Object Nets for Mobility

Michael Köhler and Berndt Farwer

Department for Informatics
University of Hamburg

Application and Theory of Petri Nets

Object Nets

- Nets within Nets
- Nets as tokens
- Recursion for Petri nets

- Object-orientation
- Mobile systems
- Multi-agents systems

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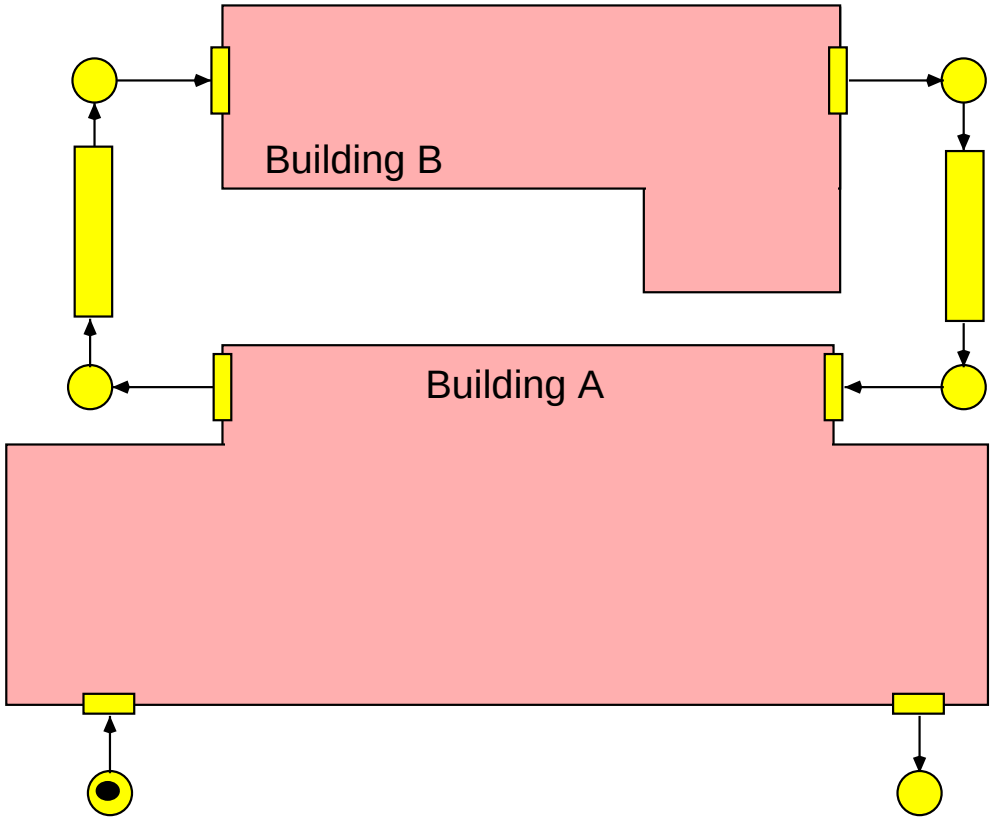
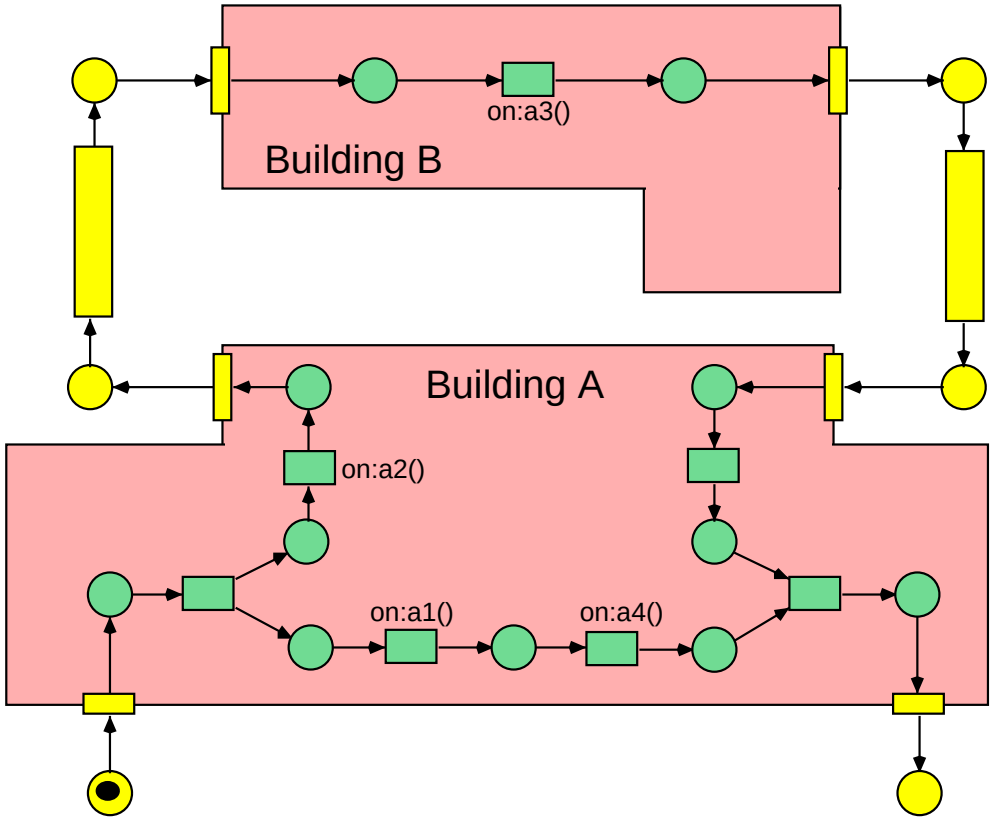
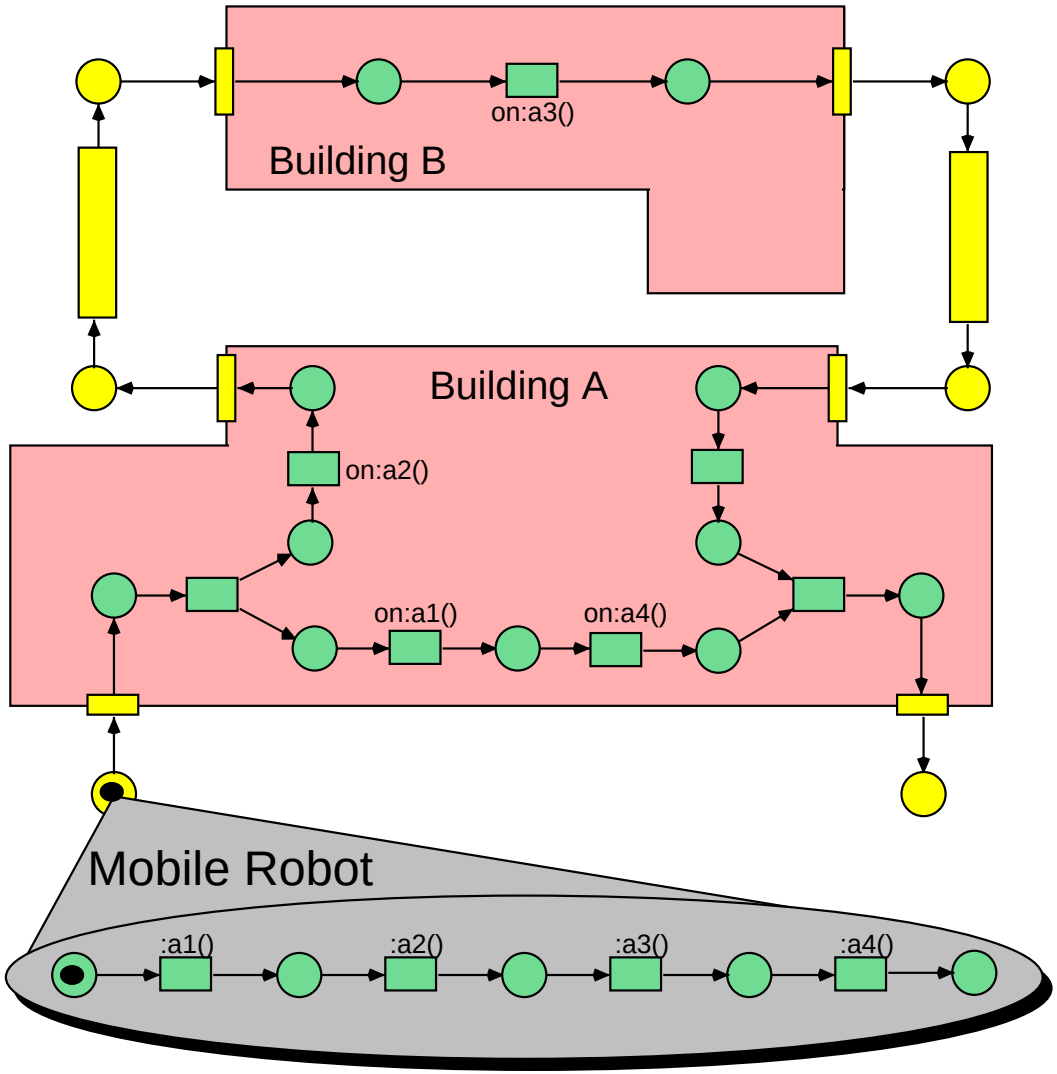
A Scenario



A Scenario



A Scenario

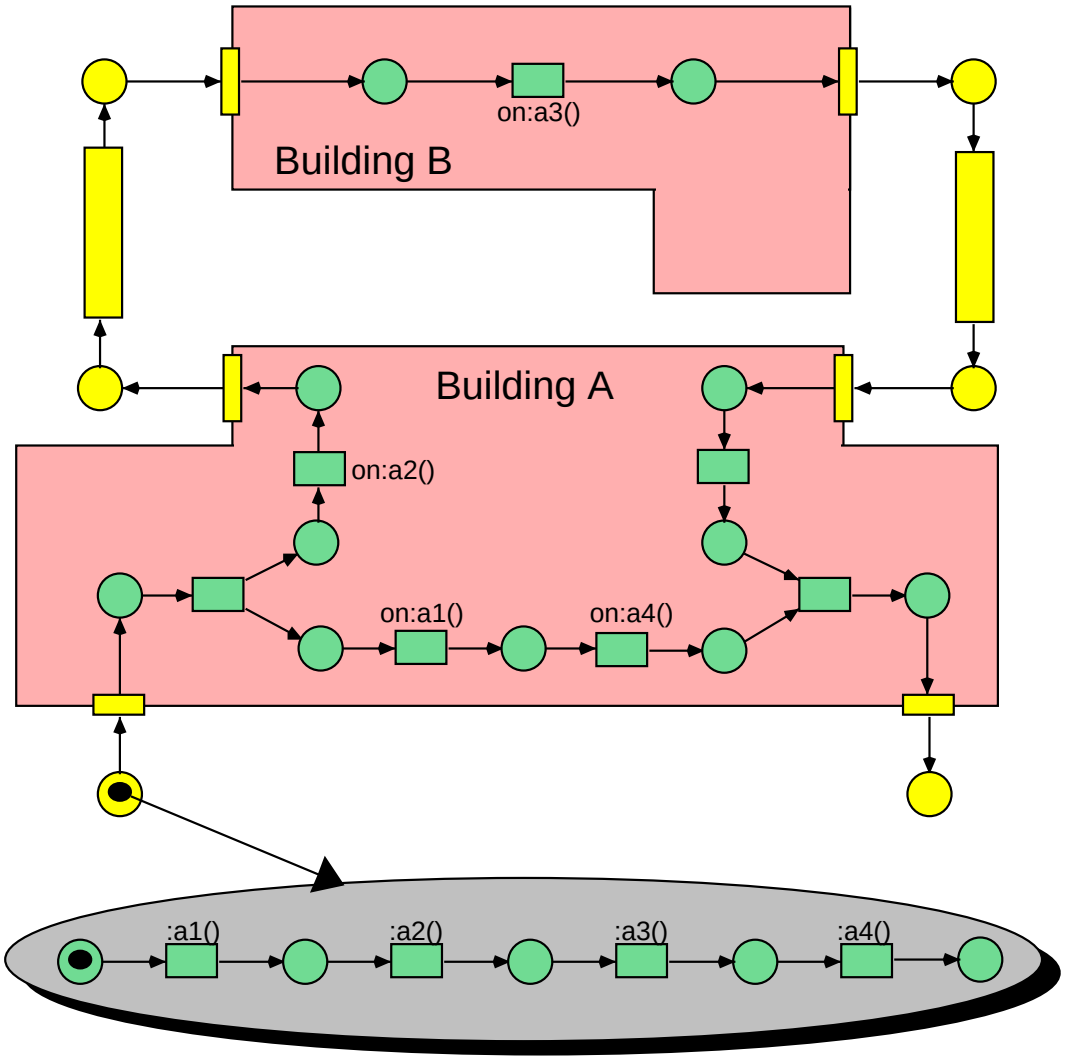

on:a3()
Building B
Building A
on:a2()
on:a1()
on:a4()
Mobile Robot
:a1()
:a2()
:a3()
:a4()

Net Tokens

How to define Net Tokens?

1. Net Tokens as **References**:
   - Assumes a global name space ($\rightarrow$ side effects)
   - Easy to implement
   - Closely related to programming languages
2. Net Tokens as **Values**:
   - Assumes a distributed space
   - Easy to understand
   - Adequate for mobility

Reference Semantics



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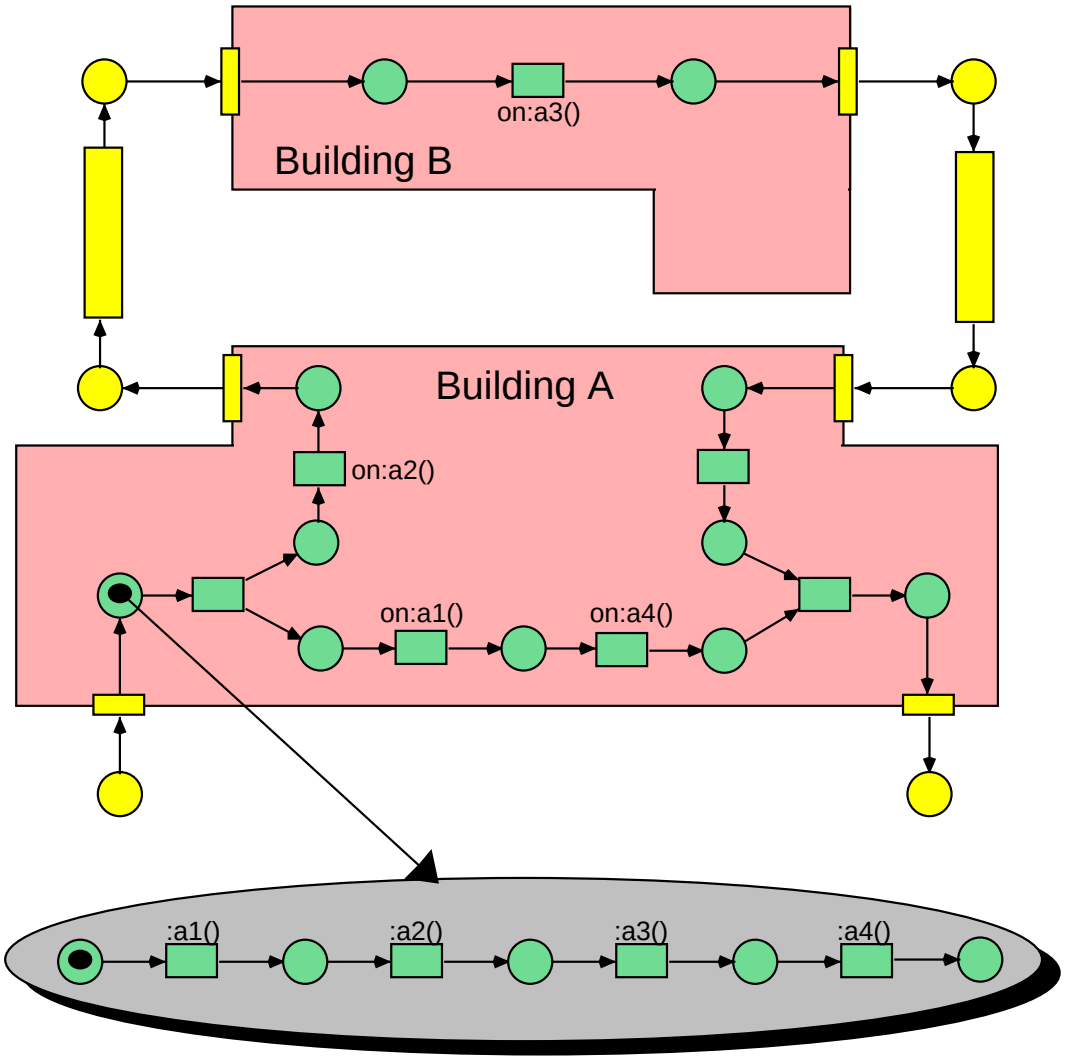

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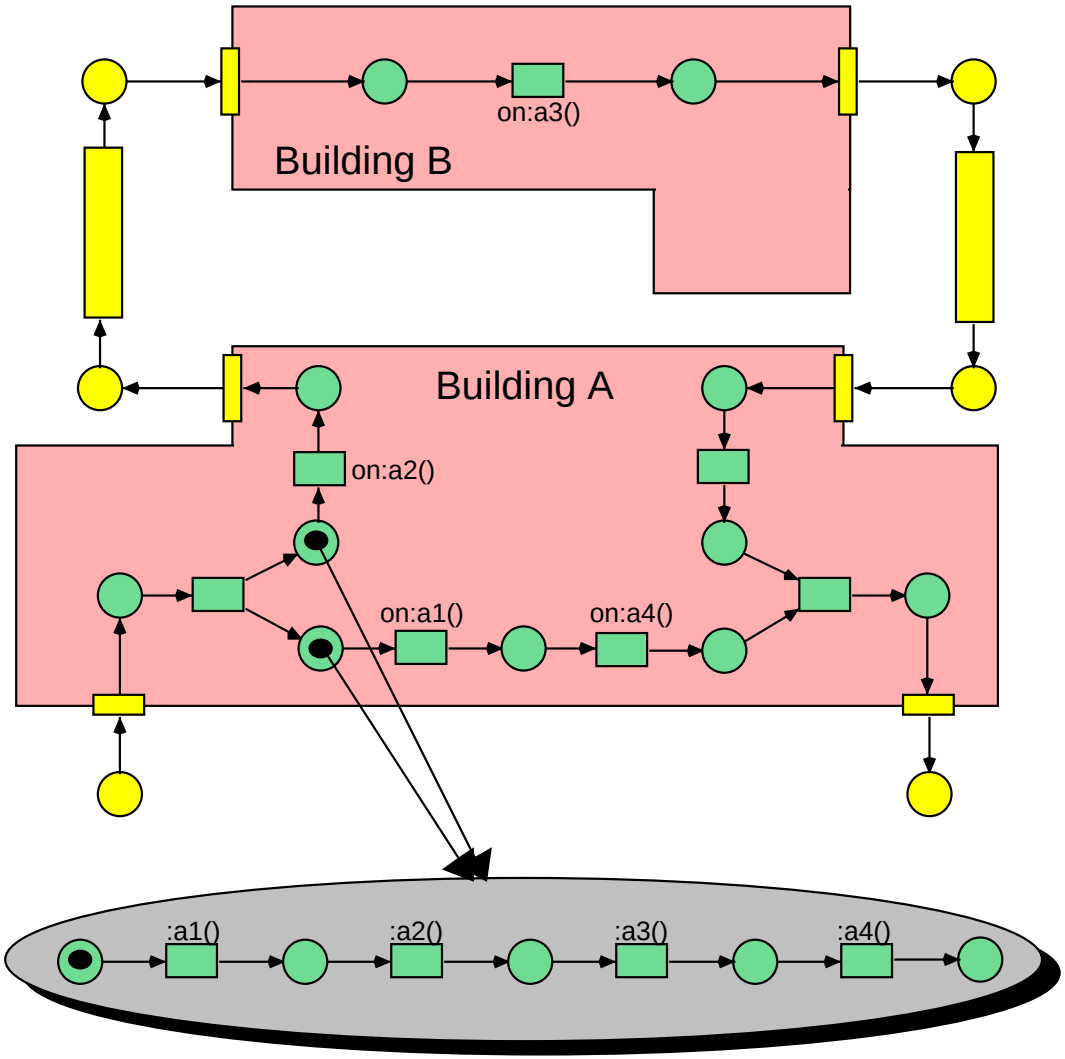

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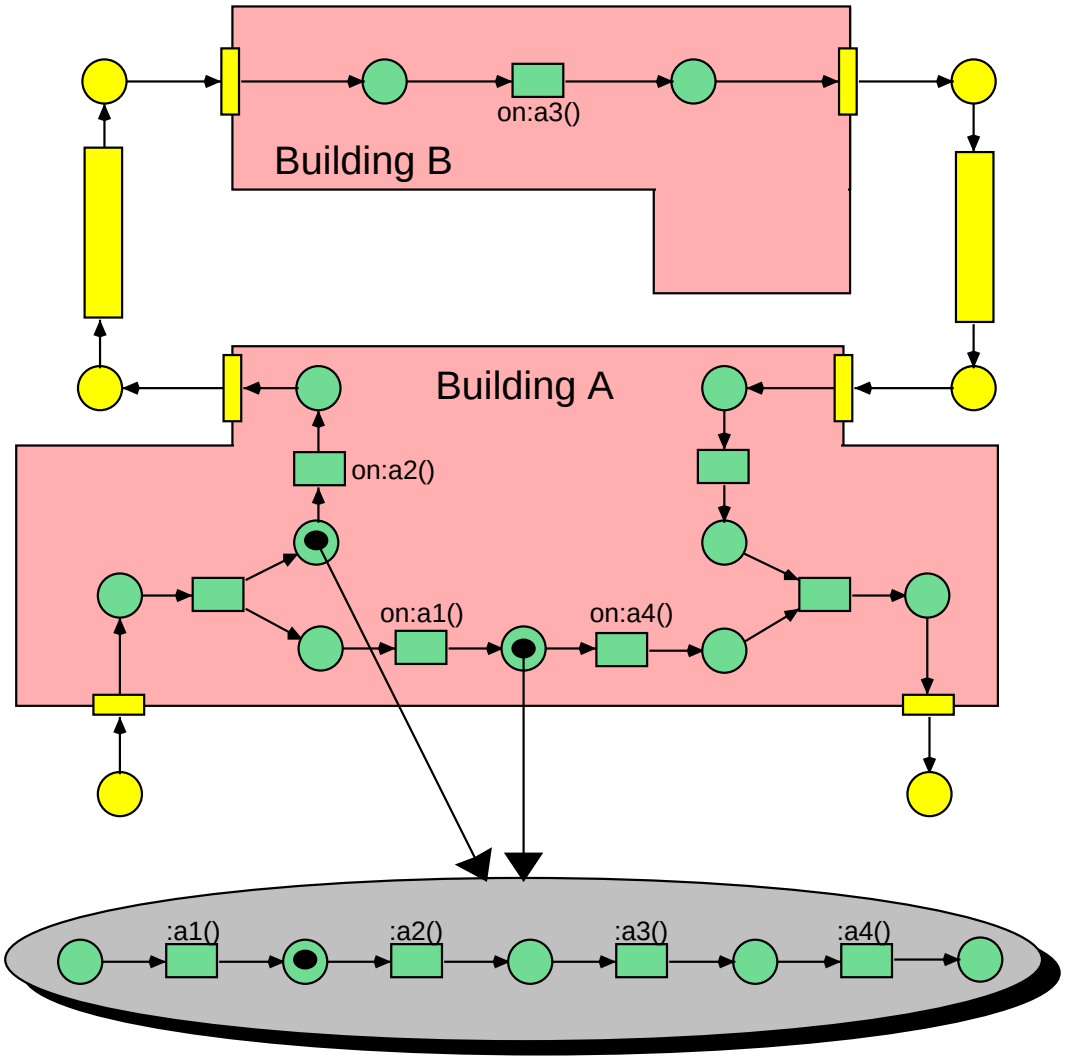

