0$$

For low synaptic input, this leads to values of τ_d^i approaching the time constant of the soma τ_m, and for high inputs τ_d^i approaches the time constant of the synapse τ_s which is usually much faster than τ_m. The effect is that a dendrite receiving strong post-synaptic input generates a sharp earlier increase of the membrane potential at the soma, whereas the potential generated from a lower input signal will be delayed. Furthermore, R_d^i is defined such that for a single spike at a synapse with strength c^{ij}, the value of the maximum of the soma membrane potential is proportional to the neuron's firing threshold, i.e. it equals $c^{ij}\theta$. Finally, the soma membrane potential u_m is:

$$\tau_m \frac{d}{dt} u_m(t) = -u_m + R_m(I_d(t) + \overline{I_s}(t))$$

where $I_d(t) = \sum_i I_d^i(t)$ is the total current from the dendritic tree, and $\overline{I_s}(t)$ is the total current from synapses attached to the soma.

The current from dendrite i generates part of the potential at the soma, which will be referred to as *partial membrane potential* and annotated as u_m^i. The *total partial membrane potential* $u_m^d = \sum_i u_m^i$ is the soma membrane potential generated from all dendrites.

The introduced active properties of the dendrites are the basis for the development presented in the next section, where the ability to control the *time* and *value* of the maxima of the membrane potentials plays a critical role in the learning algorithm.

3 Spike-Timing-Dependent Hebbian Learning

The spike-timing dependent Hebbian learning algorithm developed here adjusts the synaptic weight c^{ij} of synapse j at dendrite i, so that a post-synaptic spike occurs at the time when the partial membrane potential u_m^i is at its maximum. Immediately following a post-synaptic spike at time t' in a simulation with time step Δt, the synapse receives two weight correction signals, from the dendrite Δc_d^i and from the soma Δc_m:

$$\Delta c_d^i = \frac{2}{\pi} \arcsin\left(\frac{\Delta u_d^i(t')}{\sqrt{\Delta t^2 + \Delta {u_d^i}^2(t')}}\right), \ \Delta c_m = -\frac{2}{\pi} \arcsin\left(\frac{\Delta u_m^d(t')}{\sqrt{\Delta t^2 + \Delta {u_m^d}^2(t')}}\right)$$

where $\Delta u_d^i(t')$ and $\Delta u_m^d(t')$ are the changes in the partial mebrane potential and total partial membrane potential just before the post-synaptic spike. The correction signal Δc_d^i sent from the dendrite follows the rule: if a post-synaptic spike occurs in the rising phase of the partial membrane potential u_m^i, i.e. before it reaches its maximum, the synaptic strength will be increased so that next time the maximum will occur earlier. Respectively, the synaptic strength will be decreased if a post-synaptic spike occurs after the maximum (Figure 2 (A)). The role of the correction signal Δc_m sent from the soma is to prevent the weights of the synapses from reaching high values simultaneously, or to prevent a total decay in the synaptic strength. Its rule is opposite to the one for the dendrite. Based on the two signals, the total correction signal for the synapse is:

$$\Delta c^{ij} = \begin{cases} \frac{\Delta c_d^i + \Delta c_m}{2} & \text{if } |\Delta c_m| > \varepsilon, \\ \Delta c_d^i & \text{if } |\Delta c_m| \le \varepsilon. \end{cases}$$

where the constant ε allows the neuron to fire without generating a correction signal from the soma when the potential is sufficiently close to the maximum.

Finally, following a post-synaptic spike, the synaptic weights are updated with learning rate η according to:

$$c_{new}^{ij} = \begin{cases} c_{old}^{ij} + \eta \Delta c^{ij}(1 - c_{old}^{ij}) & \text{if } \Delta c^{ij} > 0, \\ c_{old}^{ij} + \eta \Delta c^{ij} c_{old}^{ij} & \text{if } \Delta c^{ij} < 0. \end{cases}$$

