

ON THE STRUCTURE OF CYCLOIDS

INTRODUCED BY CARL ADAM PETRI

RÜDIGER VALK

UNIVERSITY OF HAMBURG

PETRI NETS 2018

BRATISLAVA, SLOVAKIA, JUNE 29, 2018

Contents:

1. Introduction and Examples
2. From Petri-Space to Cycloids (informal)
3. Cycloids $\mathcal{C}(\alpha, \beta, \gamma, \delta)$ (formal)
4. The Initial Parameters α and β (initial marking)
5. The Final Parameters γ and δ (minimal cycles)
6. Computing Cycloid Parameters α , β , γ and δ
7. Outlook

On the Physical Basics of Information Flow

C.A.Petri

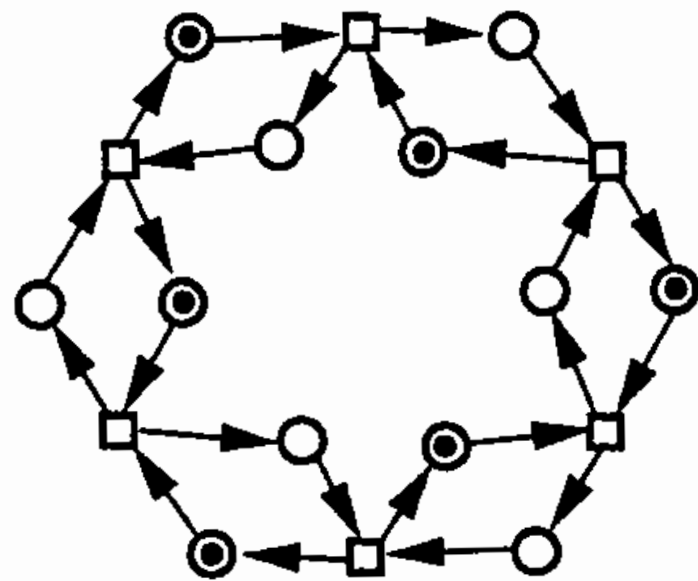
(modified and presented by Rüdiger Valk)