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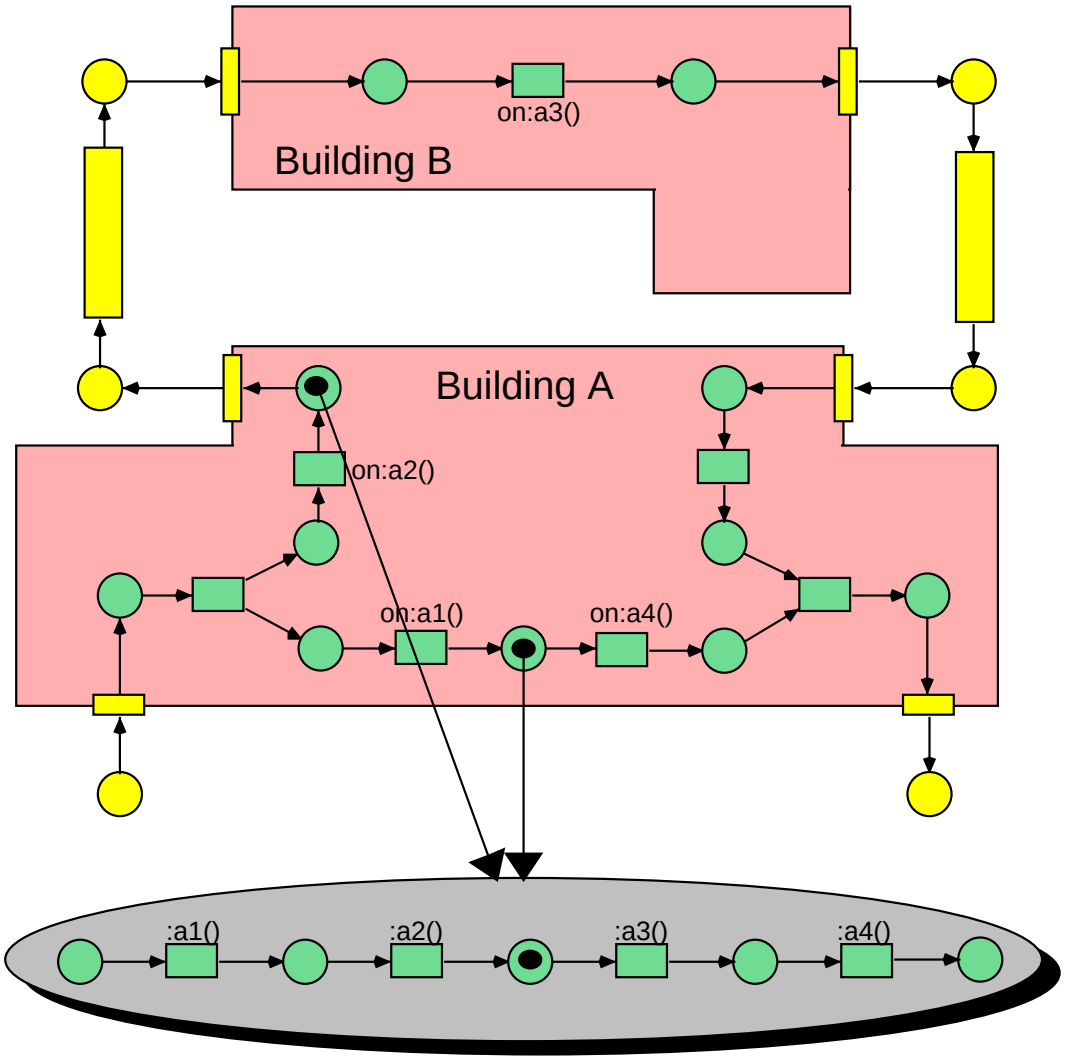

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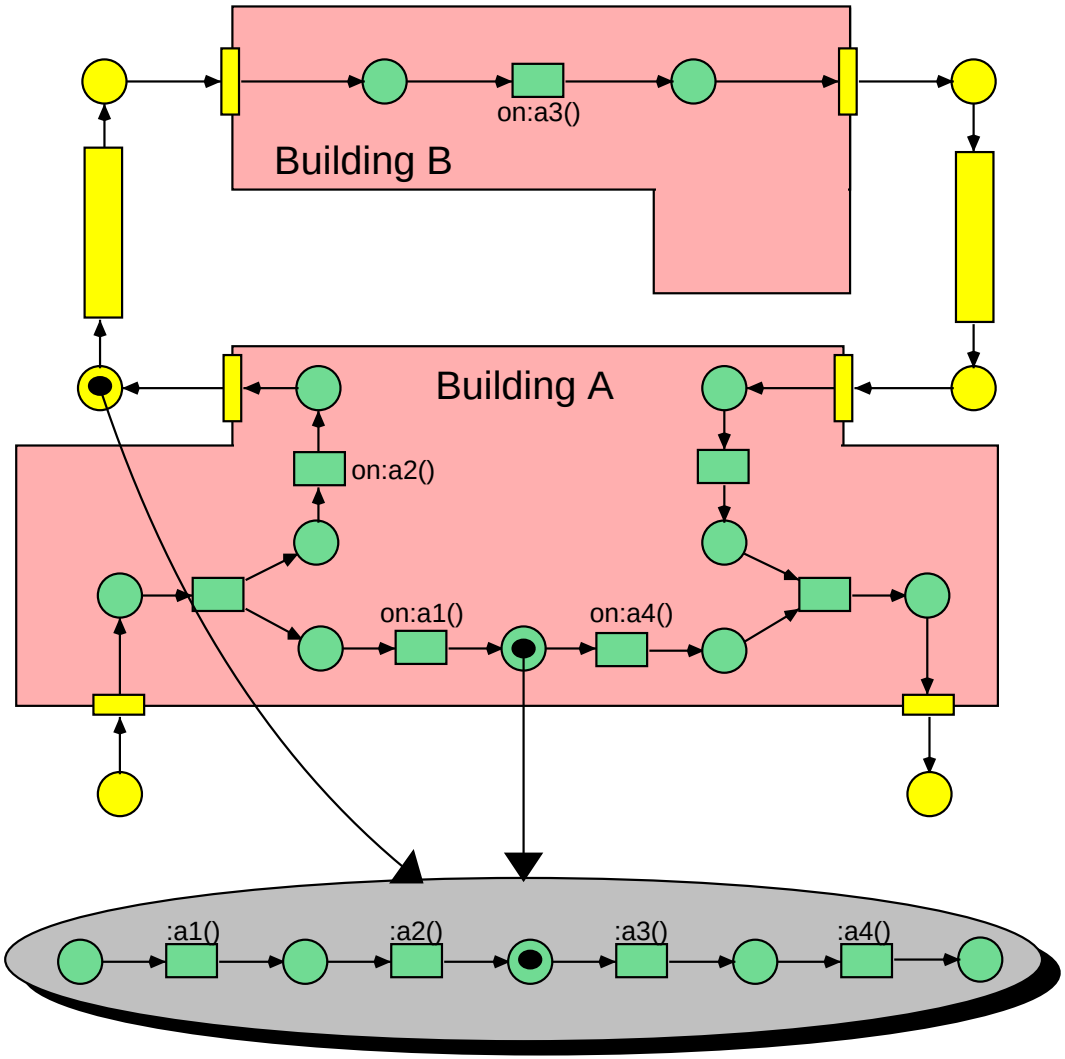
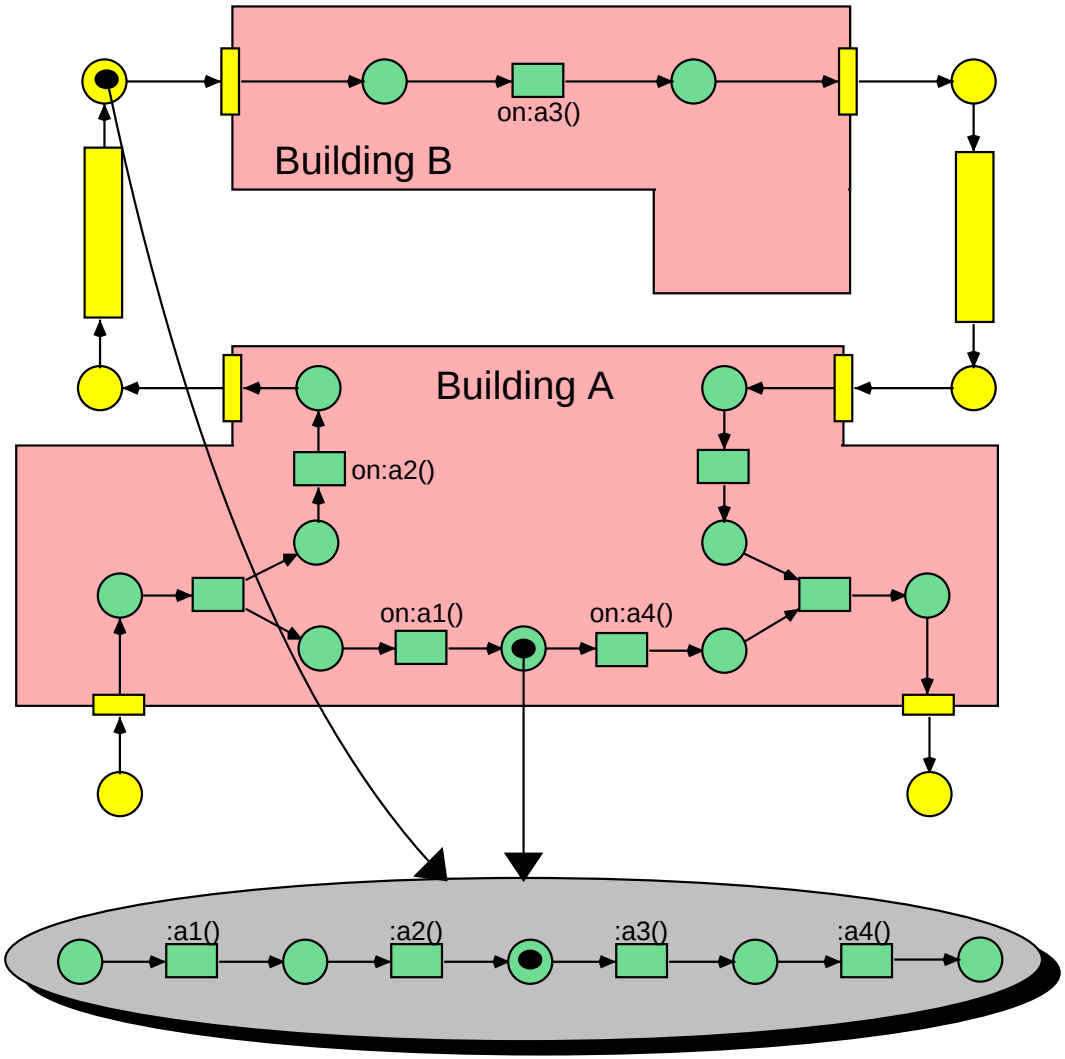


Reference Semantics



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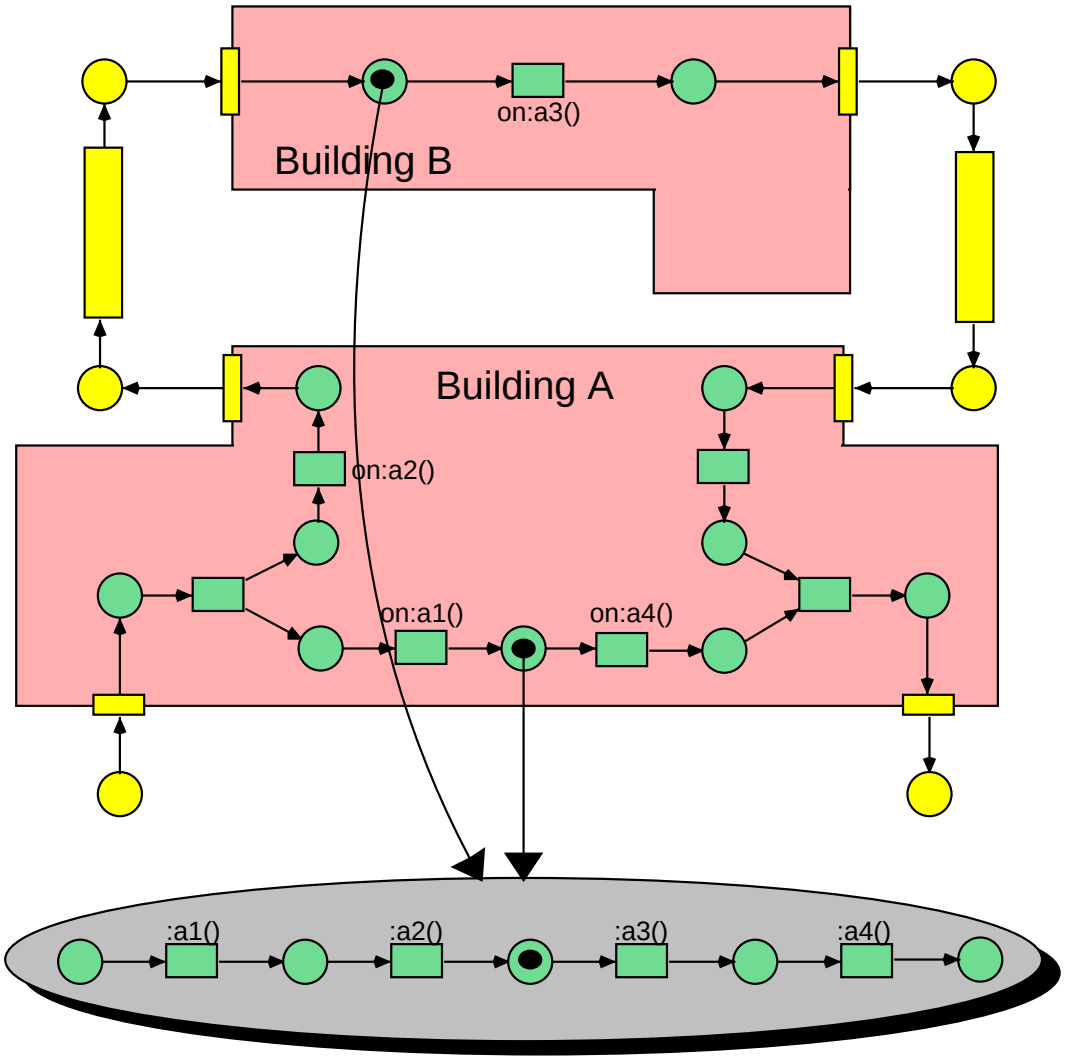

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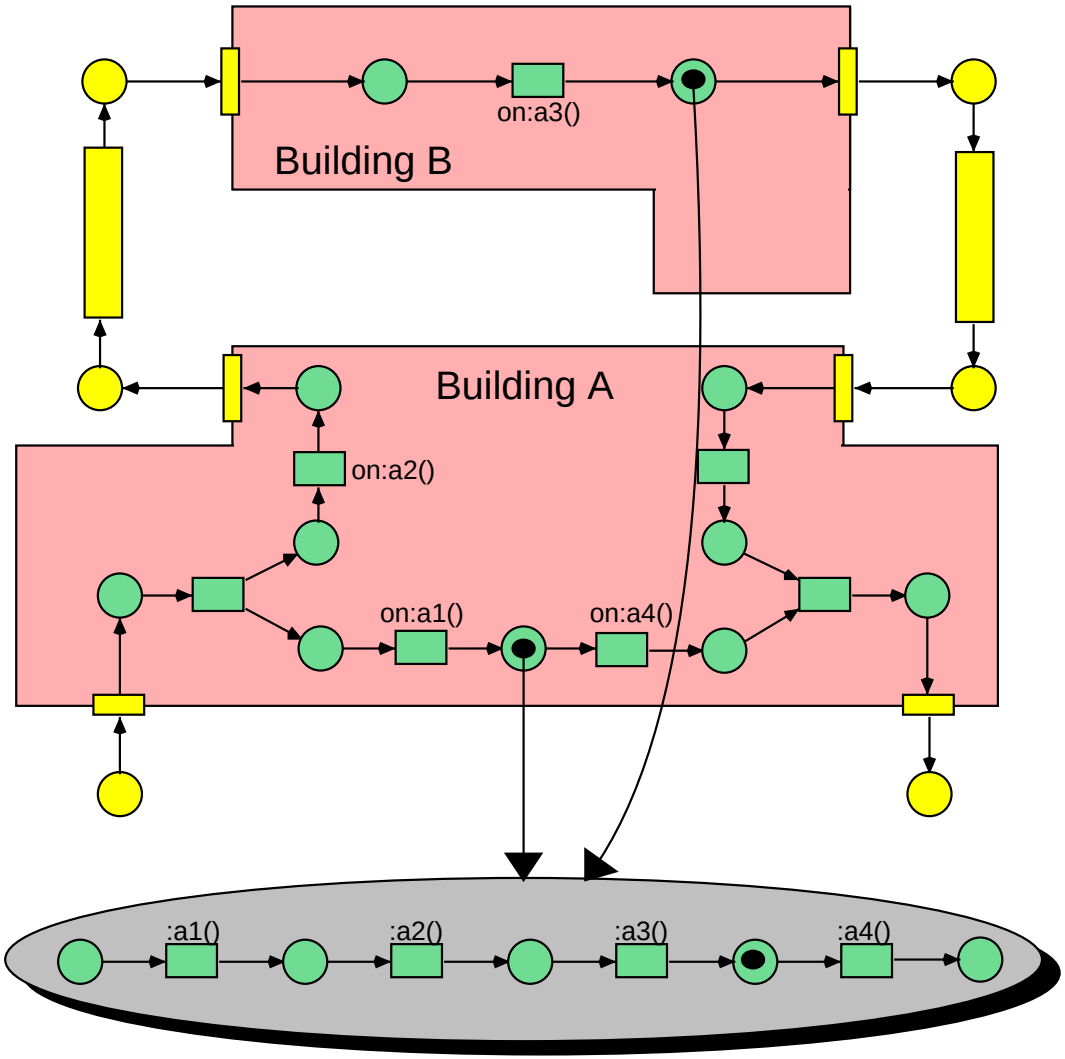

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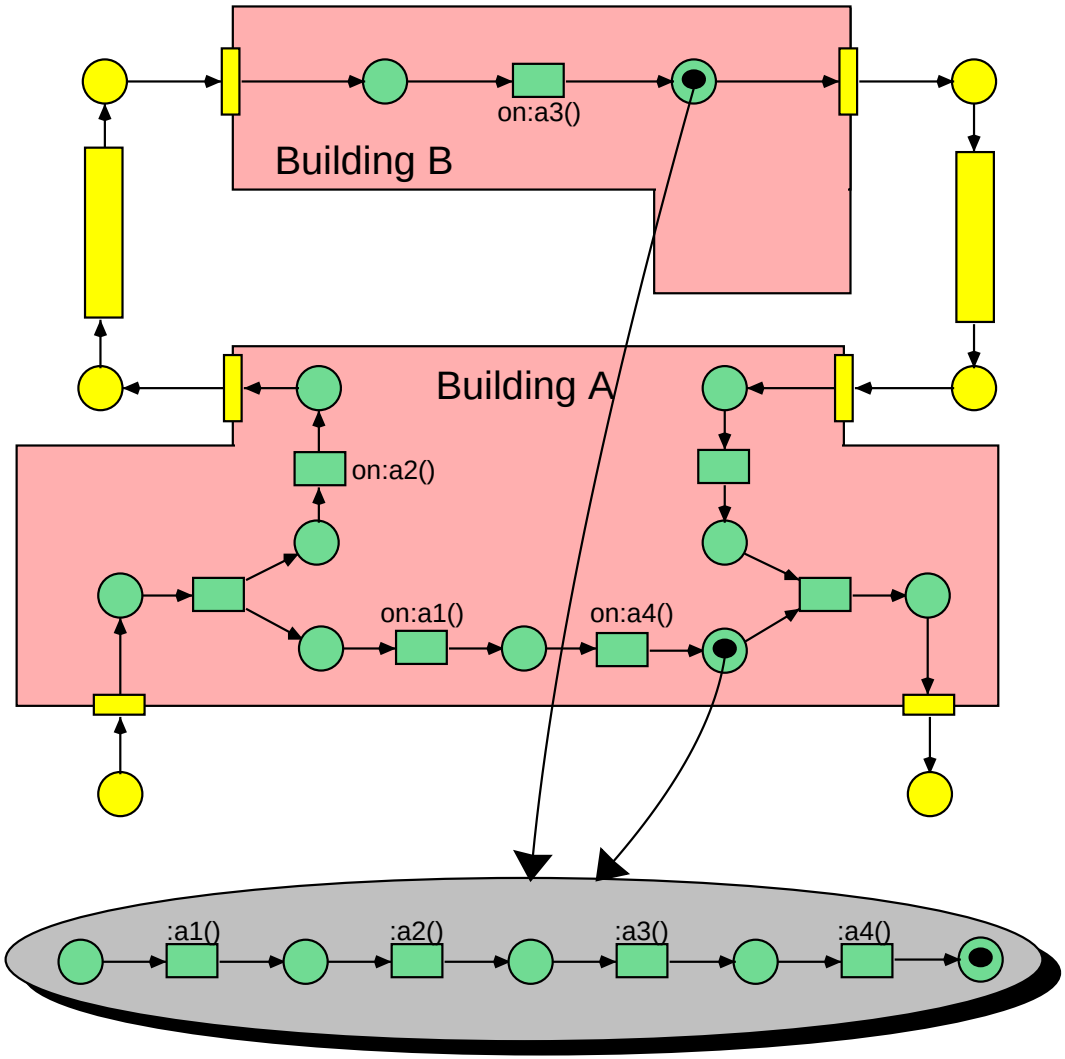

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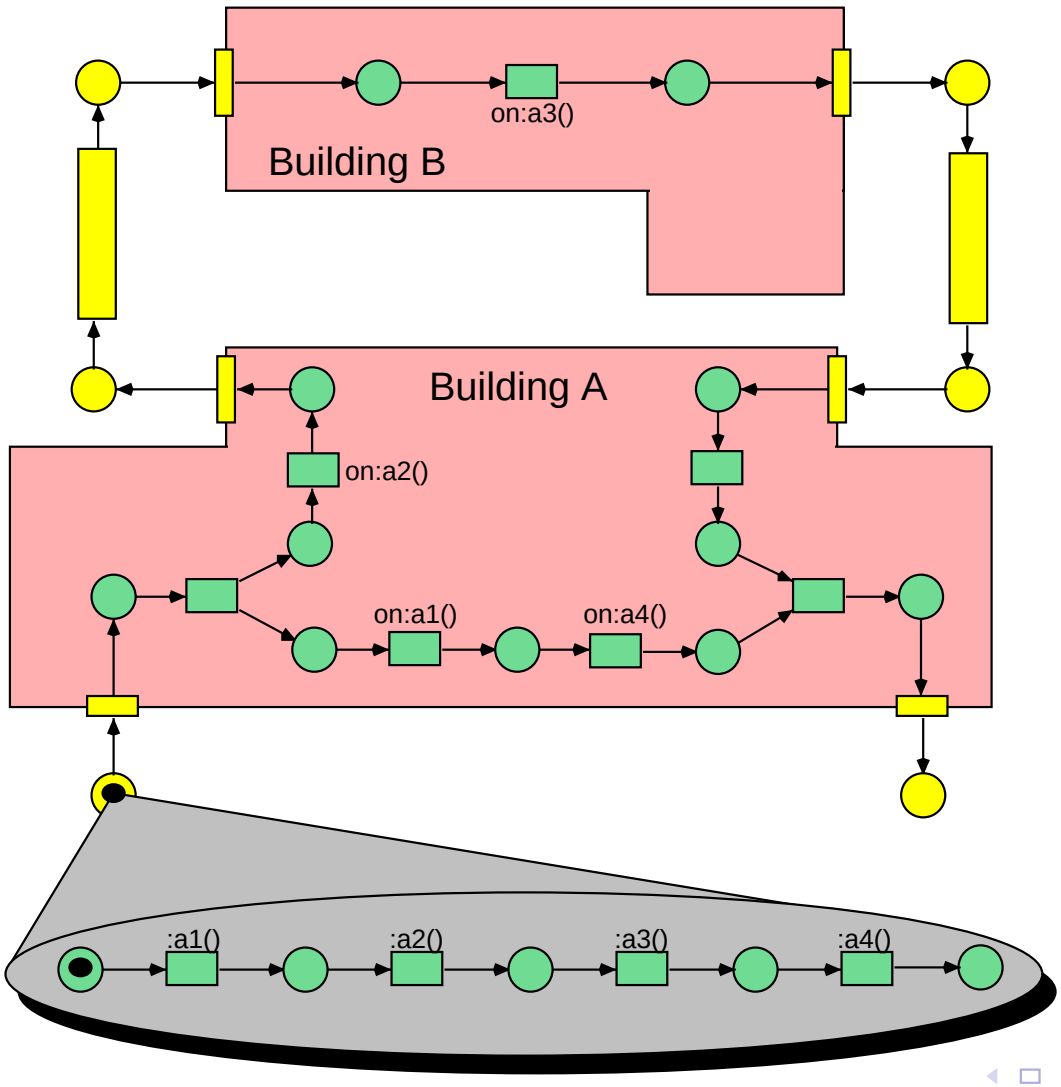
Value Semantics