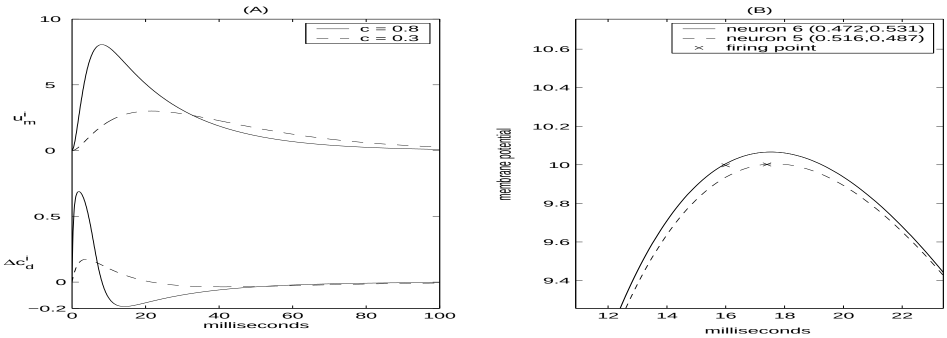

Fig. 2. (A) The correction signal Δc_d^i that would be sent from the dendrite to a synapse with weight 0.8 or 0.3 in the the event of a post-synaptic spike. $\Delta c_d^i = 0$, i.e. no change in the weight occurs, if the post-synaptic spike is at the point of maximum of u_m^i; (B) Soma membrane potential of two neurons with different weights, receiving two input spikes at different dendrites. The second spike is delayed 2 ms. If the maxima of partial membrane potentials coincide or are close in time, the neuron will reach the firing threshold earlier (neuron 6). If the maxima are not close in time, the neuron will reach the threshold later (neuron 5) or not reach it at all.

There have been several suggestions for spike-timing dependent and Hebbian learning algorithms [5–8]. The learning algorithm presented in this paper achieves very precise tuning of the synapses in response to input spikes representing information with different temporal encoding schemes. The algorithm leads to a normal distribution and intrinsic normalization of the synaptic weights, which allows competitive behaviour of the neurons with dynamic synapses in a network (Figure 2 (B)).

4 Experiments

4.1 Learning to respond to different temporal codes

The next three examples demonstrate the responses of neurons trained on temporal encoding with single spikes, spike bursts and phase coding. The neuron model and the learning algorithm are able to detect the temporal properties of the input independently on the encoding scheme being applied. The neuron in the first example receives single spikes at three synapses each belonging to a different dendrite (Figure 3 (A)). In the second example, the neuron receives two decaying spike bursts with fixed onset times (Figure 3 (B)). The third example presents a neuron responding to the phase of an input spike with respect to a global oscillation (Figure 3 (C)). The trained neurons fire near the maximum of the partial membrane potentials.

4.2 Competitive Learning

This section demonstrates an application of the neuron with active dendrites and its learning algorithm in a network of self-organizing competitive neurons. The network consist of 2 input and 10 competitive neurons. The input is encoded in the relative spike timing for the two input neurons. Each competitive neuron receives feedforward signals