Petri Nets 2008 Xian /China



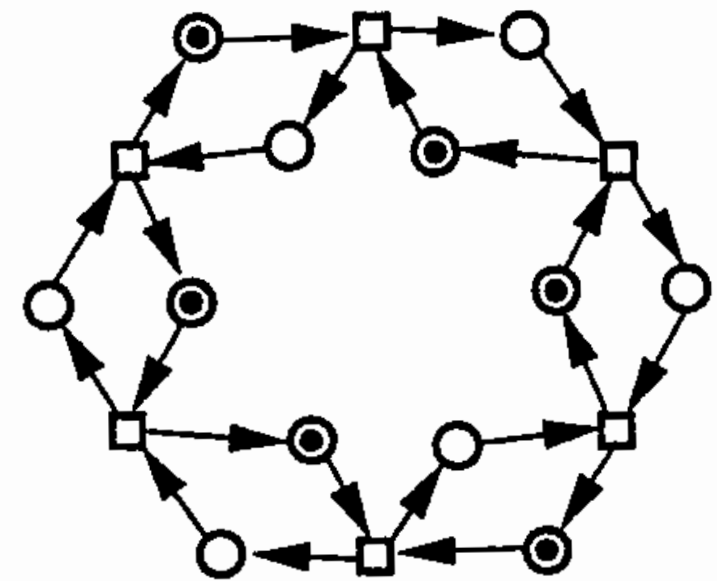
copyrights C.A. Petri, R. Valk 2008

faster



$(3,3,1,1)$ – Slowness = 0

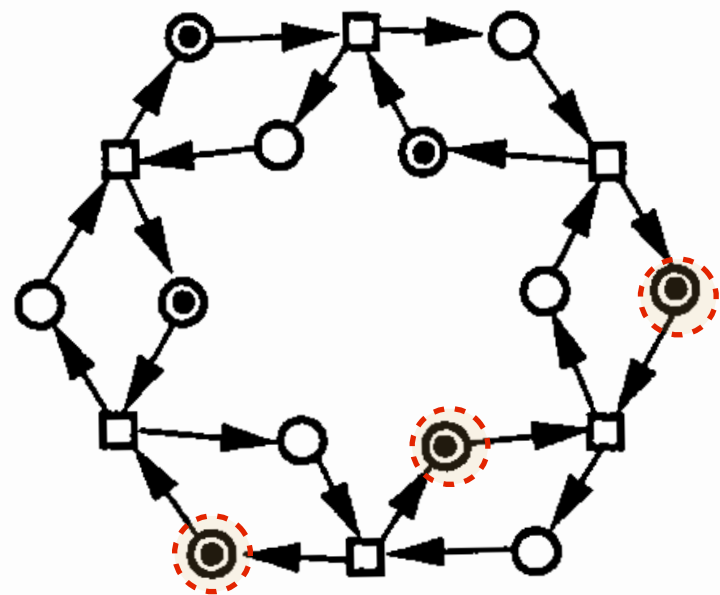
slower



$(4,2,1,1)$ – Slowness = $1/3$

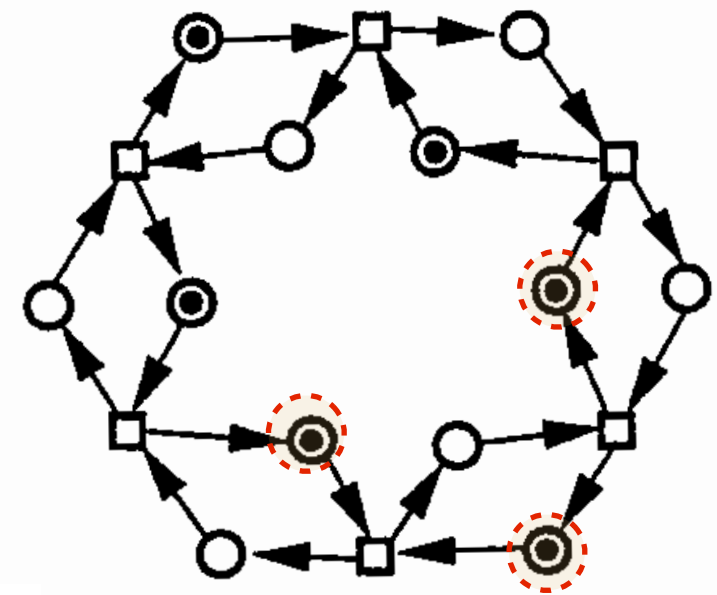
M.-O. Stehr: System Specification by Cyclic Causality Constraints.
Bericht 210, FB Informatik, Univ. Hamburg, 1998

faster



$(3,3,1,1)$ – Slowness = 0

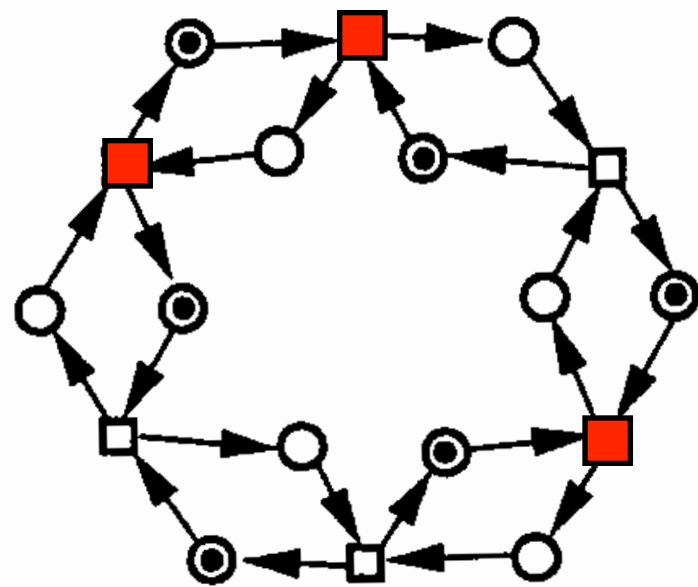
slower



$(4,2,1,1)$ – Slowness = $1/3$

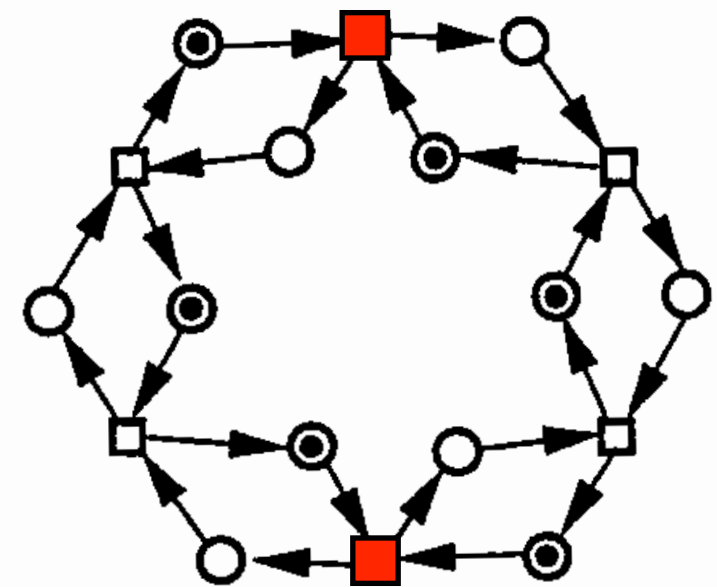
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faster