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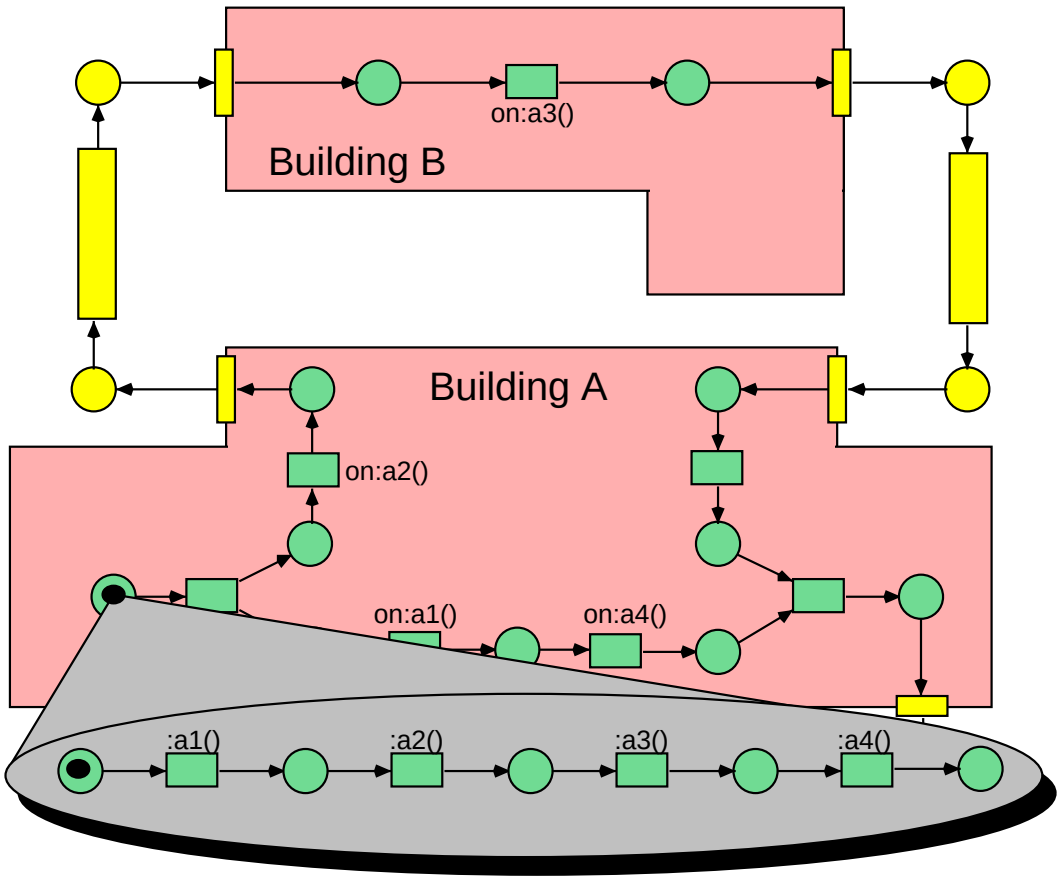
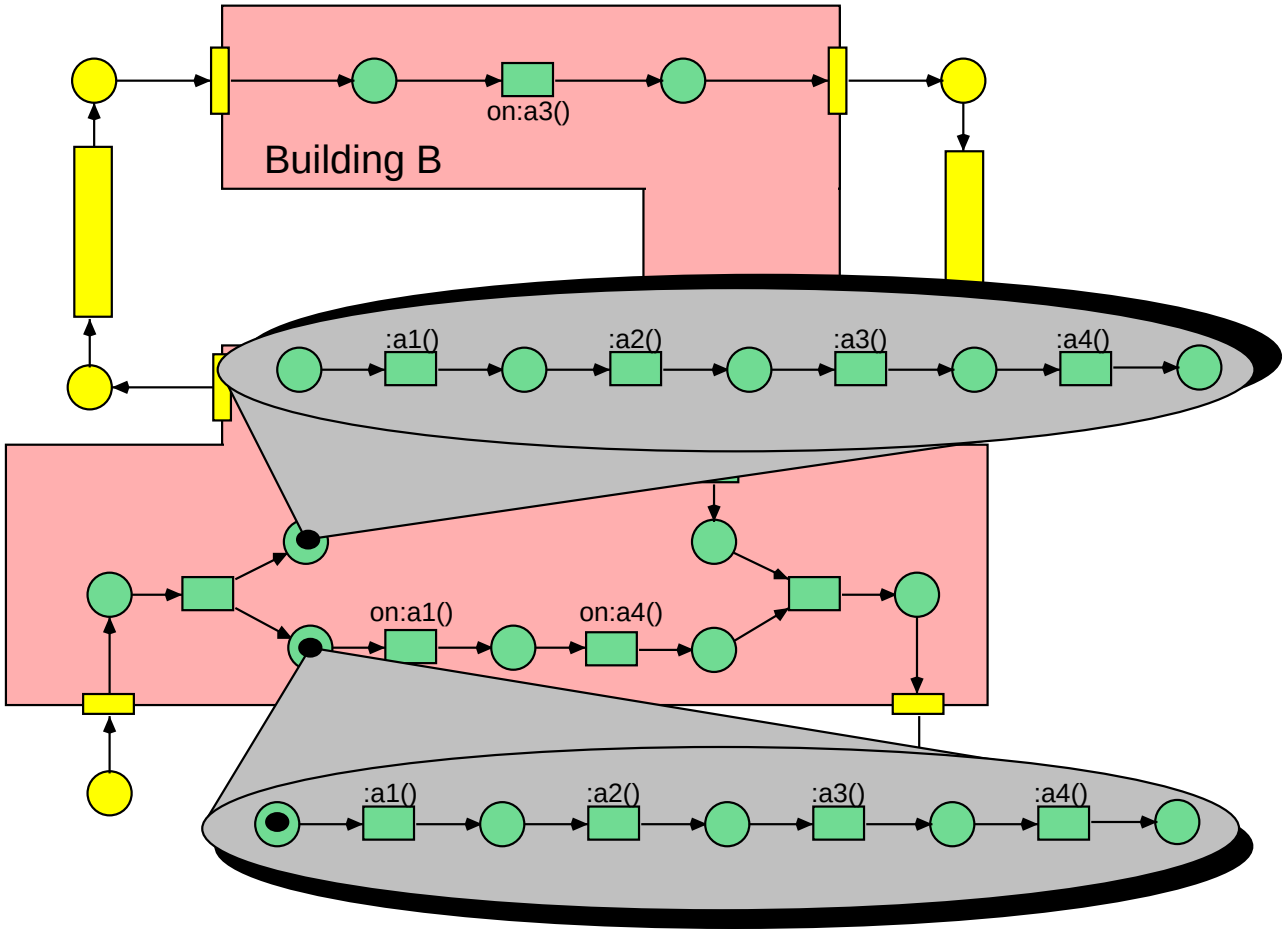


Value Semantics



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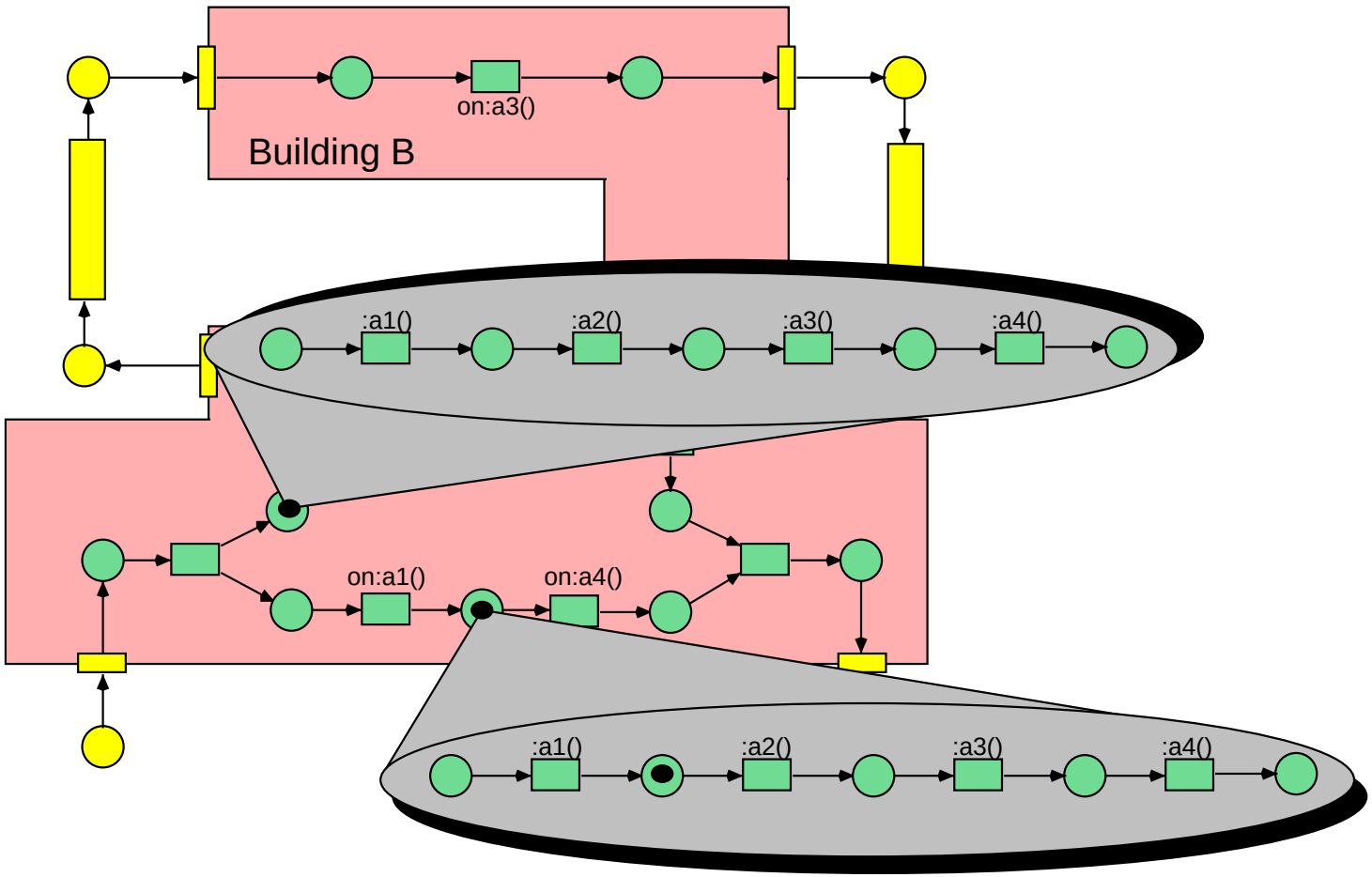


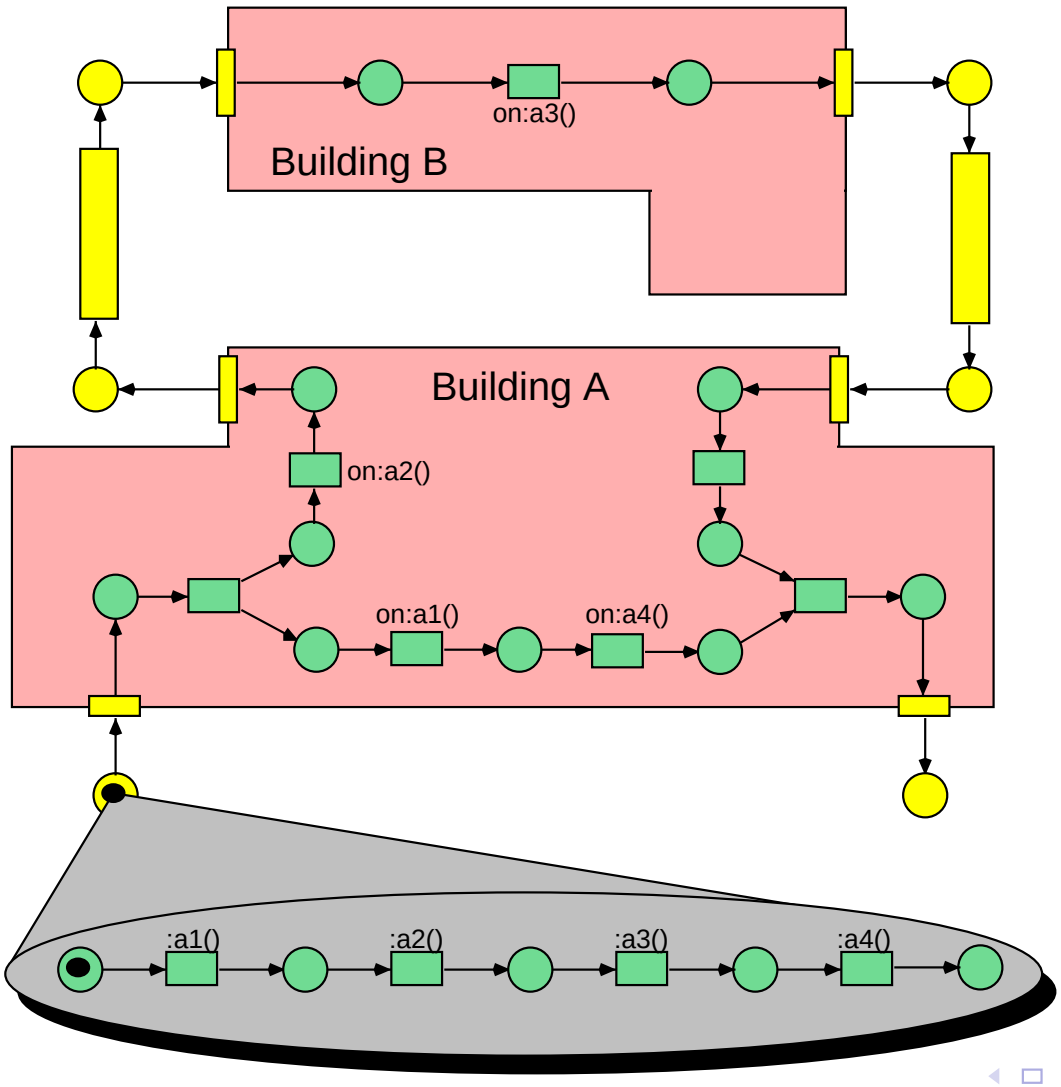
References/Values

- Reference semantics creates one single namespace.
- Reference semantics allows side-effects between different namespaces.
- Value semantics restricts access to one single place.
- Value semantics inhibits global access within the same namespace.

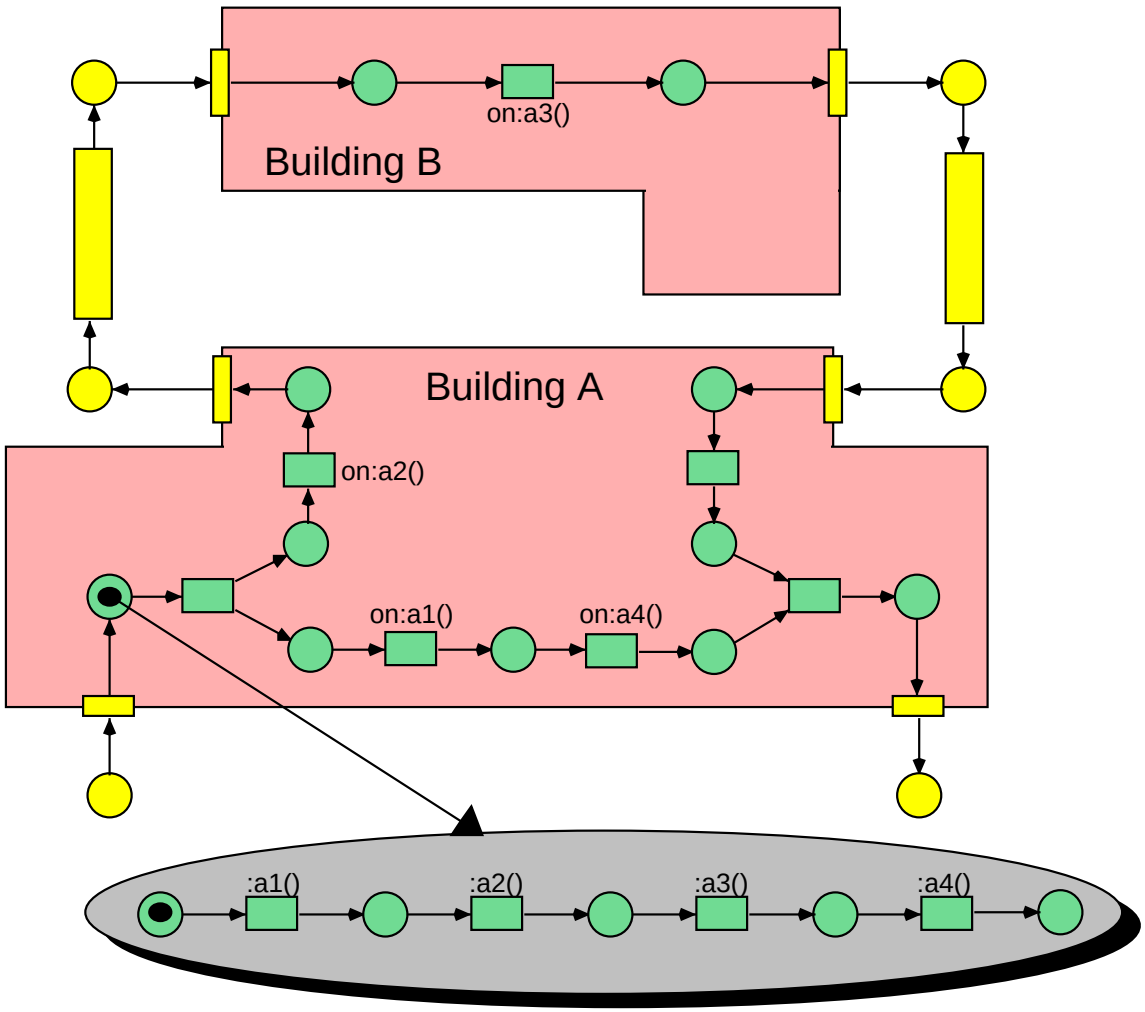
$\Longrightarrow$ A generalised semantics is needed: mobility semantics

- Mobility semantics "switches" between value and reference semantics.

Generalised Semantics



Generalised Semantics



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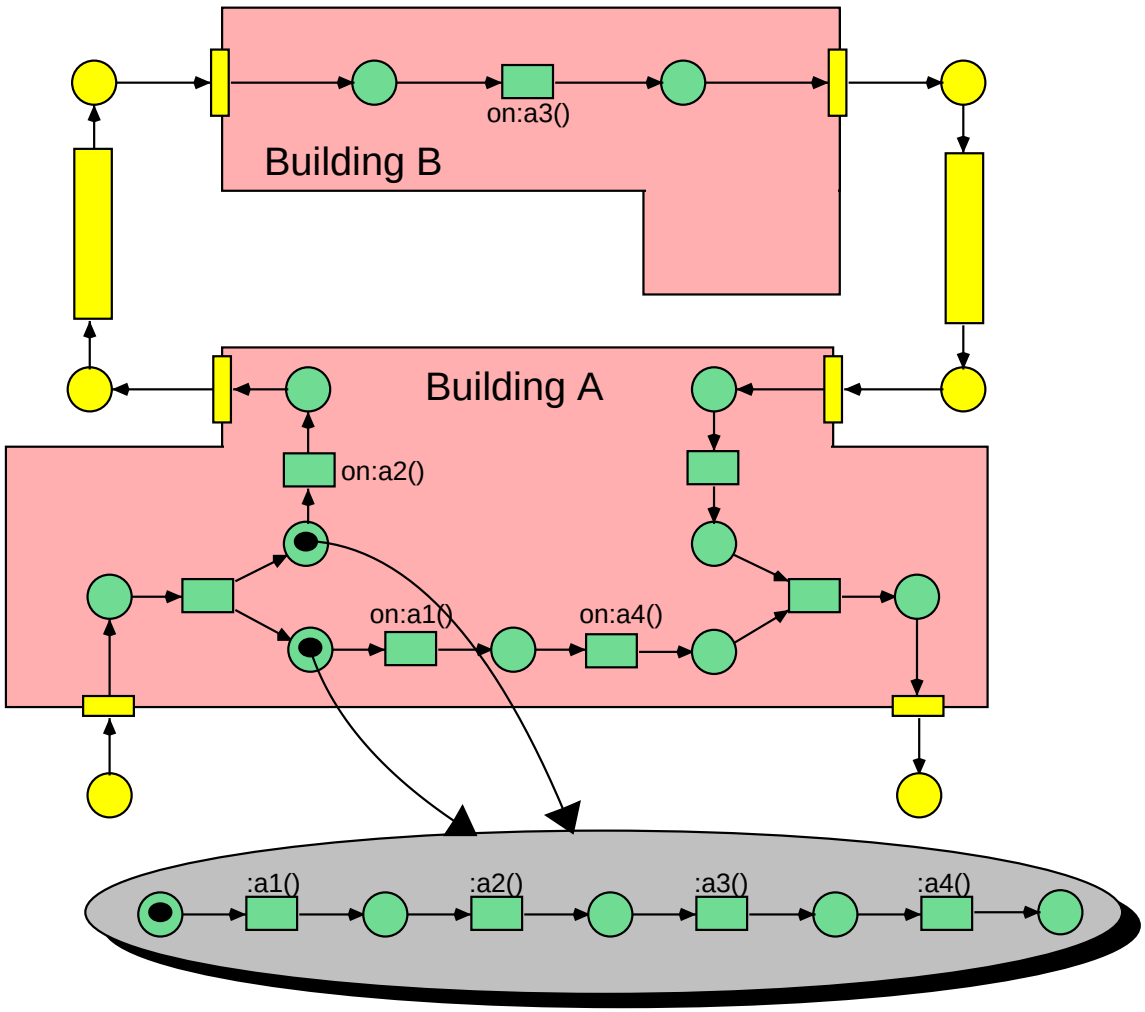
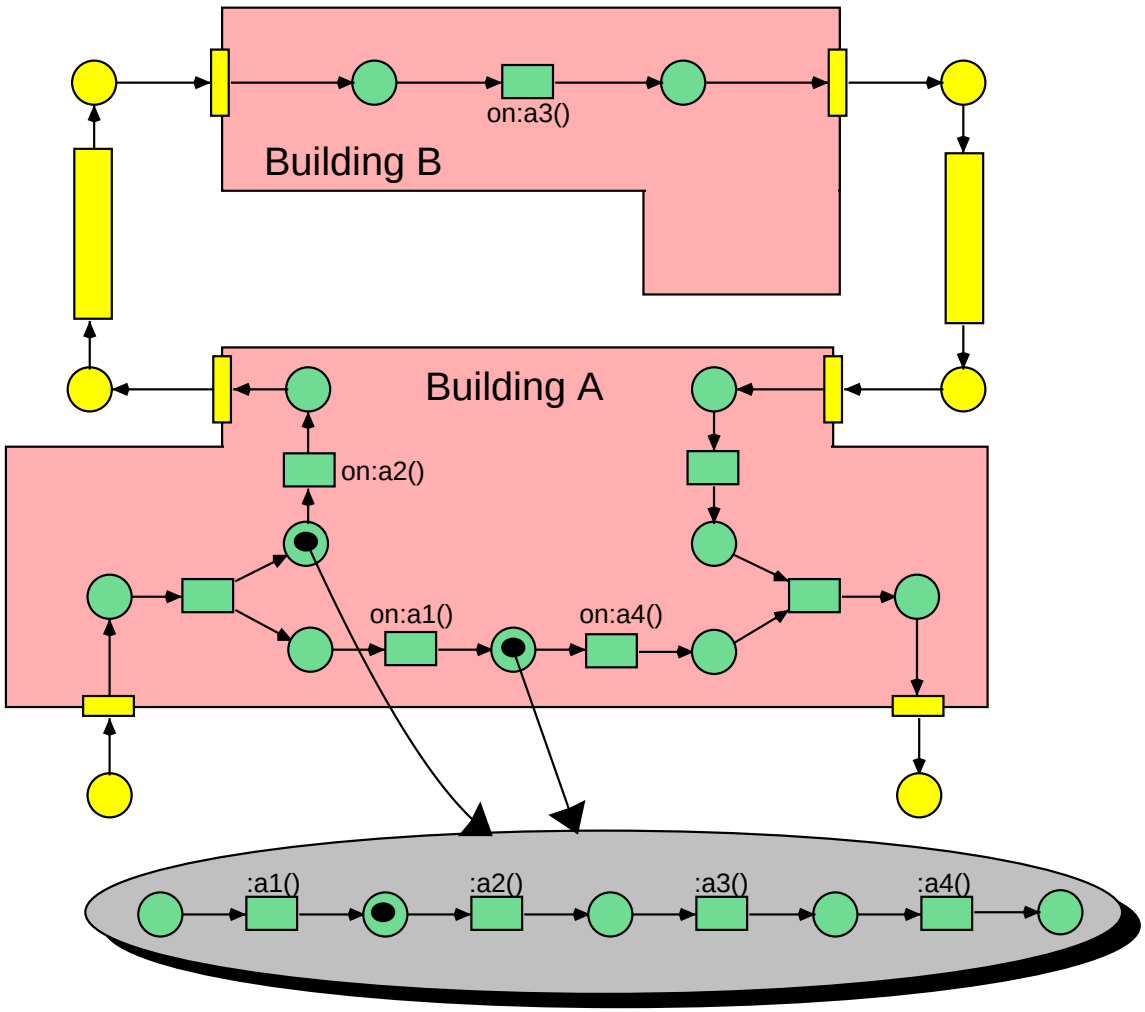
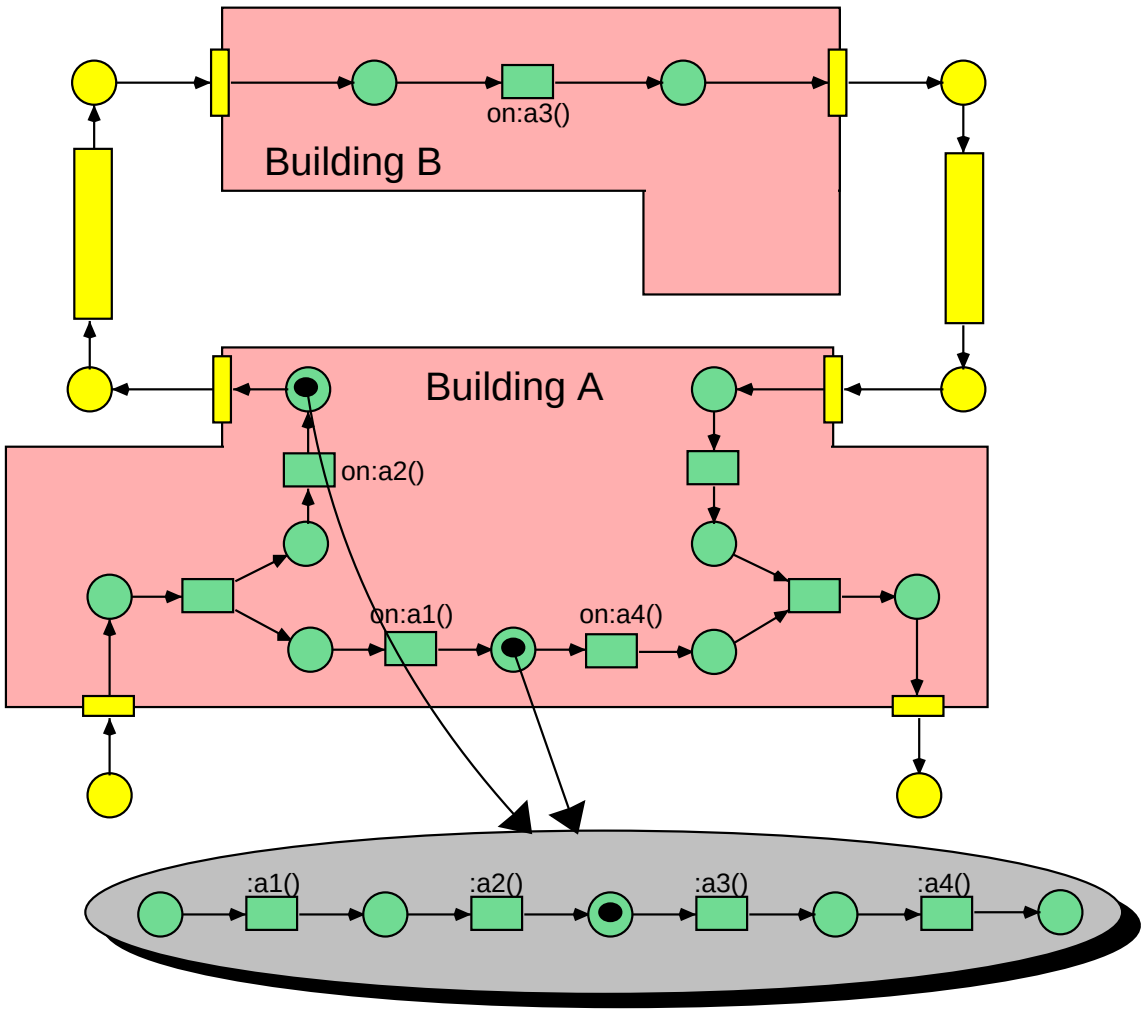