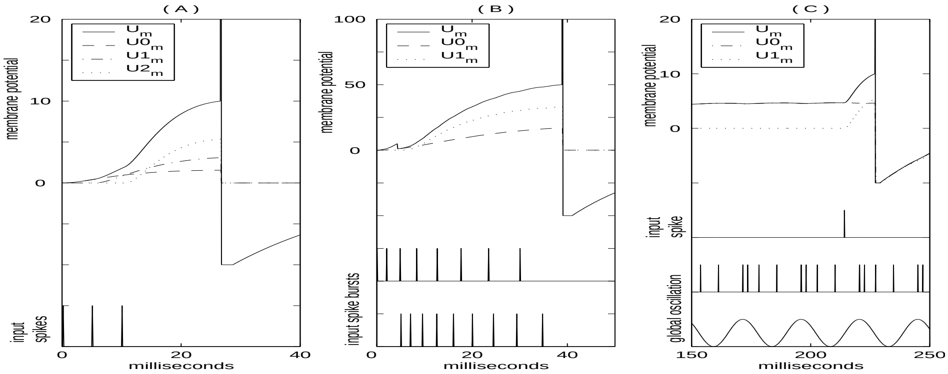

Fig. 3. (A) Response of a trained neuron receiving three input spikes at 0, 5 and 10 ms. u_m is the membrane potential at the soma, u_m^0 is the partial membrane potential generated from the dendrite receiving the first spike, u_m^1 and u_m^2 are for the dendrites receiving the second and the third spikes respectively. (B) Response of a trained neuron receiving two input spike bursts with onset times at 0 and 5 ms. u_m^0 is the partial potential generated from the dendrite receiving the first spike burst, and u_m^1 is for the dendrite receiving the second spike burst. (C) Response of a trained neuron receiving a single spike 8 ms before a peak of an oscillation with a period of 24 ms. u_m^0 is the partial membrane potential generated from the dendrite receiving as input the oscillation spike train and u_m^1 is the partial membrane potential for the dendrite receiving the single spike.

from the input neurons via excitatory synapses at different active dendrites. Furthermore, each competitive neuron receives lateral connections from all other competitive neurons via synapses attached to the soma. These synapses have a fast and strong direct influence on the soma membrane potential and are very efficient for lateral connections.

Figure 4 (A) shows the beginning of the formation of a self-organizing map after 50 epochs. After full training, a well formed self-organizing map is observed (Figure 4 (B)). Each competitive neuron responds only to a particular interval of input values. Since the competitive neurons are relatively fine-tuned to respond only to a particular interval of input values, the feedforward connections are sensitive to noise in the weights. Such noise will destroy the map. On the other hand, due to the fine tuning, the competitive neurons are very robust to noise in the lateral connections. The network was tested with the lateral inhibition removed, and showed relatively little overlap of the responses of the different neurons in the map (Figure 4 (C)). The responses of the trained neurons exhibit clear selectivity to the input. A zoomed-in example of the response of neurons 5 and 6 without lateral connections is shown in Figure 2 (B).

5 Conclusions

The developed new model of a neuron with active dendrites and spike-timing dependent Hebbian learning algorithm are viewed as a contribution towards novel efficient computing models of networks with artificial spiking neurons. The introduction of the active dendrites plays a critical role in achieving a learning algorithm which goes be-

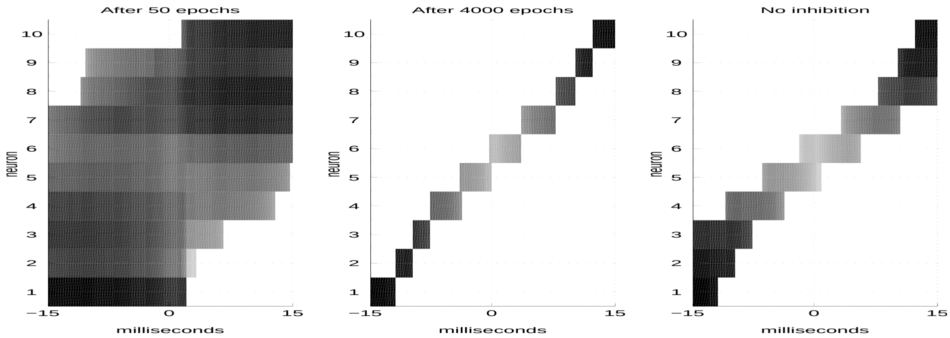

Fig. 4. Self-organization of competitive neurons with active dendrites. The three graphs show the response of the network to two input spikes with relative delays from the interval $[-15, 15]$ ms. The darker color indicates faster response of the competitive neuron to the particular input. Lighter colors indicate later post-synaptic spikes. White areas indicate no post-synaptic response. Left (A): Early self-organized formation after 50 training epochs; Middle (B): A well formed self-organizing map where each competitive neuron responds only to a particular interval of input values; Right (C): Response of the trained network with all lateral connections removed.

yond the relative timing of the pre- and post-synaptic spikes to incorporate functions of the membrane potential at the dendrite and at the soma, and the synaptic strength. The algorithm trains the neurons independently on the temporal code being used at the input and achieves precise selective responses. The presented experiments show details of the functionalities of the neuron, the learning algorithm and their application in training a network of competitive neurons. Further work will build upon these encouraging results and concentrate on applying the model of a neuron with active dendrites and the spike-timing dependent learning algorithm in the development of more complex neural structures such as cell assemblies and synfire chains.

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