Generalised Semantics



Generalised Semantics



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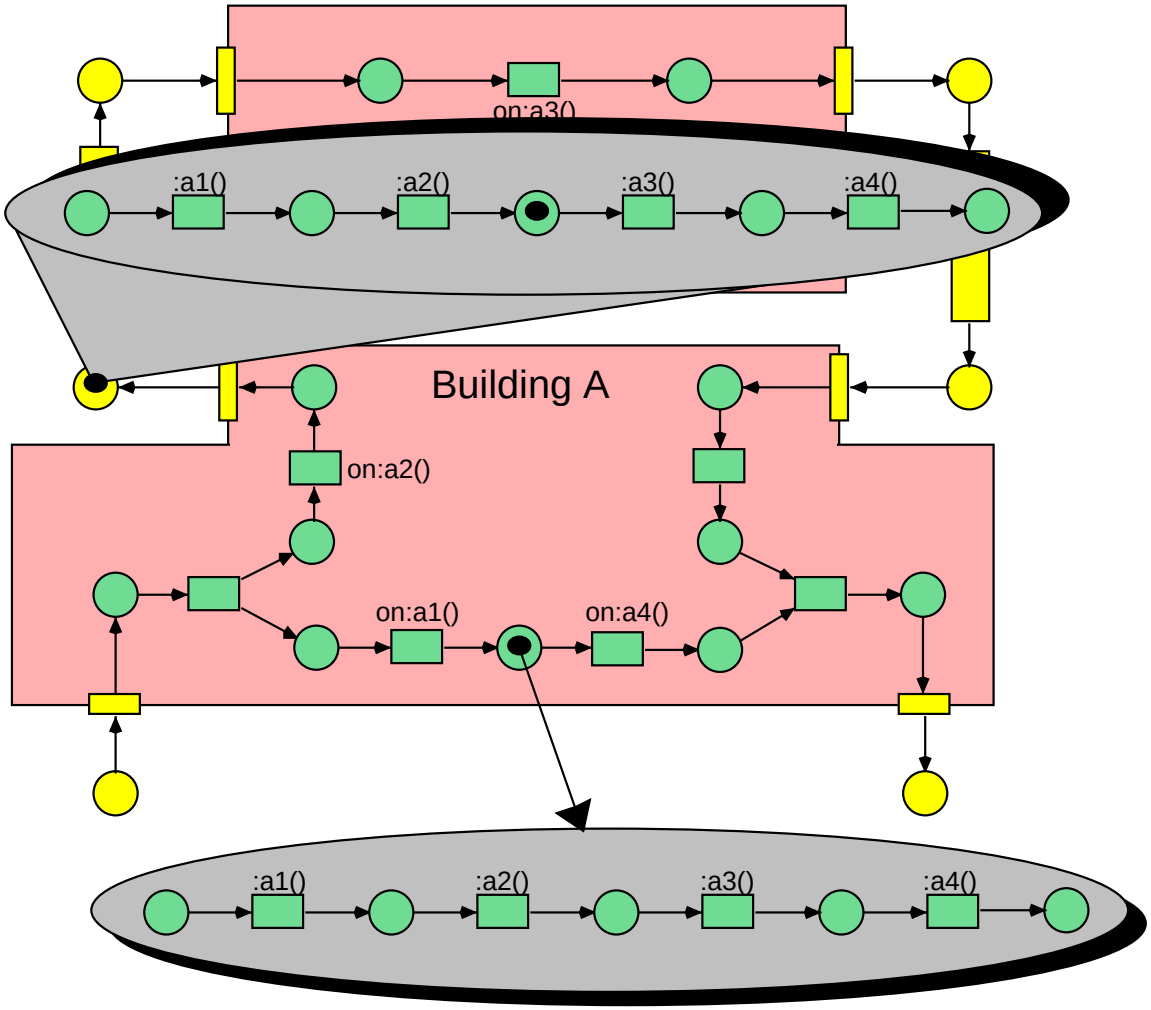

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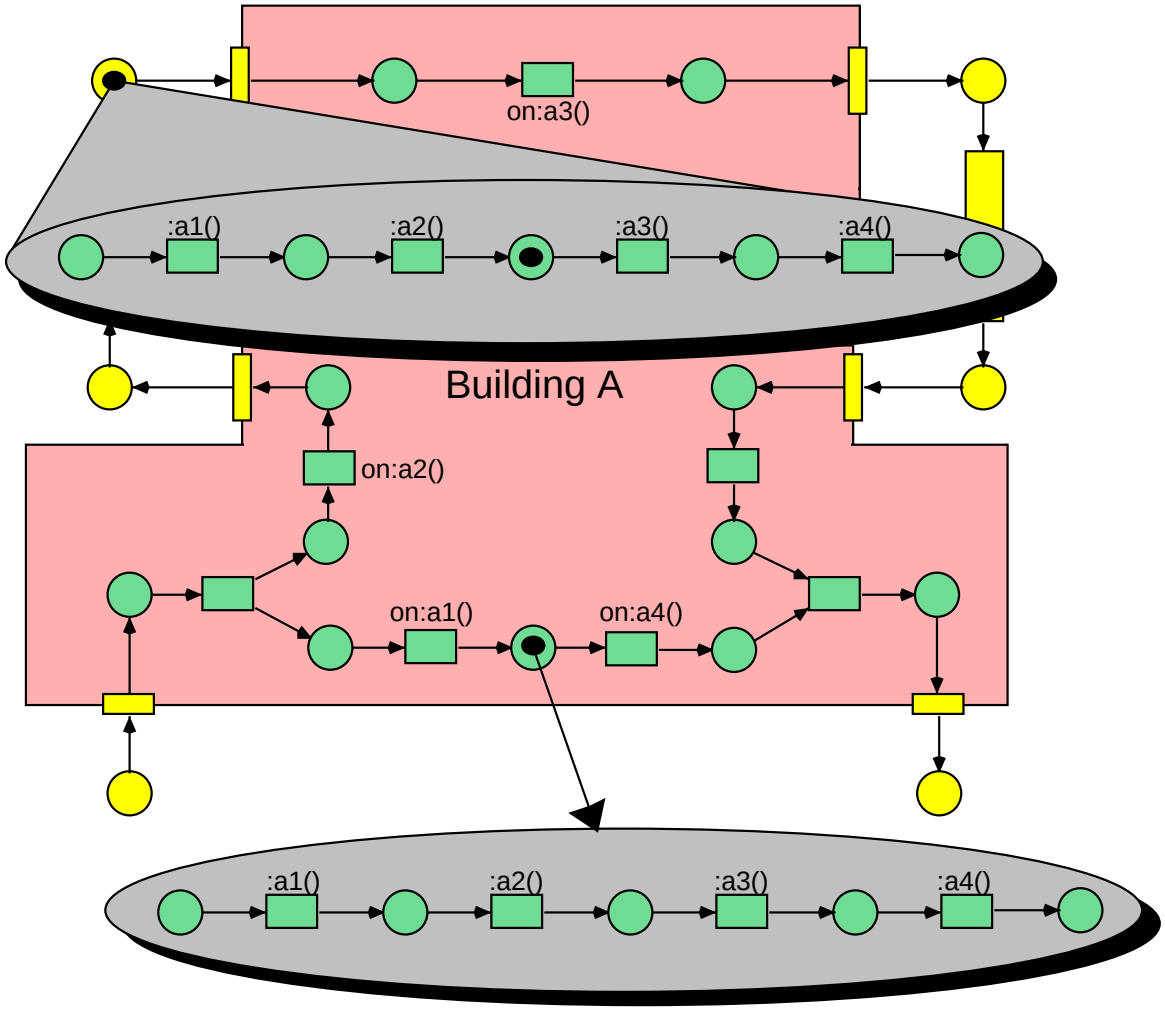

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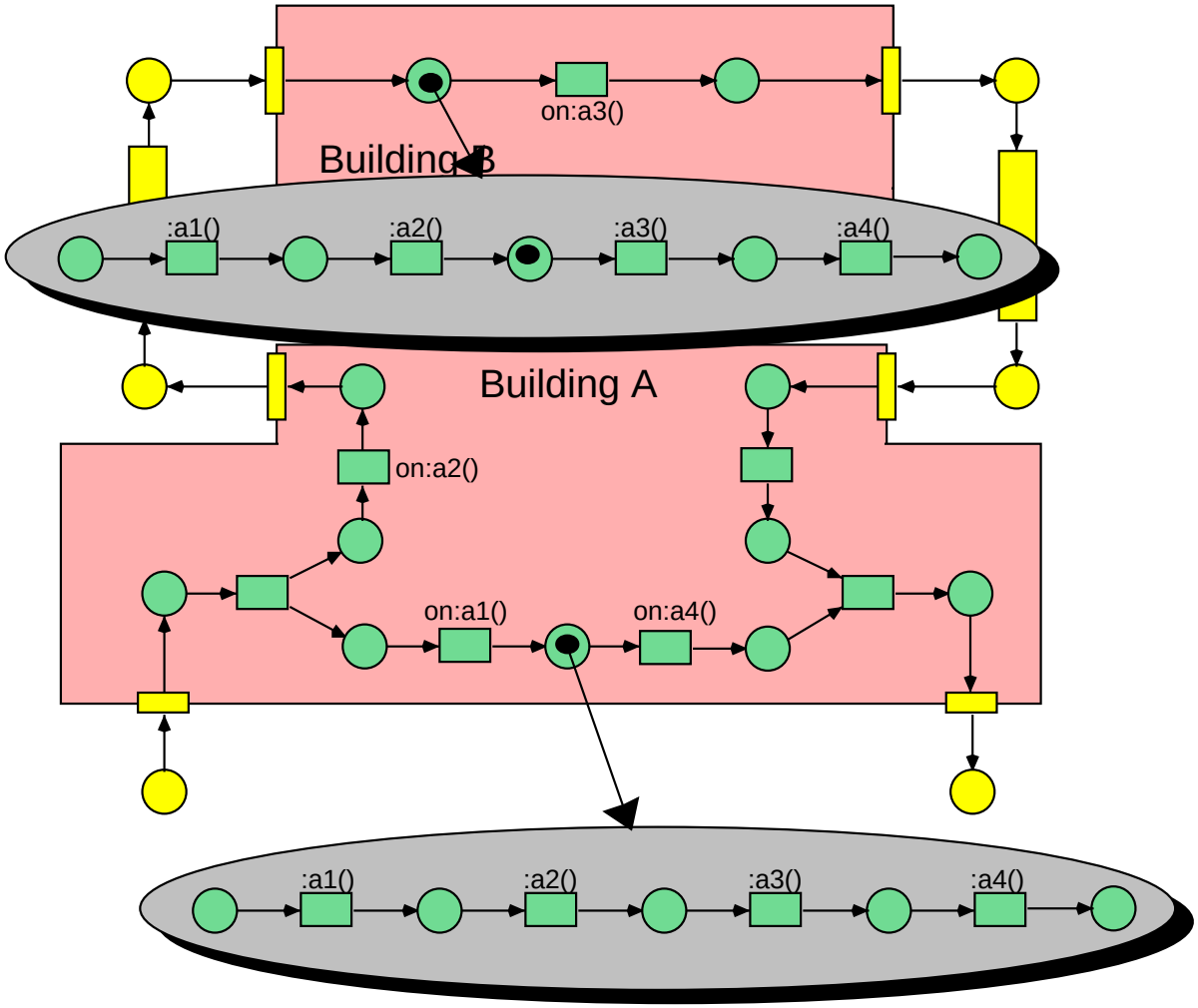

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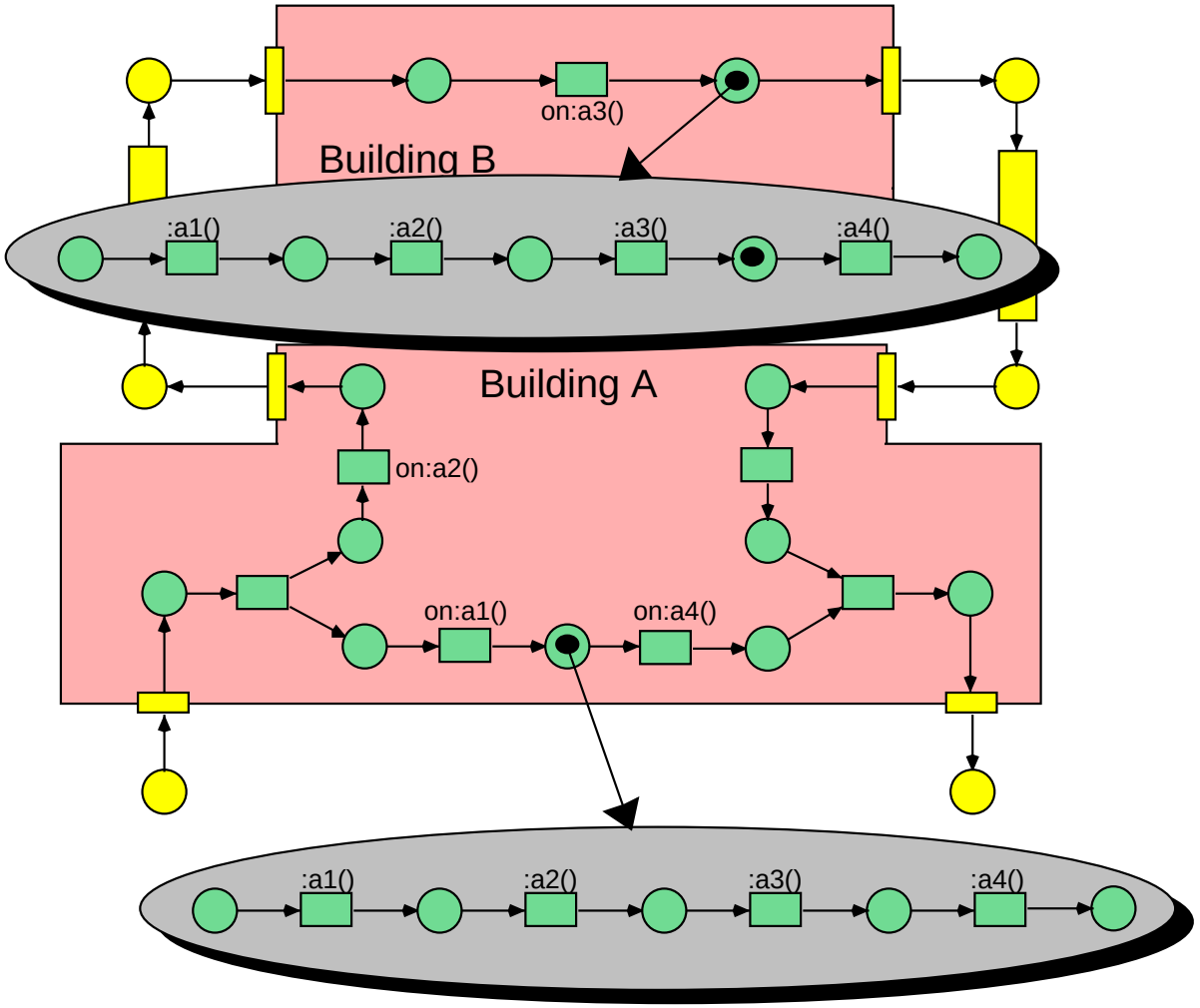

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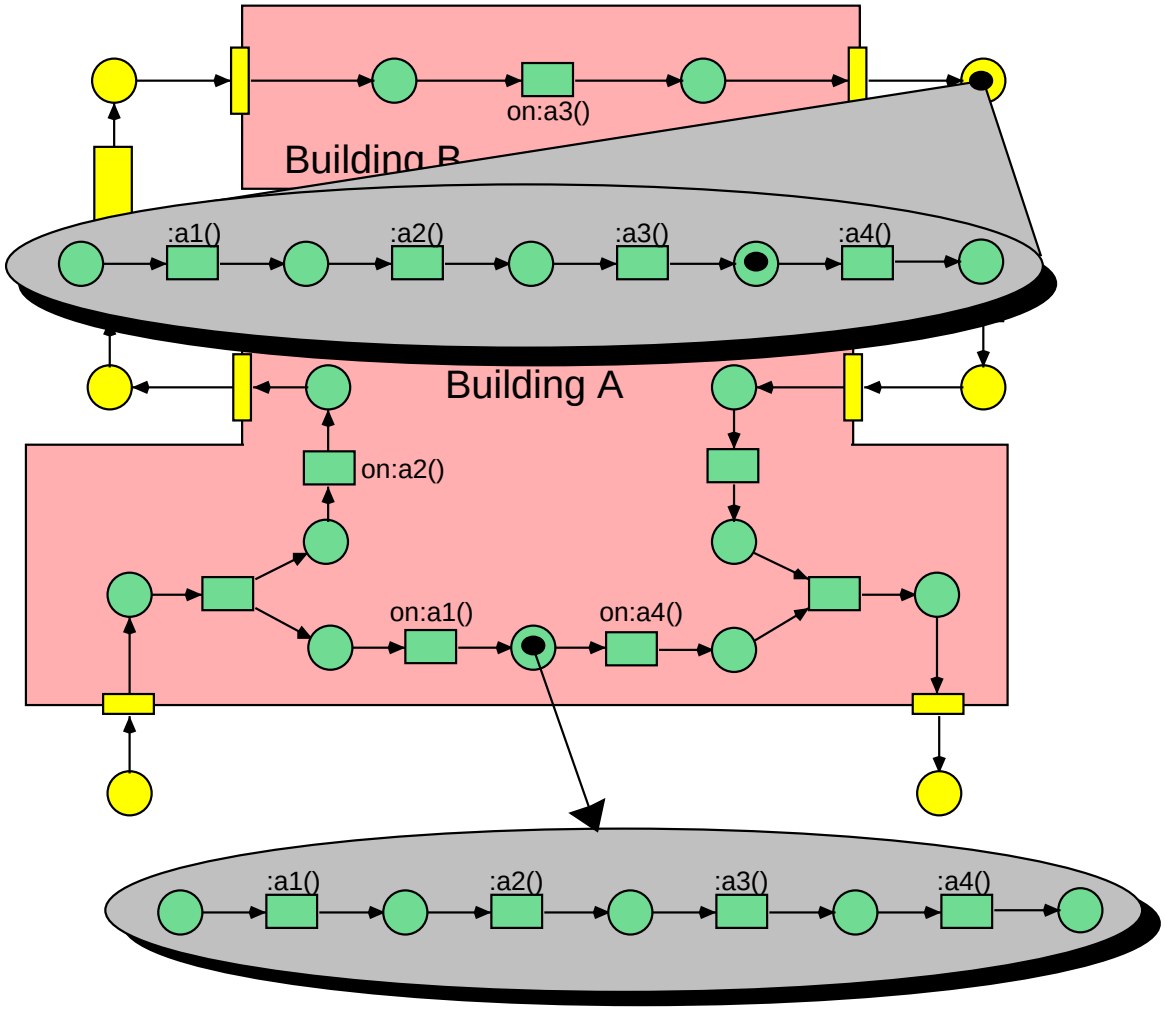

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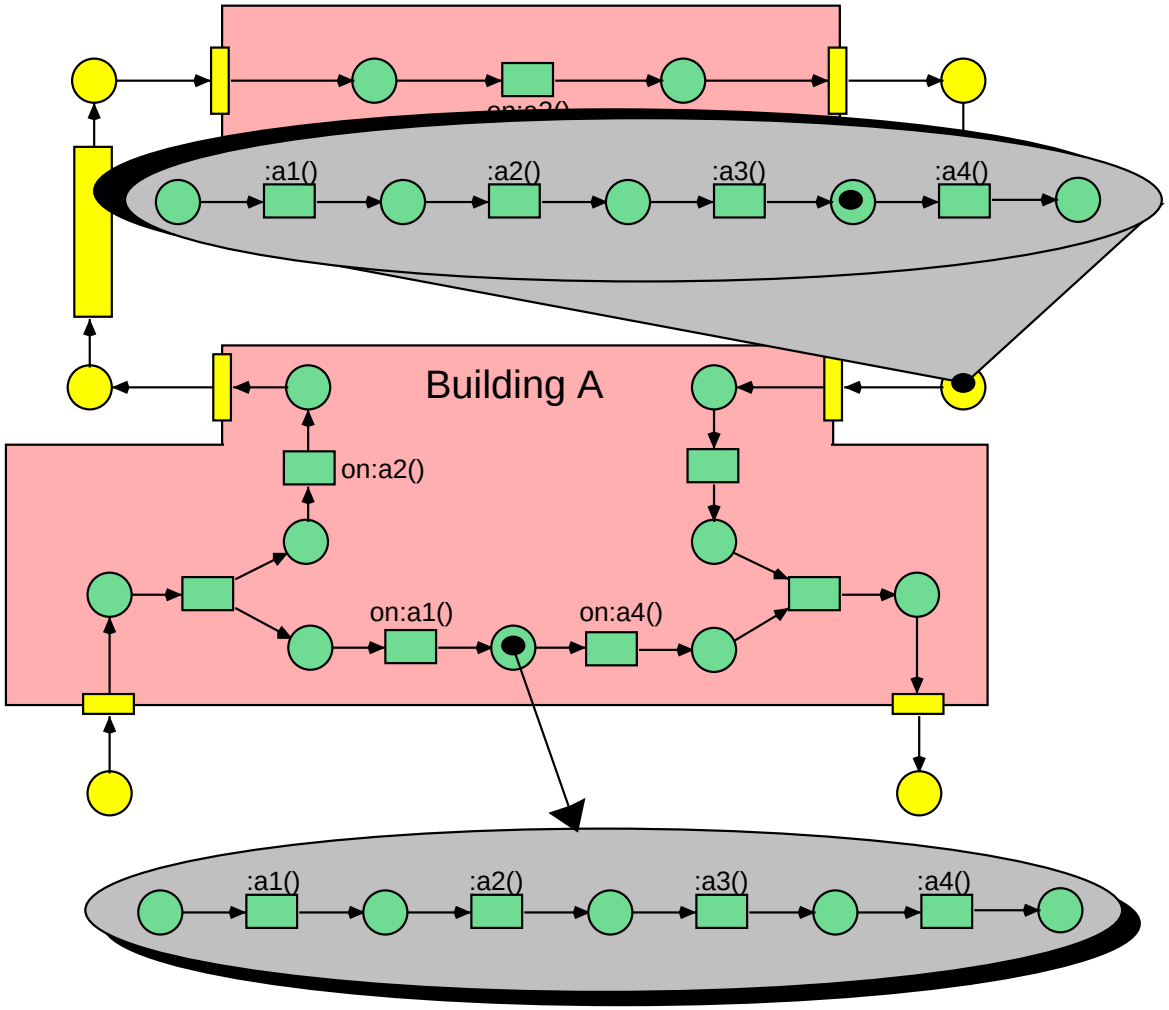

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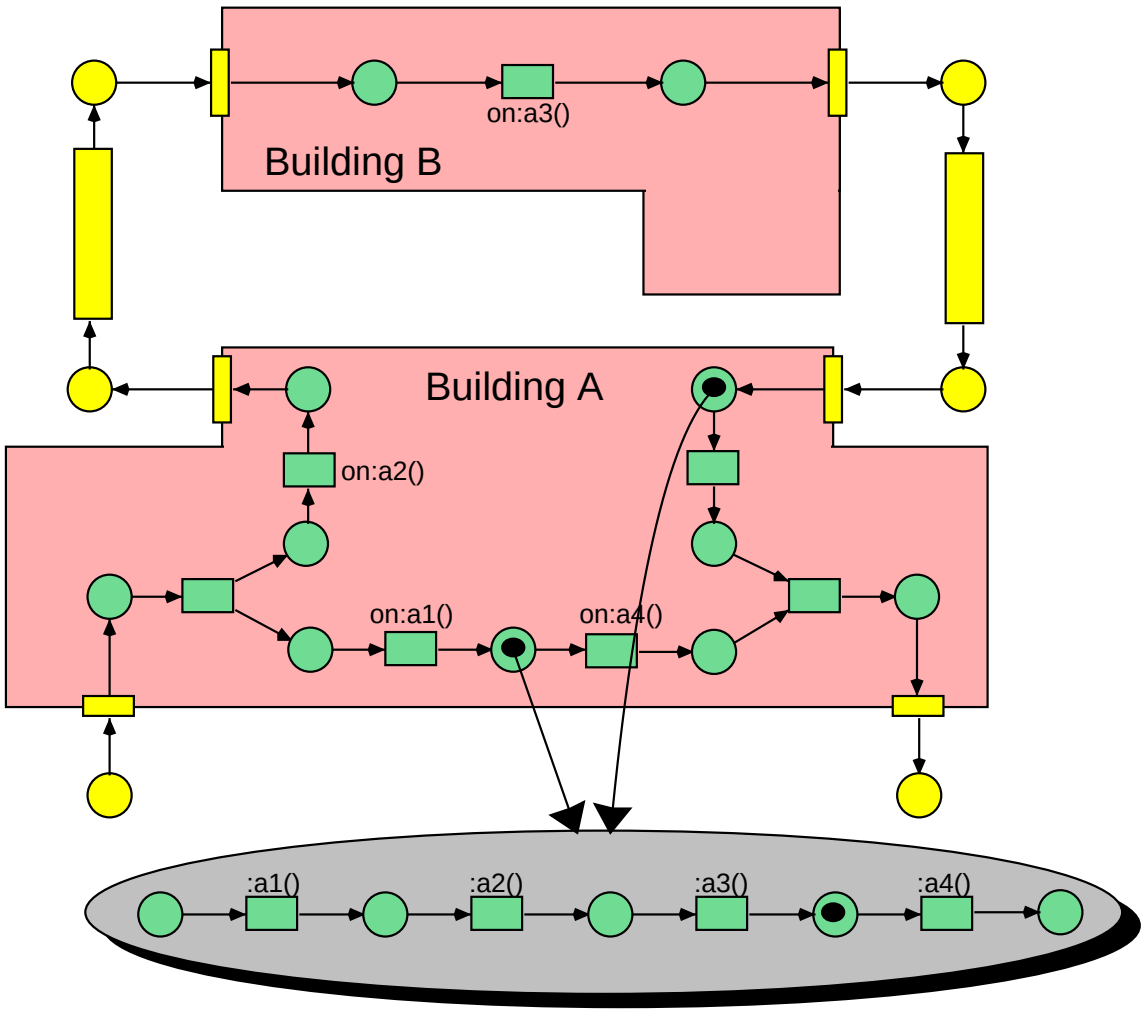

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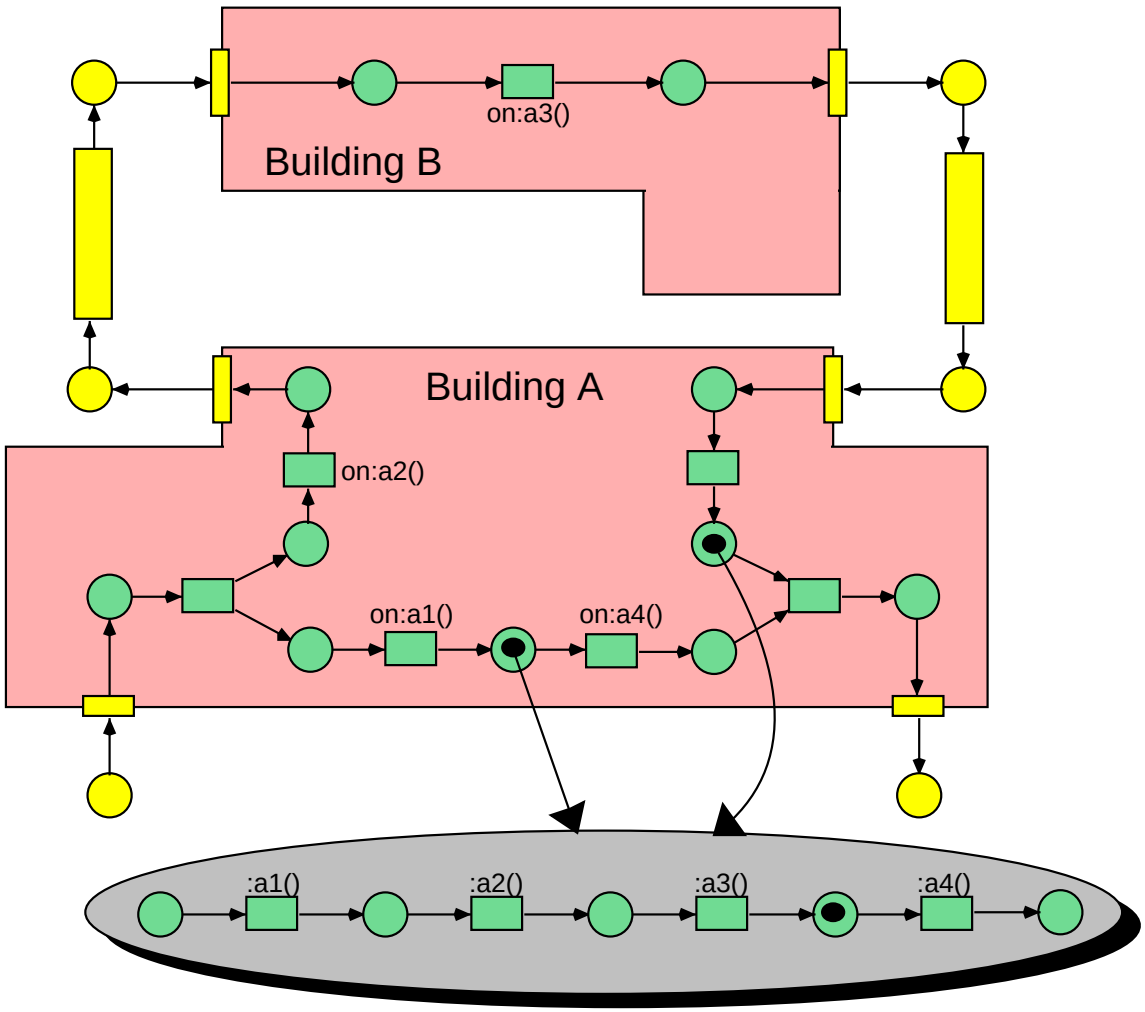

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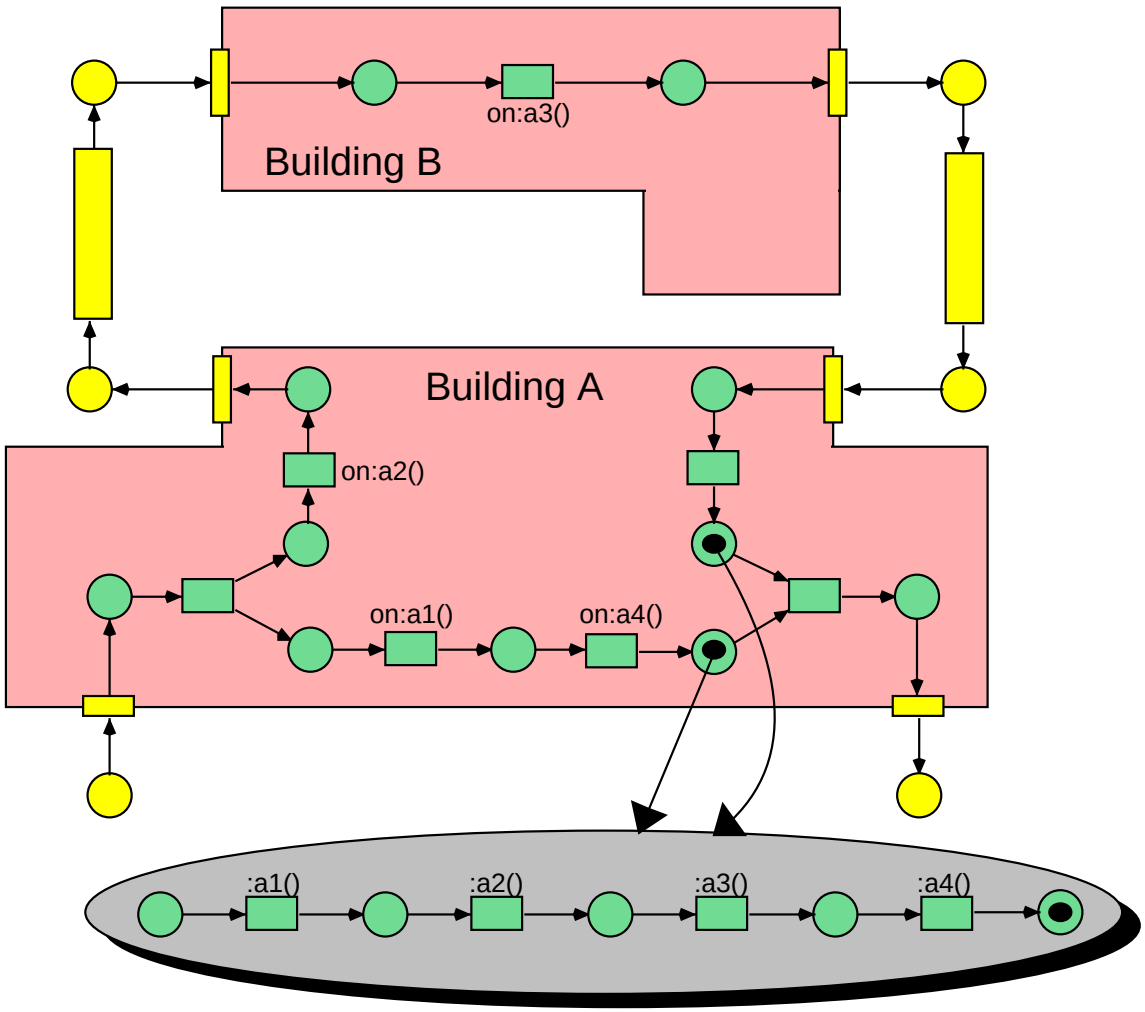

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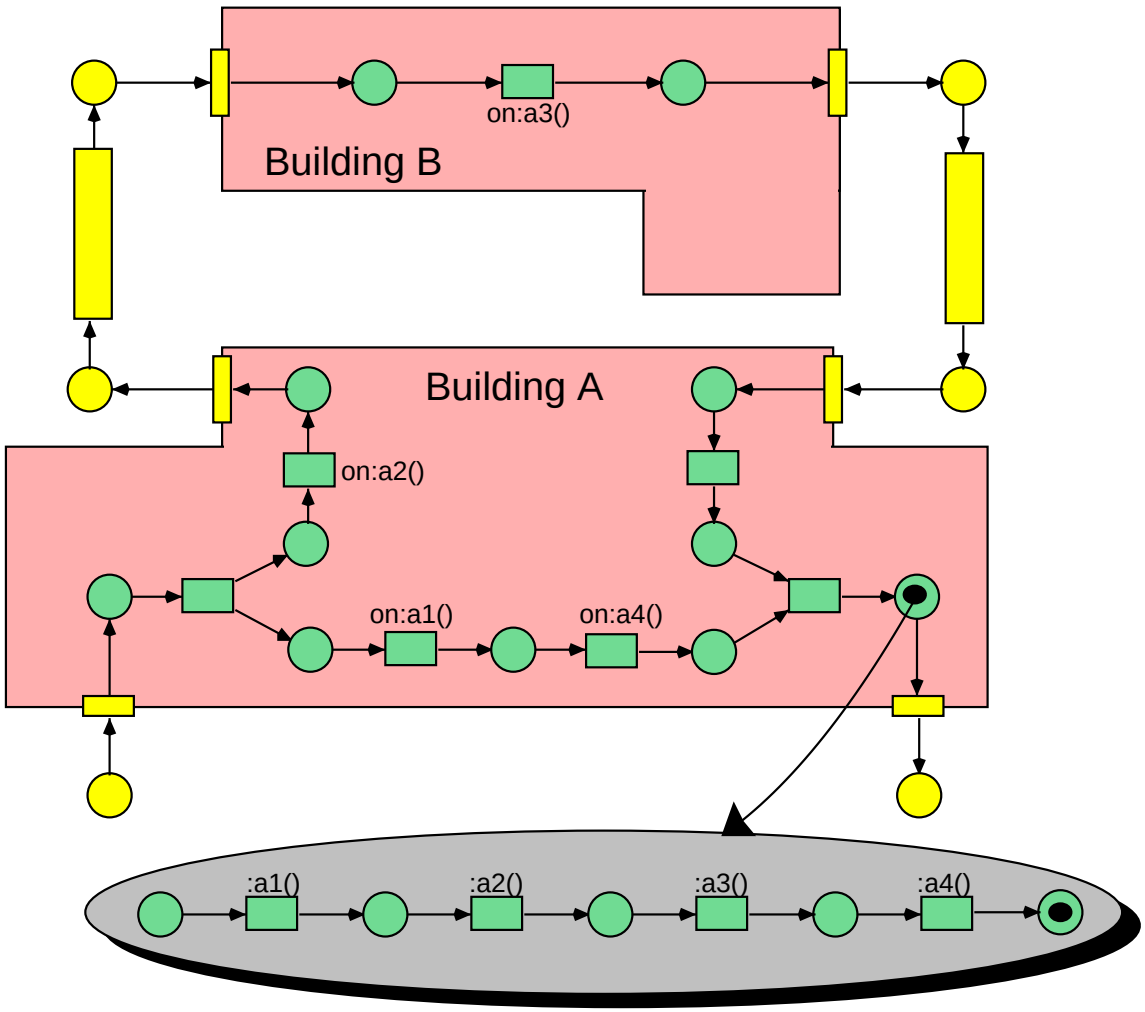

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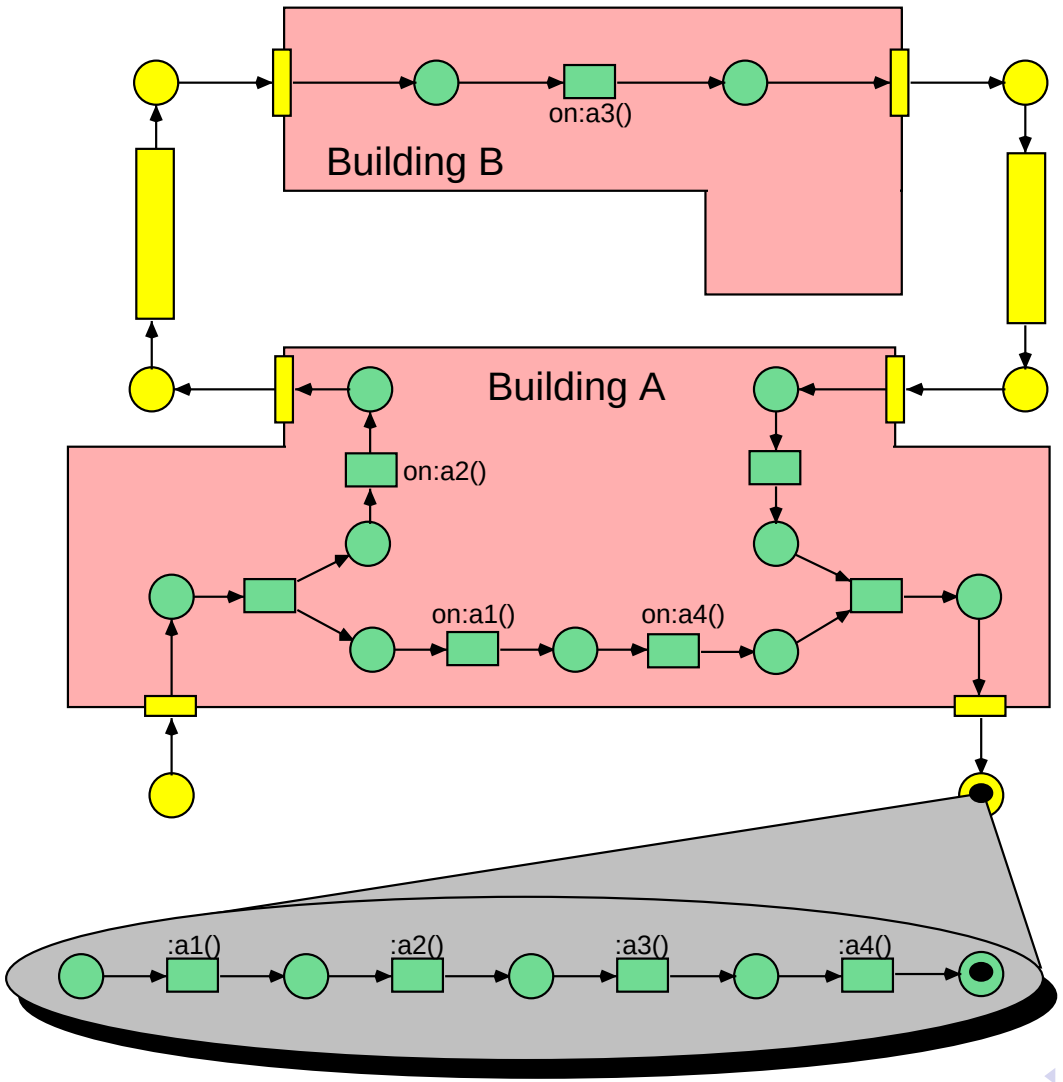
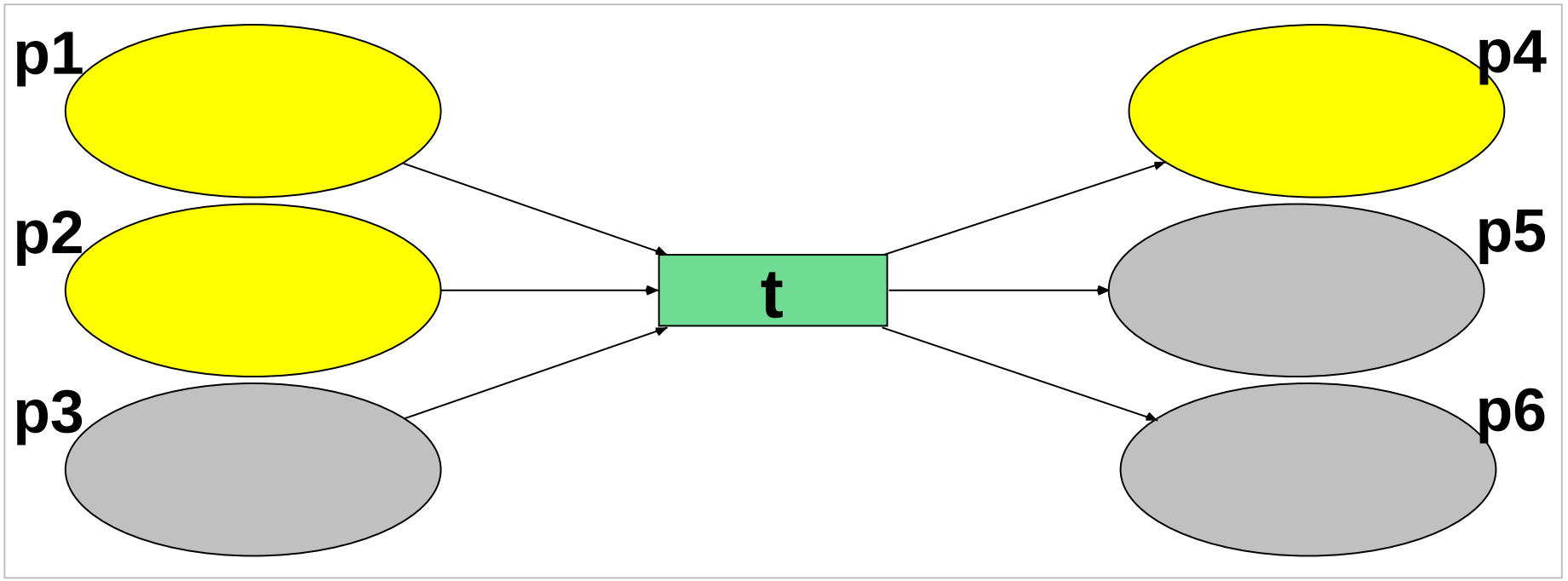


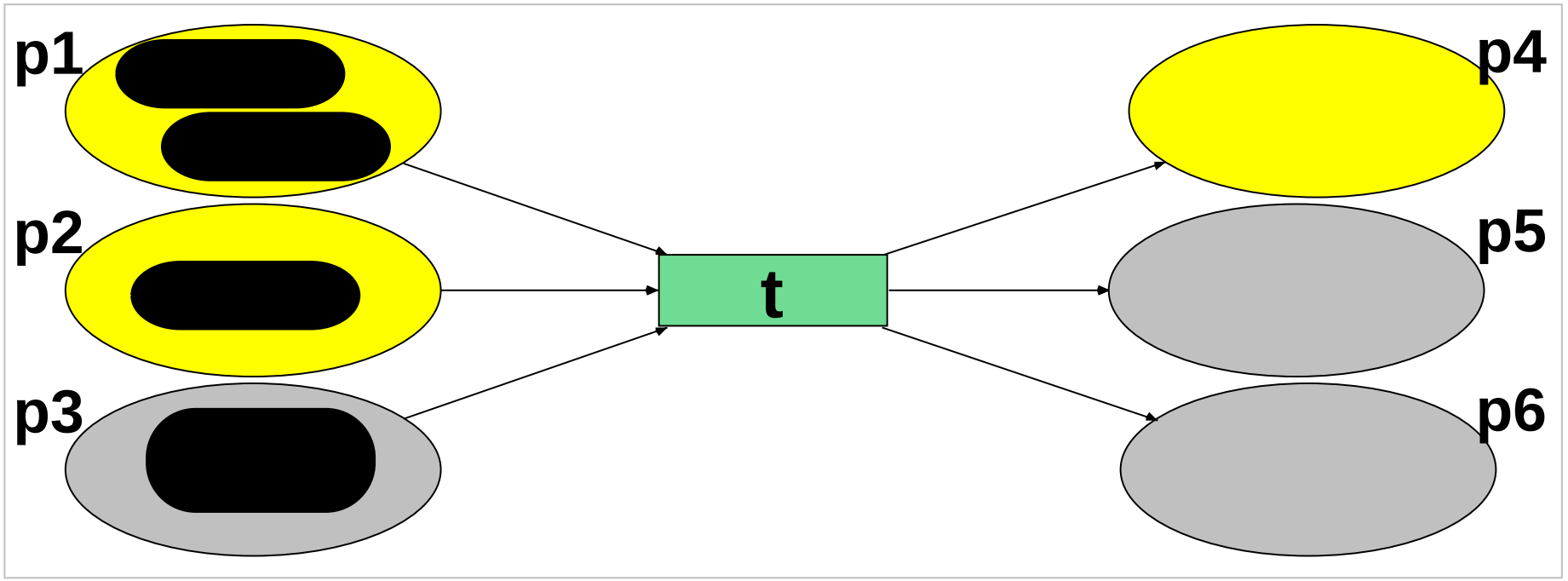
Example



Places p with different types:

$$\begin{aligned} d(p_1) &= d(p_1) = d(p_4) = N_1 \\ d(p_3) &= d(p_5) = d(p_6) = N_2 \end{aligned}$$

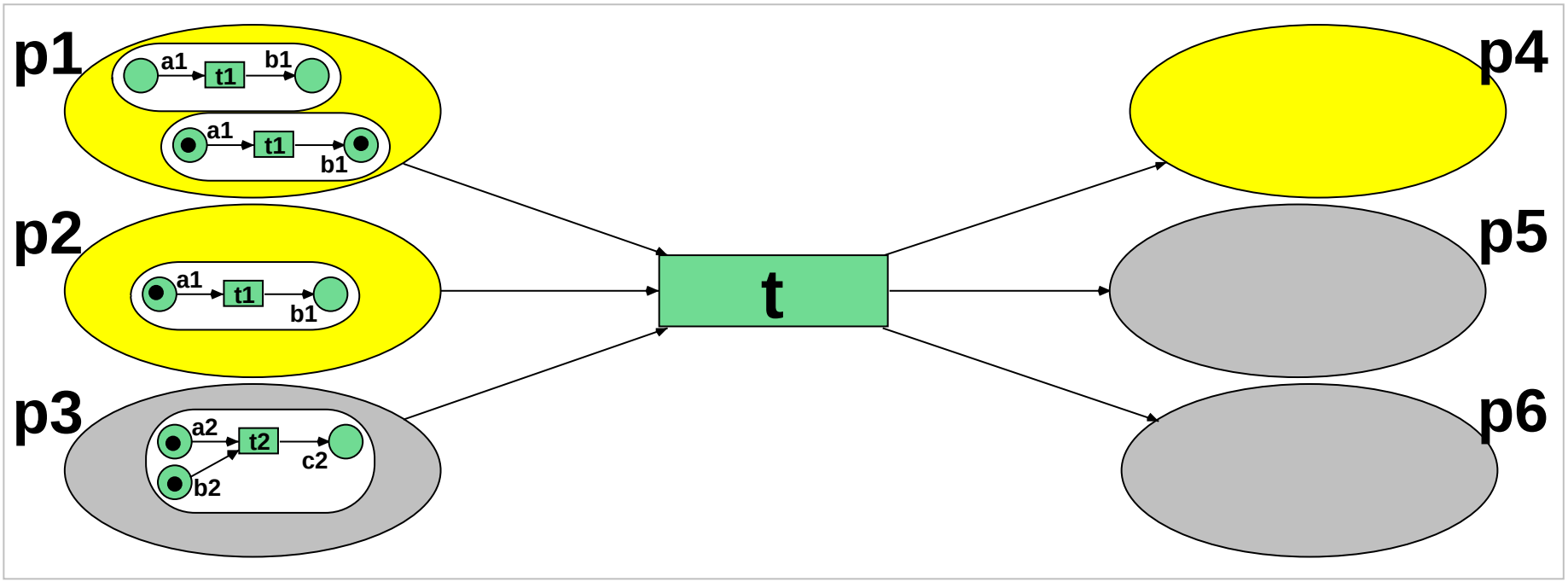
Example



Marking of the system net:

$$\begin{aligned} \mu &= p_1 \\ &+ p_1 \\ &+ p_2 \\ &+ p_3 \end{aligned}$$

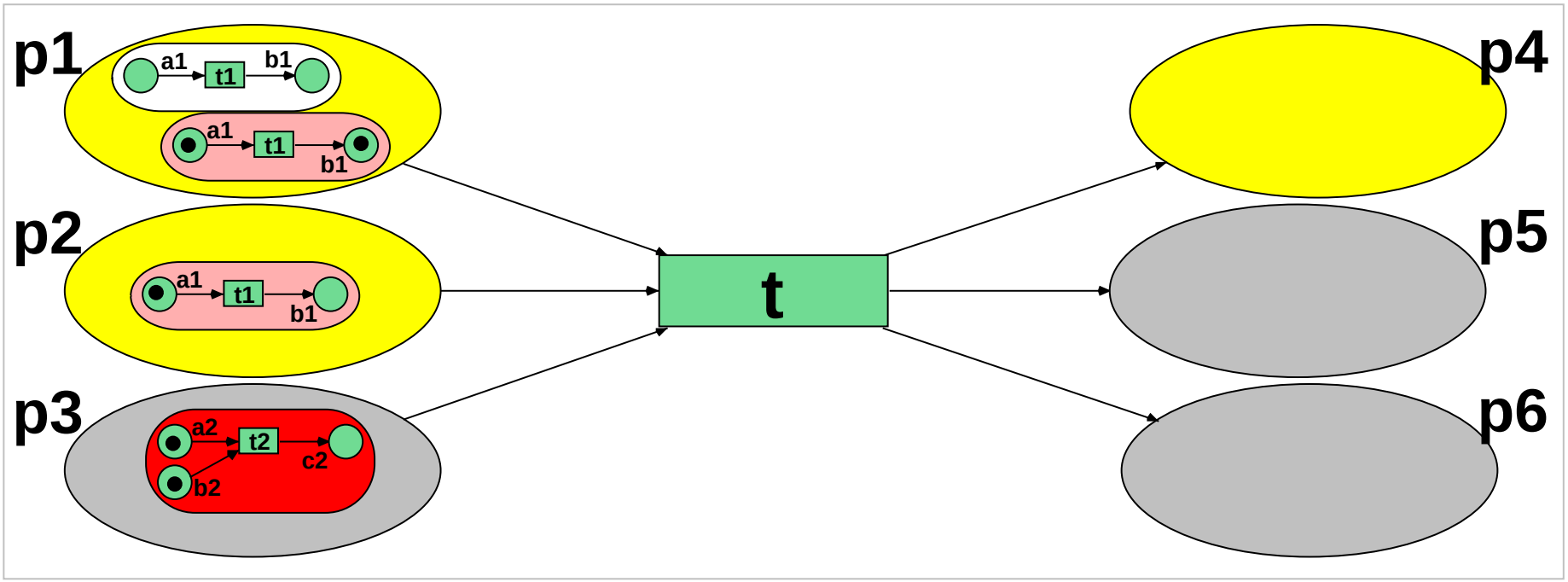
Example



Tokens are net-tokens, which have independent (nested) markings:

$$\begin{aligned} \mu &= (p_1, \mathbf{0}) \\ &+ (p_1, a_1 + b_1) \\ &+ (p_2, a_1) \\ &+ (p_3, a_2 + b_2) \end{aligned}$$

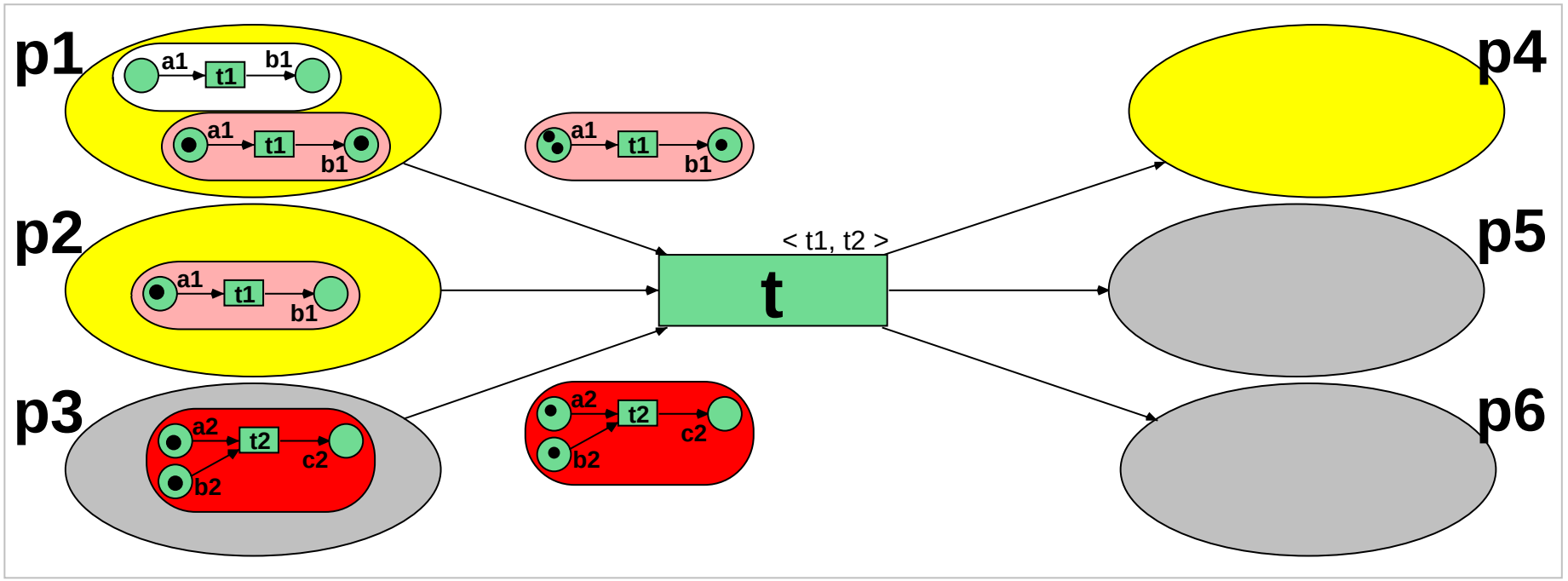
Example



Firing of t selects a subset λ of μ:

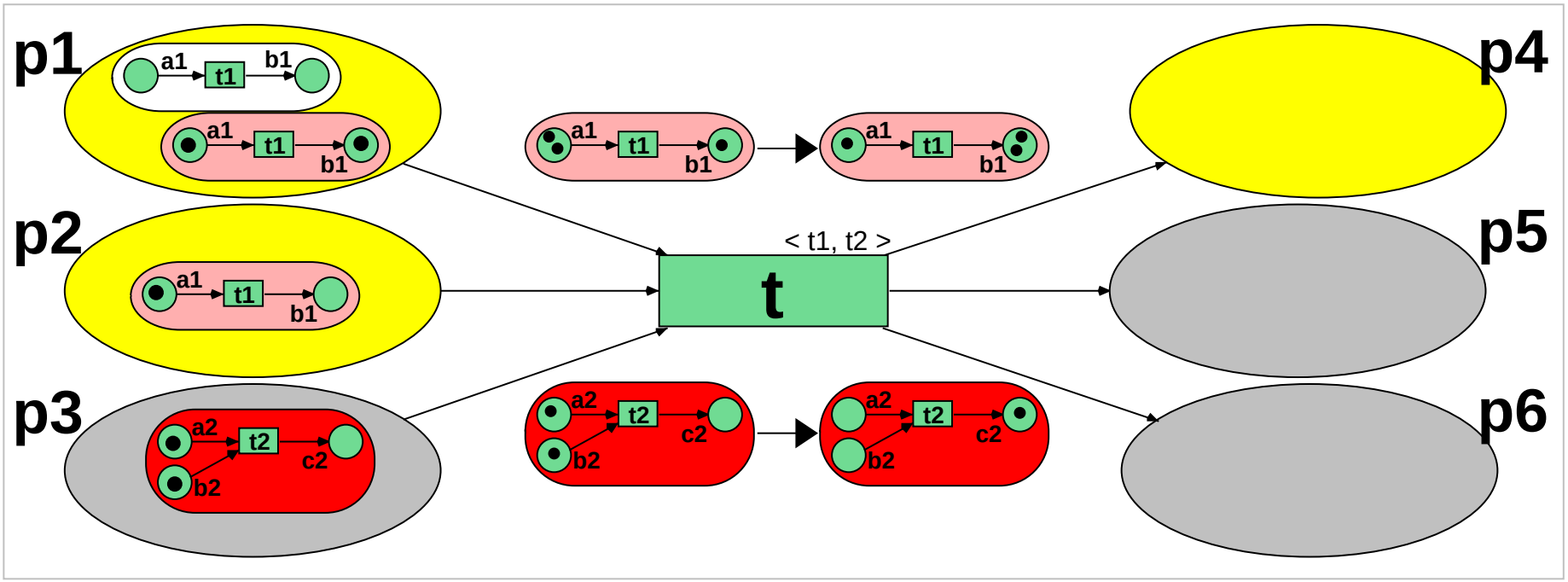
$$\begin{aligned} \lambda &= \cdots \\ &+ (p_1, a_1 + b_1) \\ &+ (p_2, a_1) \\ &+ (p_3, a_2 + b_2) \end{aligned}$$

Example



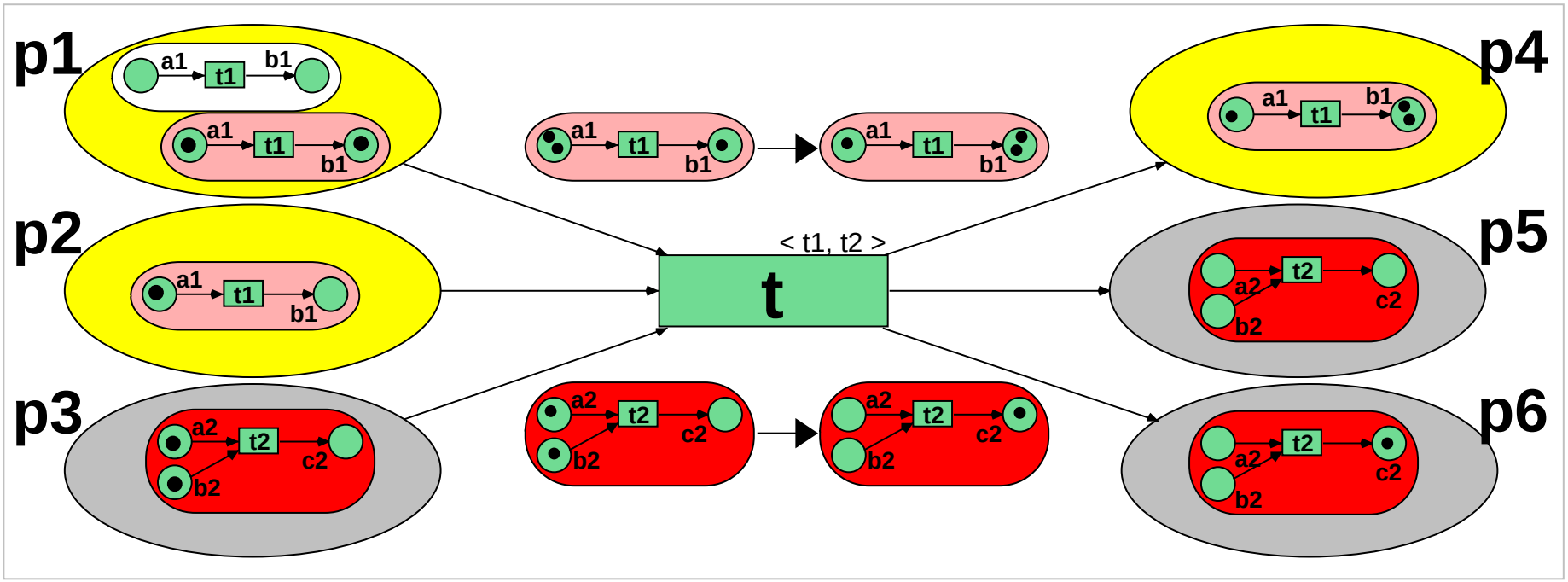
The markings of the net-tokens are added.

Example



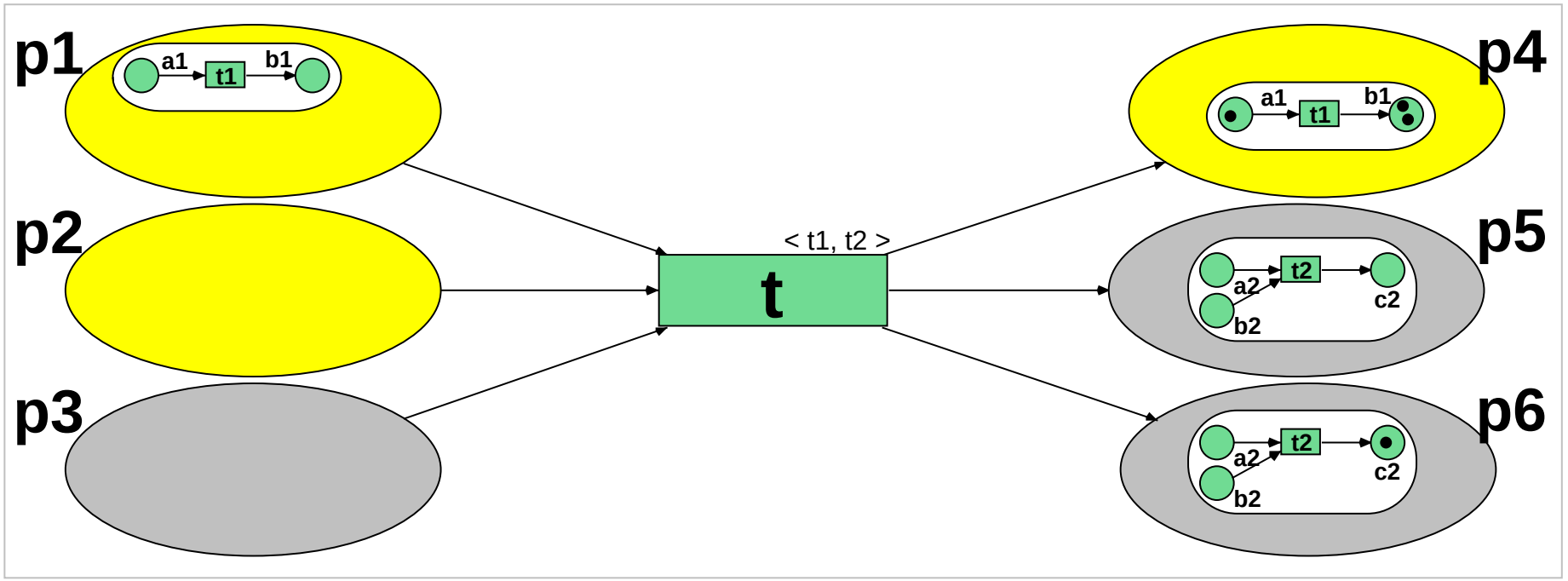
Synchronisation $\theta = (t, \langle t_1, t_2 \rangle)$ of t with t_1 and t_2.

Example



The resulting markings are distributed resulting in ρ.

Example



Successor marking $\mu' = \mu - \lambda + \rho$:

$$\mu \xrightarrow[OS]{(t,\langle t_1,t_2\rangle)(\lambda,\rho)} \mu'$$

Definition

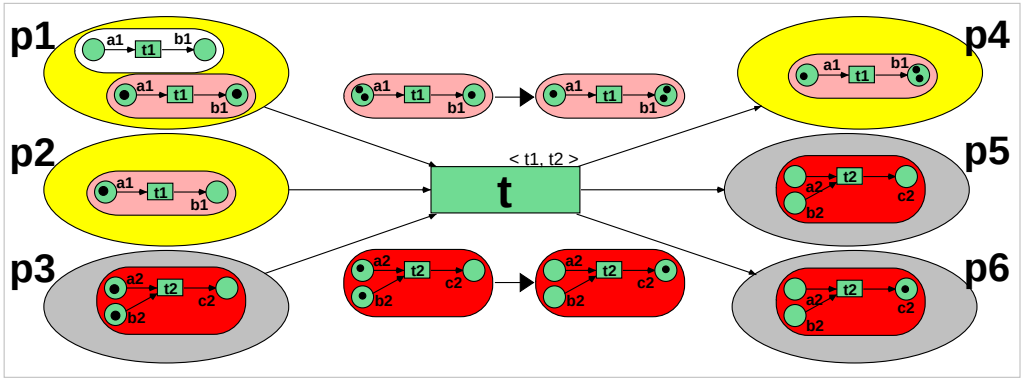
An elementary object system has two nesting levels.

Definition

An *elementary object system* is a tuple $OS = (\widehat{N}, \mathcal{N}, d, \Theta, \mu_0)$

1. $\widehat{N}$ is a P/T-net, called the *system-net*.
2. $\mathcal{N} = \{N_1, \ldots, N_n\}$ is a set of P/T-nets, called *object-nets*.
3. $d : \widehat{P} \to \{\bullet\} \cup \mathcal{N}$ is the typing of the system-net places.
4. $\Theta \subseteq \mathcal{T}$ defines synchronised transitions (system/object).
5. $\mu_0 \in \mathcal{M}$ is the initial marking.

Enabling Predicate



The enabling predicate $\phi((\widehat{\tau}, C), \lambda, \rho)$ holds iff:

1. System: $\Pi^1(\lambda) = \mathbf{pre}(\widehat{\tau})$
2. System: $\Pi^1(\rho) = \mathbf{post}(\widehat{\tau})$
3. Object: $\forall N \in \mathcal{N} : \Pi^2_N(\lambda) \geq \mathbf{pre}(C(N))$
4. Object:
   $\forall N \in \mathcal{N} : \Pi^2_N(\rho) = \Pi^2_N(\lambda) - \mathbf{pre}(C(N)) + \mathbf{post}(C(N))$

Projection Equivalence

The equivalence $\cong$ relates nested markings which coincide in their projections:

$$\alpha \cong \beta \iff \begin{aligned} \Pi^1(\alpha) &= \Pi^1(\beta) \wedge \\ \forall N \in \mathcal{N} : \Pi^2_N(\alpha) &= \Pi^2_N(\beta) \end{aligned}$$

Lemma

The enabling predicate is invariant w.r.t. $\cong$:

$$\phi((\widehat{\tau}, C), \lambda, \rho) \iff \forall \lambda', \rho' : \lambda' \cong \lambda \wedge \rho' \cong \rho \Longrightarrow \phi((\widehat{\tau}, C), \lambda', \rho')$$

Firing Rule

Definition

The event $(\widehat{\tau}, C) \in \mathcal{T}$ is *enabled* in mode (λ, ρ) if:

$$\lambda \leq \mu \wedge \exists \lambda', \rho' : \quad \begin{array}{c} (\lambda' \cong \lambda) \wedge \\ (\rho' \cong \rho) \wedge \\ \phi((\widehat{\tau}, C), \lambda', \rho') \end{array}$$

Successor marking: $\mu' := \mu - \lambda + \rho$.

Notation: $\mu \xrightarrow[OS]{(\widehat{\tau}, C)(\lambda, \rho)} \mu'$.

Synchronisation and autonomous firing

Synchronisation is the general case.

- System-autonomous firing: All the object nets fire the idle event ϵ_N with $\mathbf{pre}(\epsilon_N) = \mathbf{post}(\epsilon_N) = \mathbf{0}$.
- Object-autonomous firing: The object net fires in the place $\widehat{p}$ which is expressed by the idle event of the system net: $\epsilon_{\widehat{p}}$ where $\widehat{\mathbf{pre}}(\epsilon_{\widehat{p}}) = \widehat{\mathbf{post}}(\epsilon_{\widehat{p}}) = \widehat{p}$.

1. Object Nets
   - Reference Semantics
   - Value Semantics
   - New: Mobility Semantics
2. Elementary Object Systems
   - Synchronous Firing
   - Enabling Predicate
   - Firing Rule
3. Mobile EOS
   - Conversion
   - Expressiveness

Mobile EOS

- For Eos each place is its own name space.
- Clusters of places belonging to the same name space.
- An equivalence $\leftrightarrow$ on the object net's places $P = \bigcup_{N \in \mathcal{N}} P_N$ is called a *conversion equivalence*.

Definition

A *mobile* EOS is a pair $(OS, \leftrightarrow)$ where OS is an Eos and $\leftrightarrow$ is a conversion equivalence on the object nets' places $P = \bigcup_{N \in \mathcal{N}} P_N$.

Conversion Equivalence

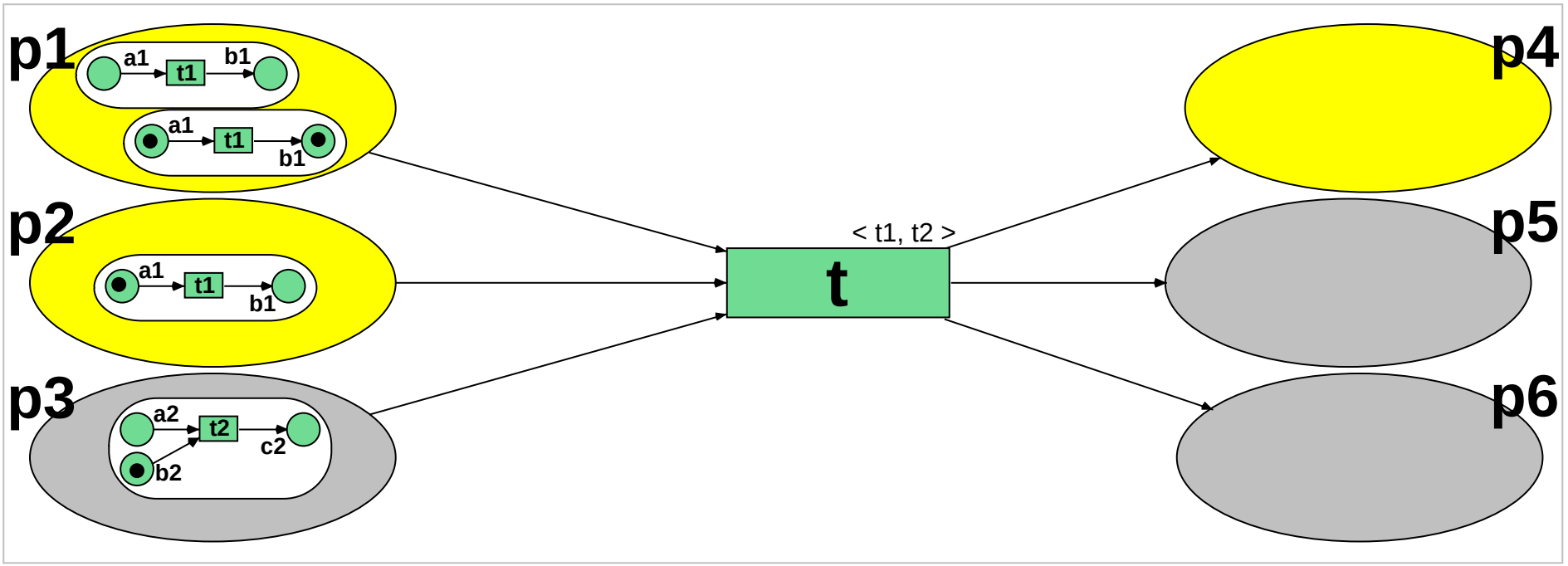
Conversion equivalence for nested markings:

$$\alpha \leftrightarrow \beta \quad \Longleftrightarrow \quad \begin{aligned} \Pi^1(\alpha) &= \Pi^1(\beta) \wedge \\ \forall N \in \mathcal{N} : \Pi^2_N(\alpha) &= \Pi^2_N(\beta) \pmod{\leftrightarrow} \end{aligned}$$

Lemma

For all conversions $\leftrightarrow$ we have $\alpha \cong \beta \Longrightarrow \alpha \leftrightarrow \beta$.
In particular, for $\leftrightarrow = id_P$ we have $\alpha \cong \beta \iff \alpha \leftrightarrow \beta$.

Example



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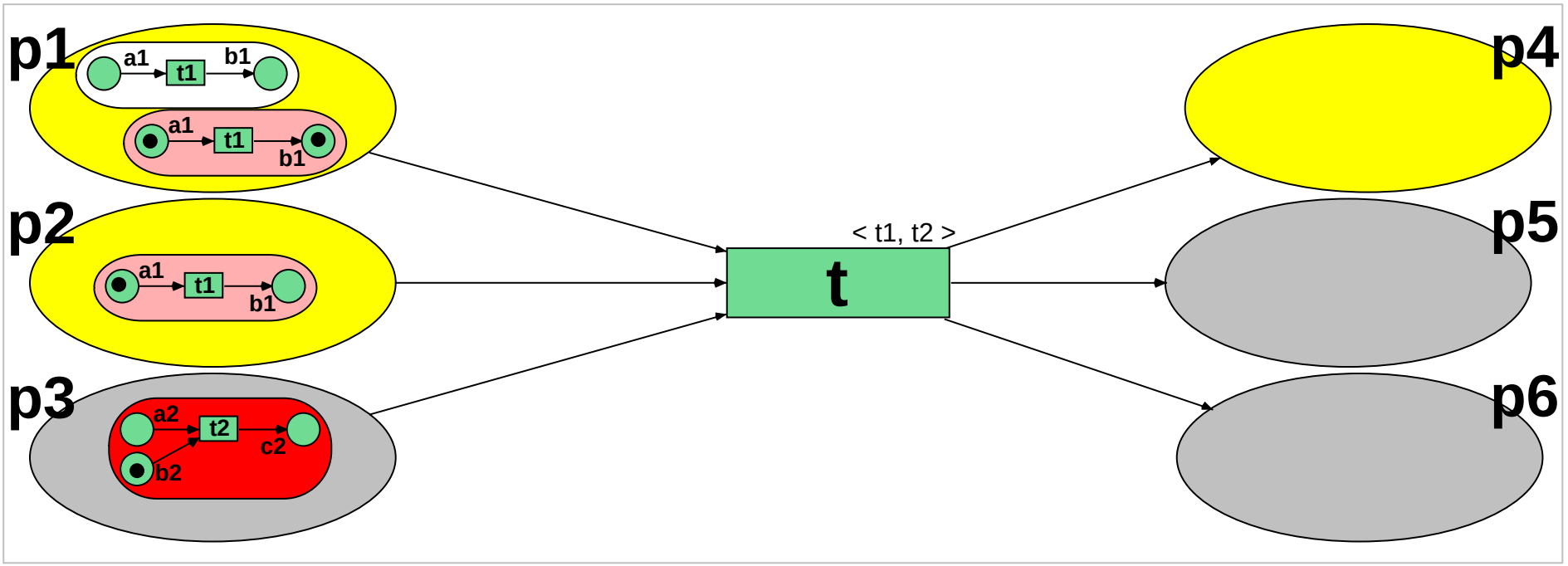
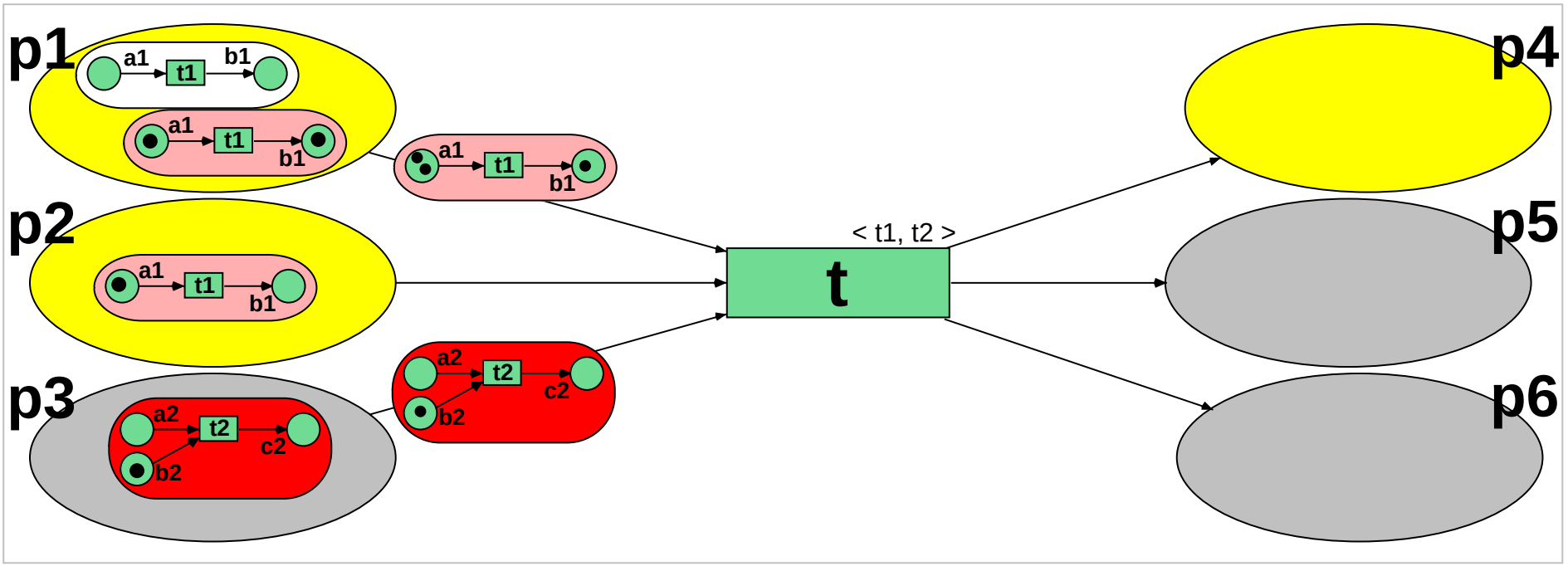
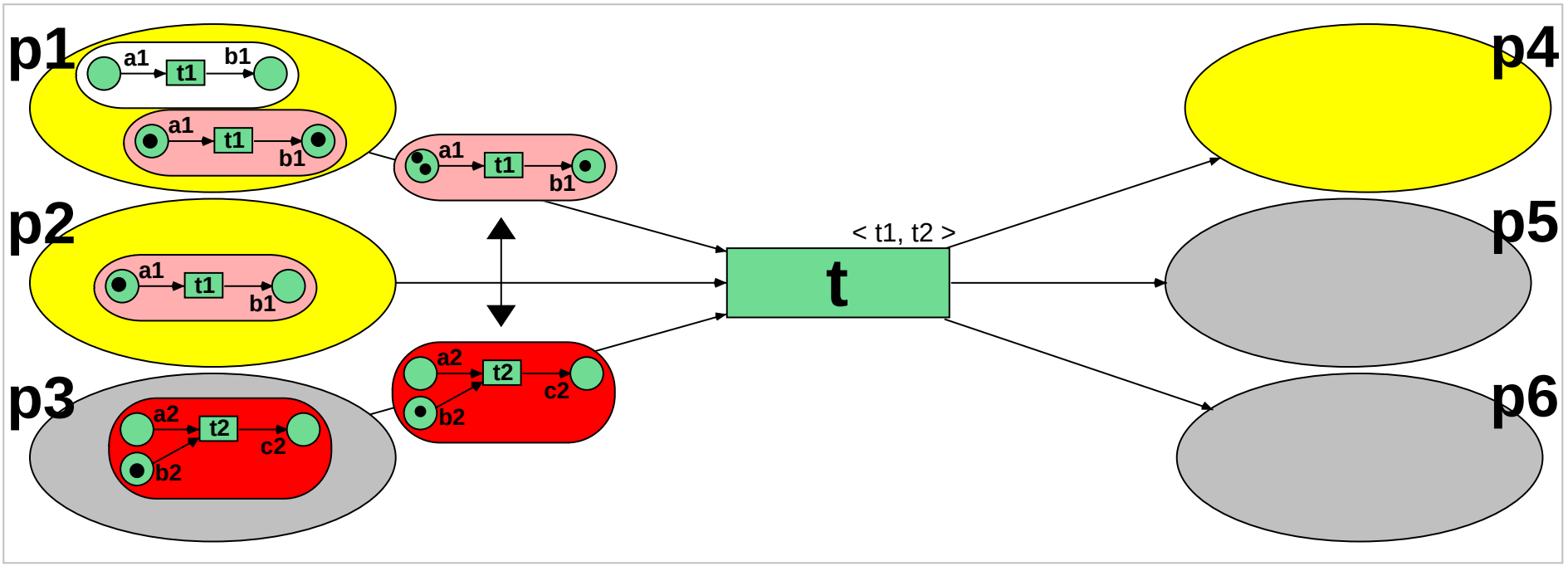


Example



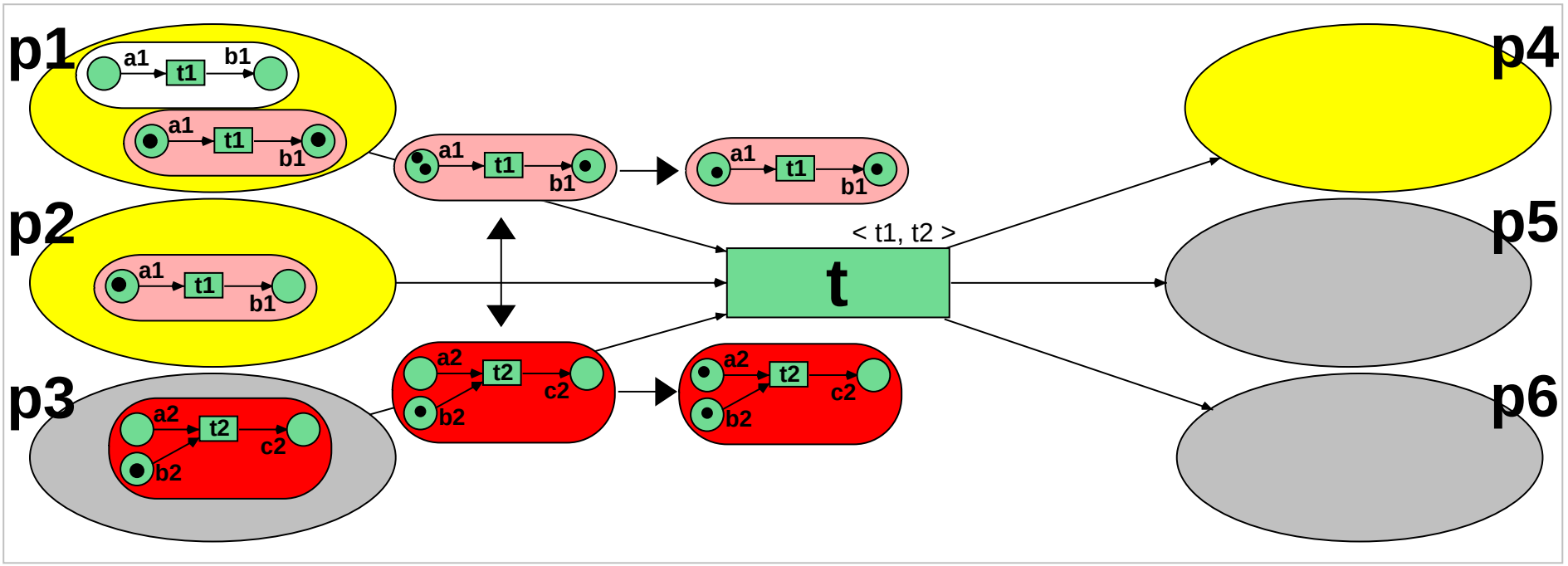
Synchronisation with t_2 is disabled.

Example



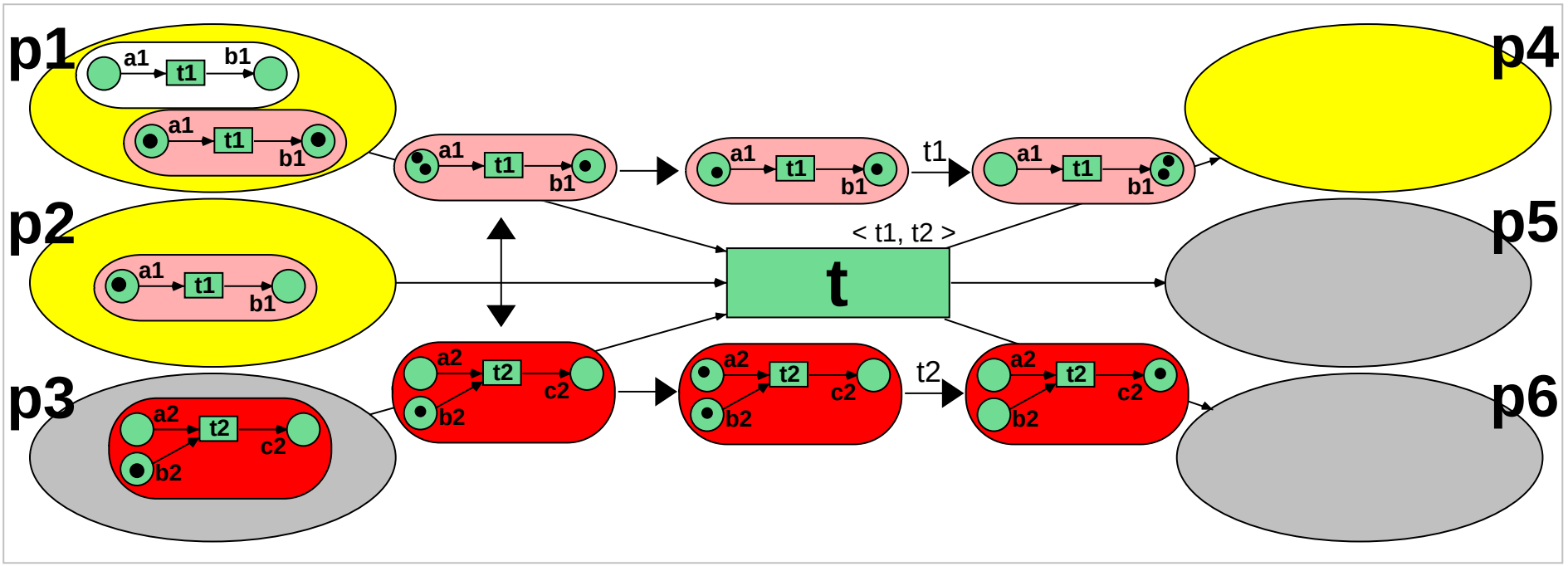
Conversion: $a_1 \leftrightarrow a_2$

Example



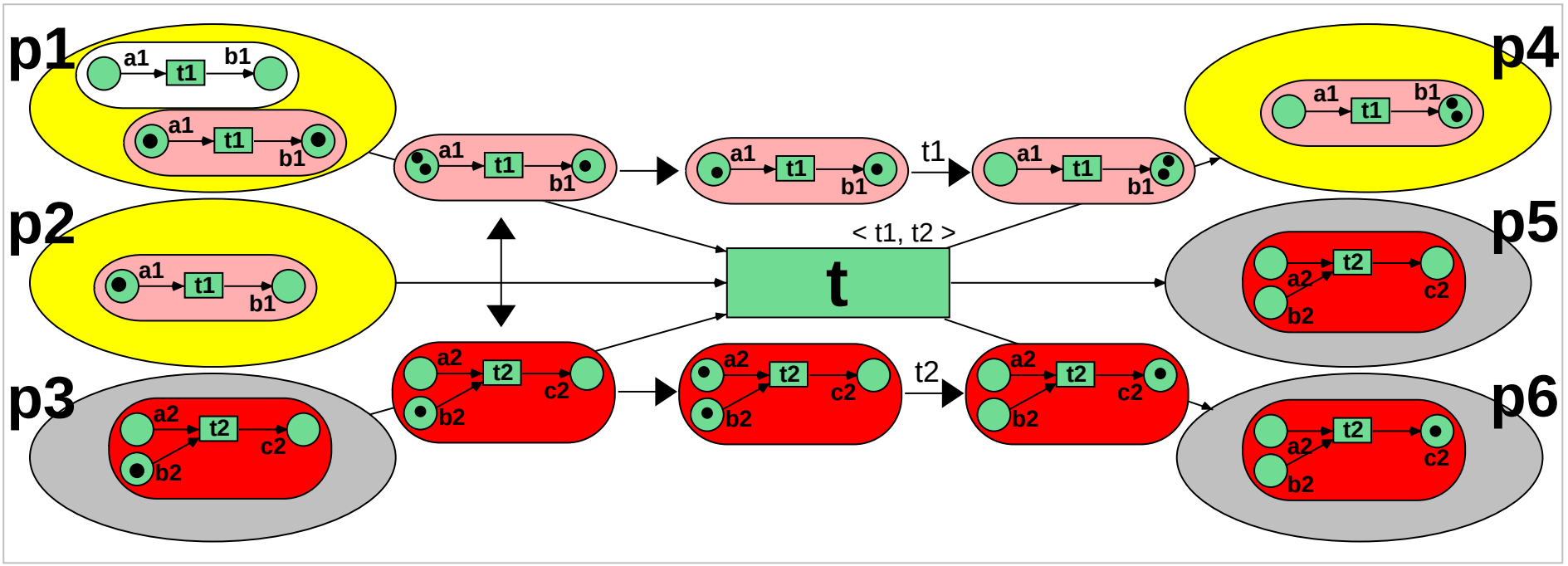
One token is converted from a_1 to a_2.

Example



Synchronisation is enabled.

Example



Firing Rule

Definition

$(\widehat{\tau}, C) \in \mathcal{T}$ is $\leftrightarrow$-*enabled* in mode (λ, ρ) if:

$$\lambda \leq \mu \wedge \exists \lambda', \rho' : \quad (\lambda' \leftrightarrow \lambda) \wedge (\rho' \leftrightarrow \rho) \wedge \phi((\widehat{\tau}, C), \lambda', \rho')$$

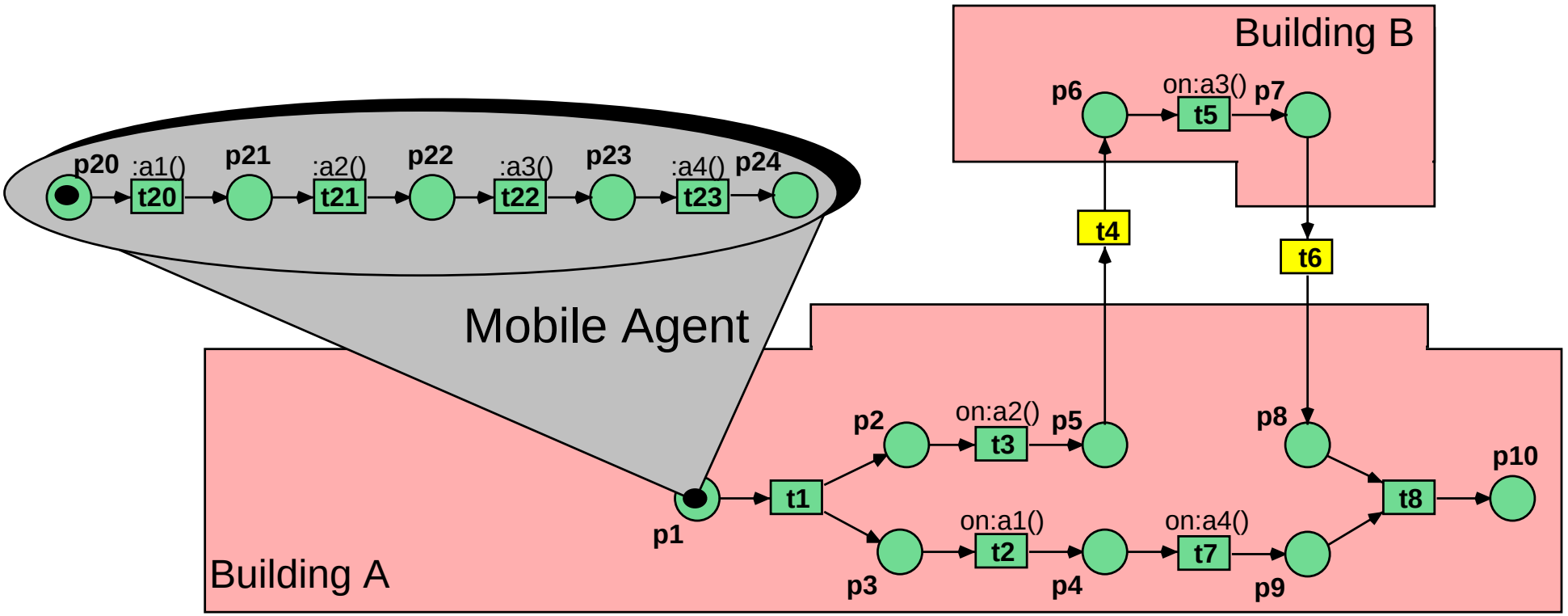
The successor marking is defined as $\mu' := \mu - \lambda + \rho$.

The mobile EOS (OS, id_P) behaves like the Eos OS:

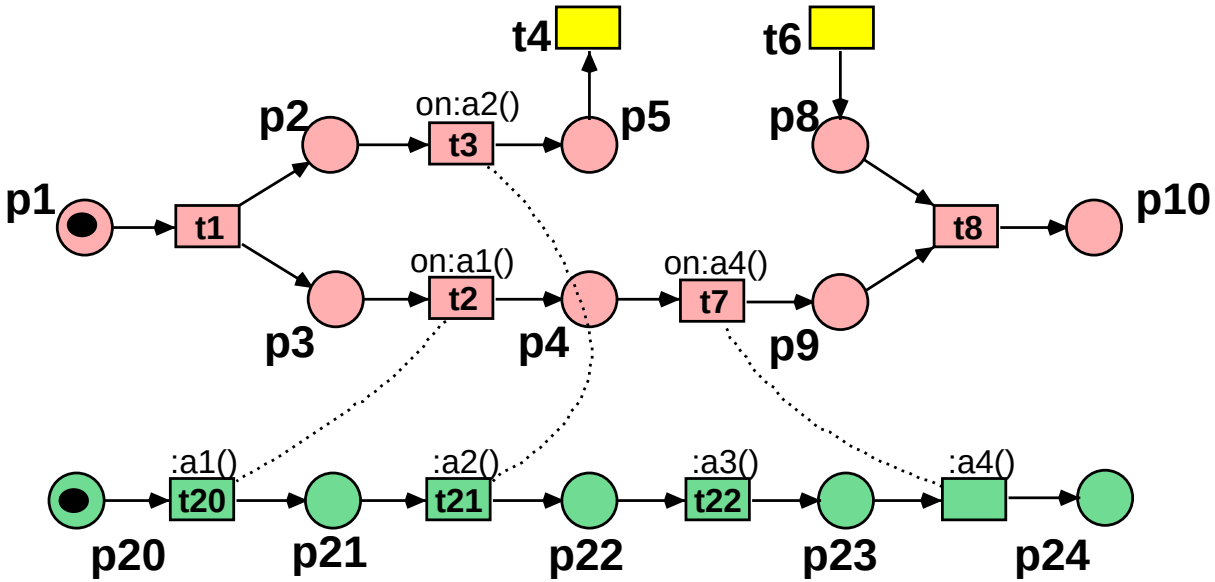
Theorem

The transition $(\widehat{\tau}, C) \in \mathcal{T}$ is enabled in mode (λ, ρ) for OS iff it is id_P-enabled in mode (λ, ρ) for (OS, id_P).

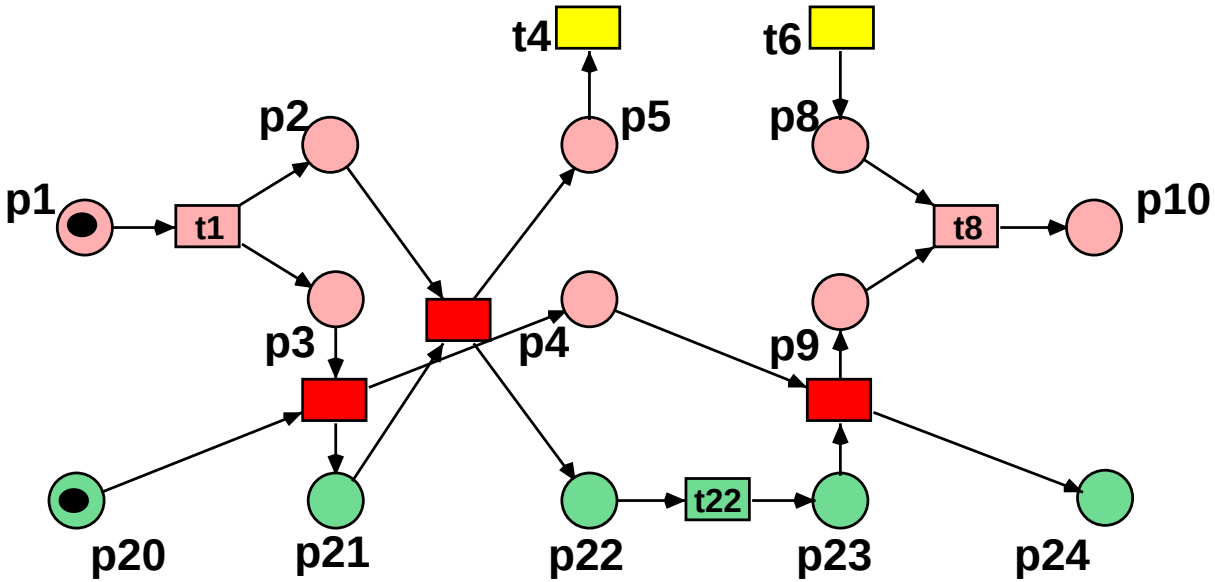
Simulation of Namespaces



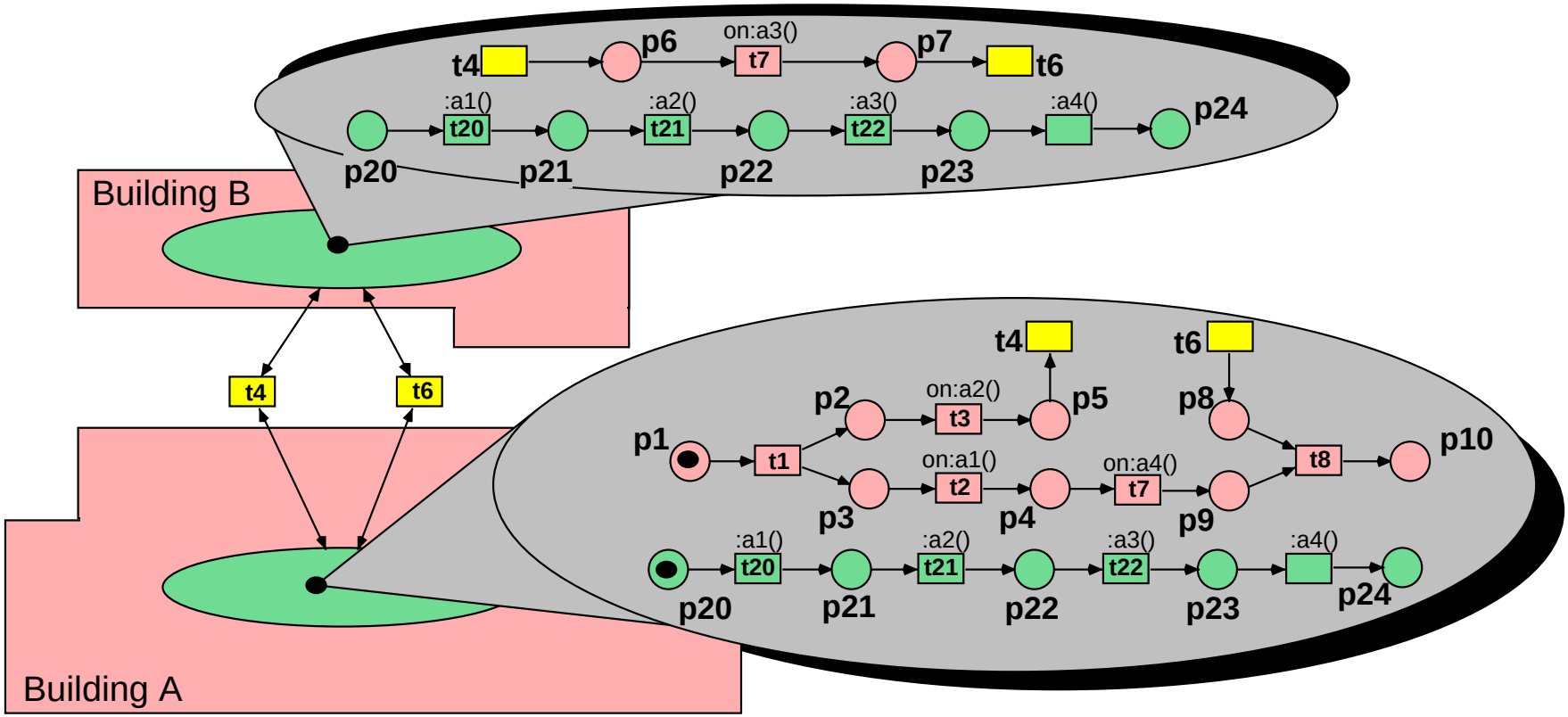
Simulation of Namespaces



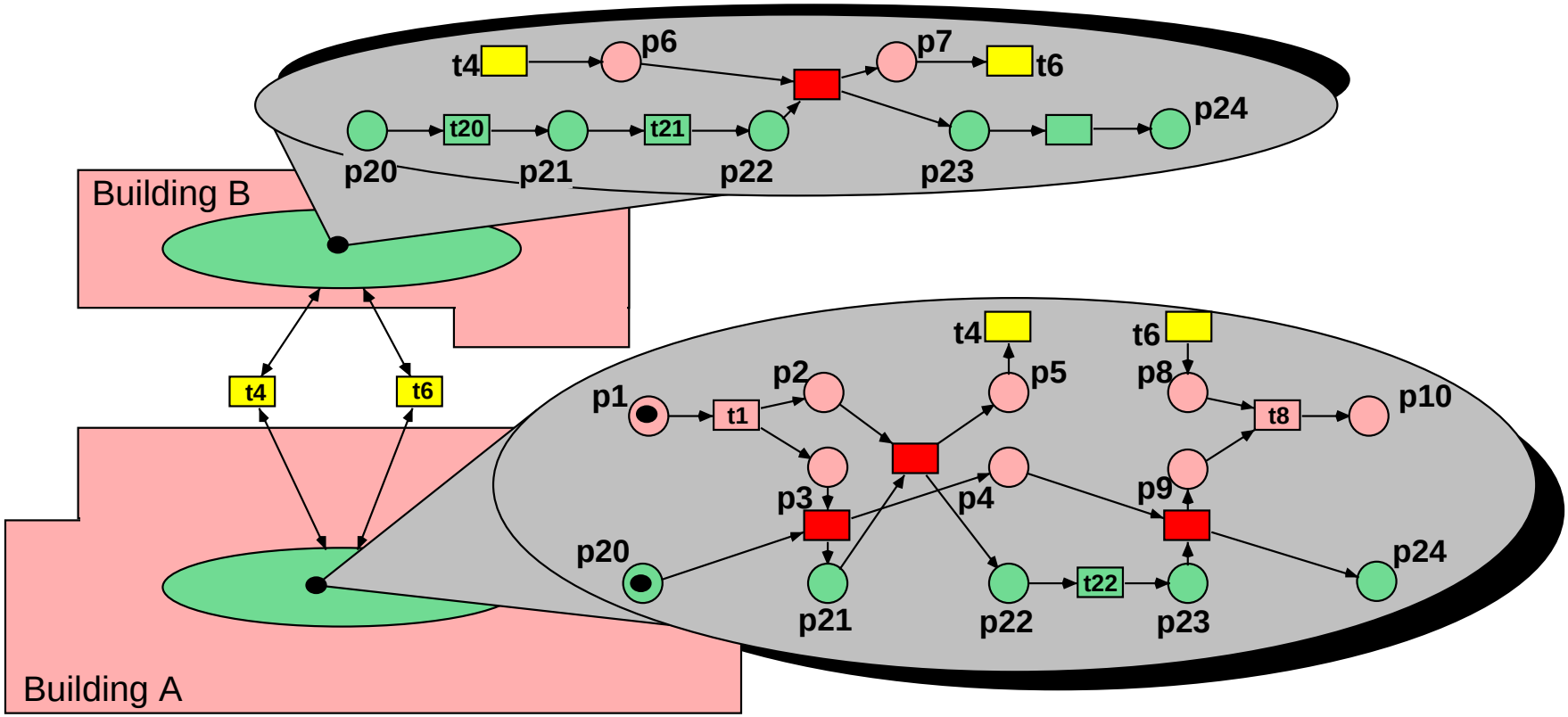
Simulation of Namespaces



Simulation of Namespaces



Simulation of Namespaces



Expressiveness

Theorem

Mobile Eos *can simulate Petri nets with inhibitor arcs.*

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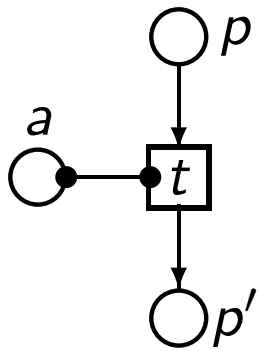
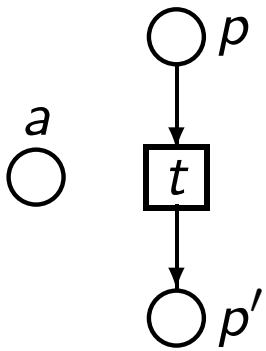
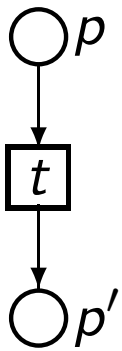


(a) Inhibitor net (N, A)



(b) Net N

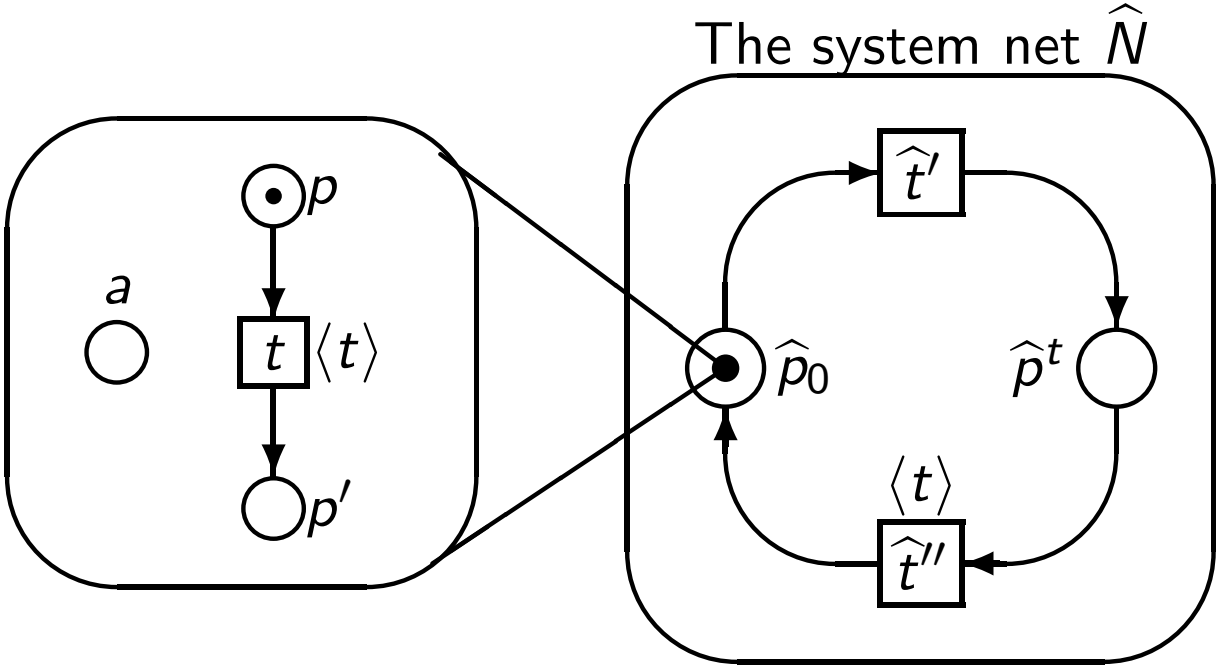


(c) Net N_t

Expressiveness

Theorem

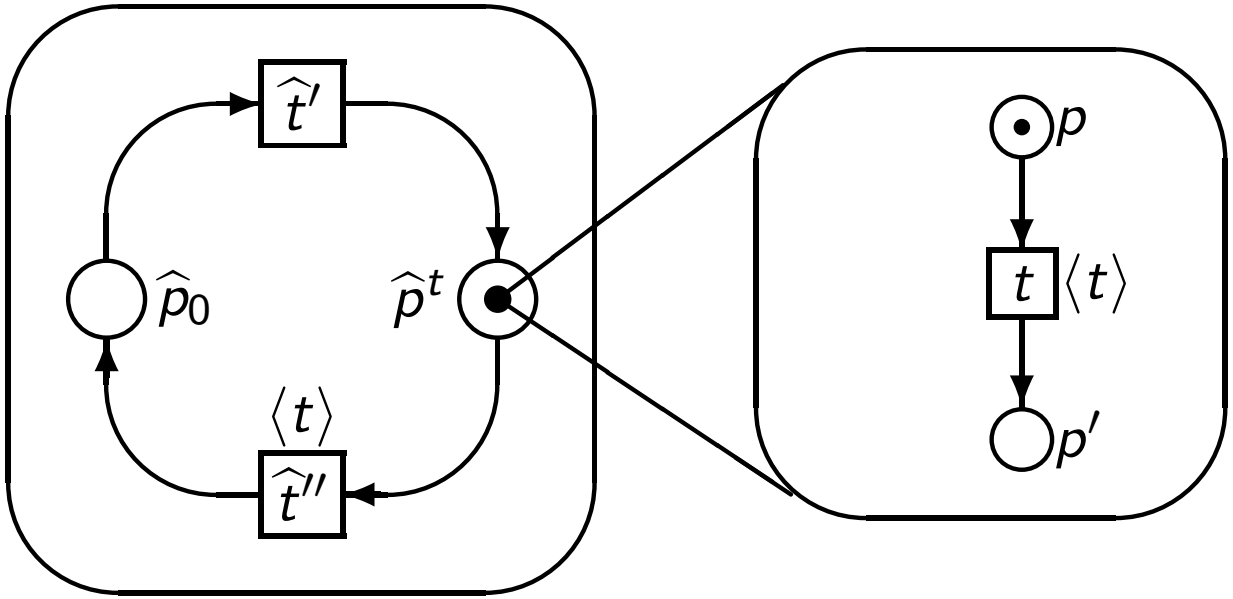
Mobile Eos *can simulate Petri nets with inhibitor arcs.*



Expressiveness

Theorem

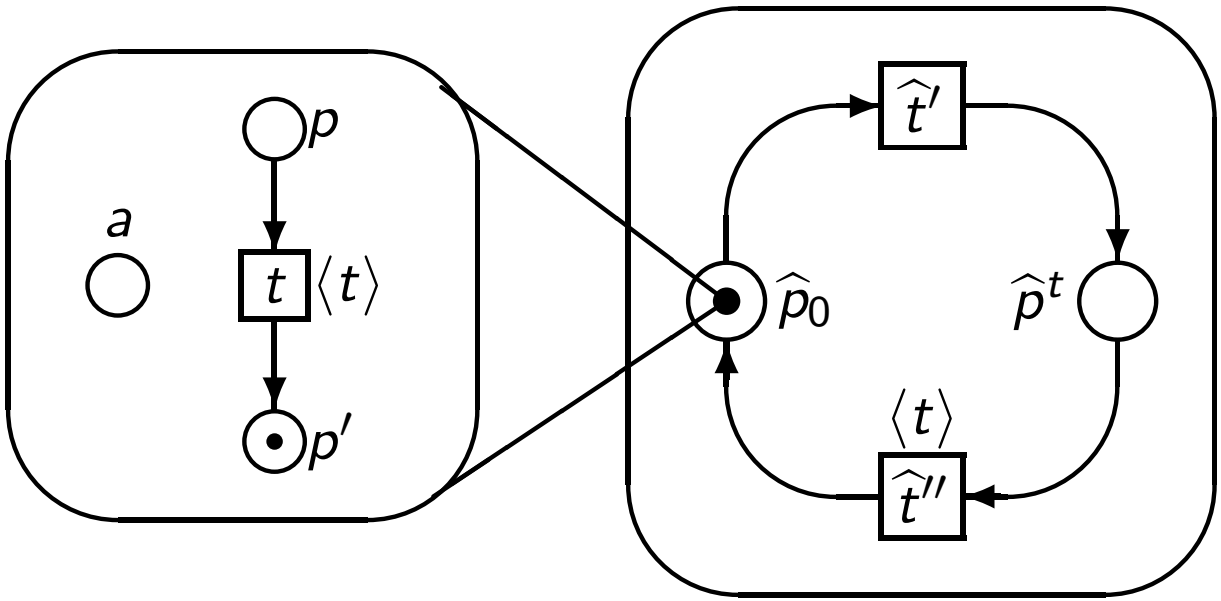
Mobile Eos *can simulate Petri nets with inhibitor arcs.*



Expressiveness

Theorem

Mobile Eos *can simulate Petri nets with inhibitor arcs.*



Expressiveness

Theorem

Mobile Eos *can simulate Petri nets with inhibitor arcs.*

Since boundedness is decidable for Eos and undecidable for inhibitor nets, we have that mobile Eos are more expressive than Eos.

Conclusion

- Object Nets
- Reference Semantics
- Value Semantics
- Here: Mobility Semantics
- Mobile EOS
- Conversion


Michael Köhler, Daniel Moldt, and Heiko Rölke.
Modelling mobility and mobile agents using nets within nets.
In W. v. d. Aalst and E. Best, editors, *International Conference on Application and Theory of Petri Nets 2003*, volume 2679 of *Lecture Notes in Computer Science*, pages 121–140. Springer-Verlag, 2003.


Michael Köhler and Heiko Rölke.
Properties of Object Petri Nets.
In J. Cortadella and W. Reisig, editors, *International Conference on Application and Theory of Petri Nets 2004*, volume 3099 of *Lecture Notes in Computer Science*, pages 278–297. Springer-Verlag, 2004.


Michael Köhler and Heiko Rölke.
Reference and value semantics are equivalent for ordinary Object Petri Nets.

In P. Darondeau and G. Ciardo, editors, *International Conference on Application and Theory of Petri Nets 2005*, volume 3536 of *Lecture Notes in Computer Science*, pages 309–328. Springer-Verlag, 2005.