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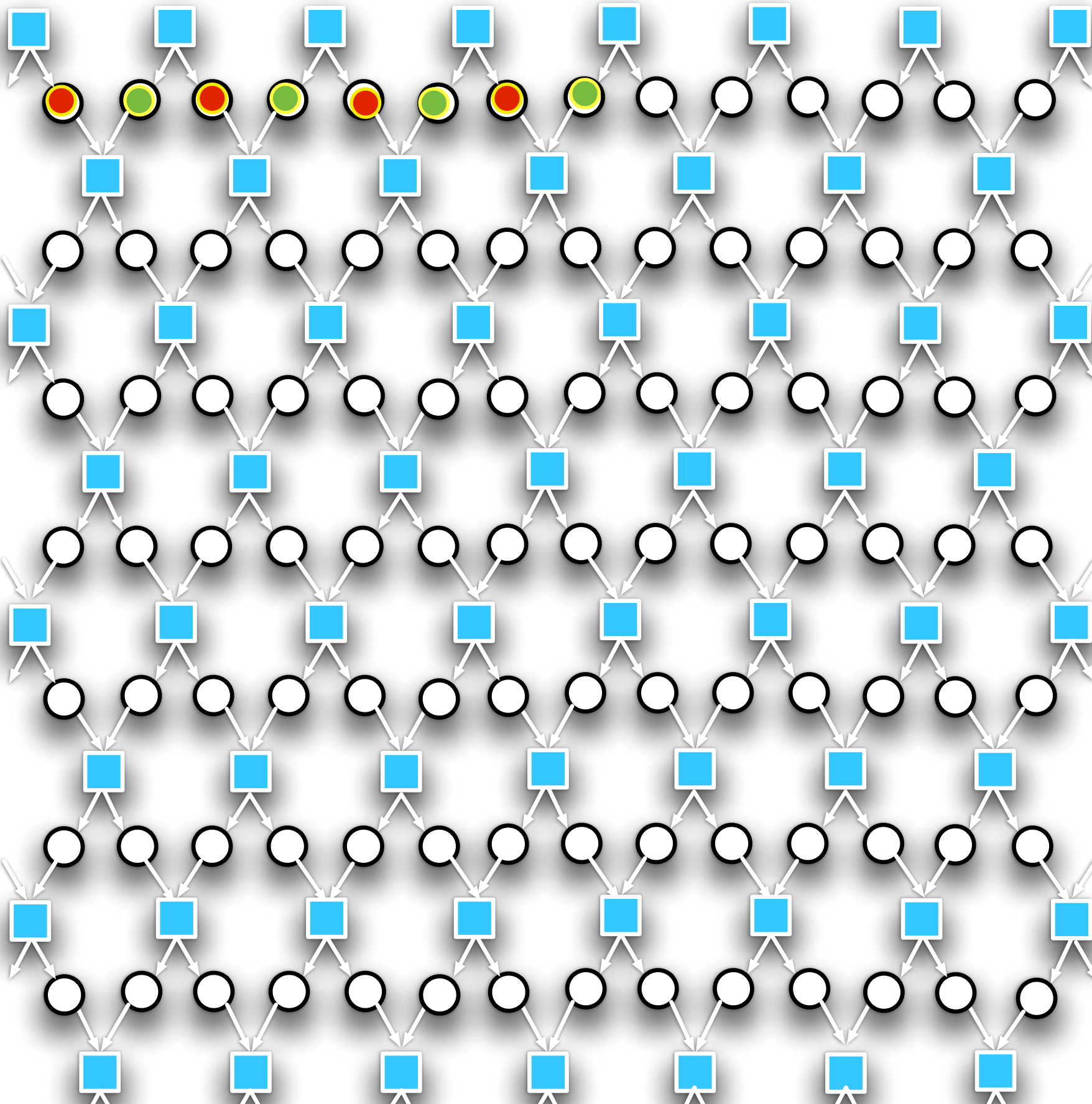


cars



gap item





cars



gap item

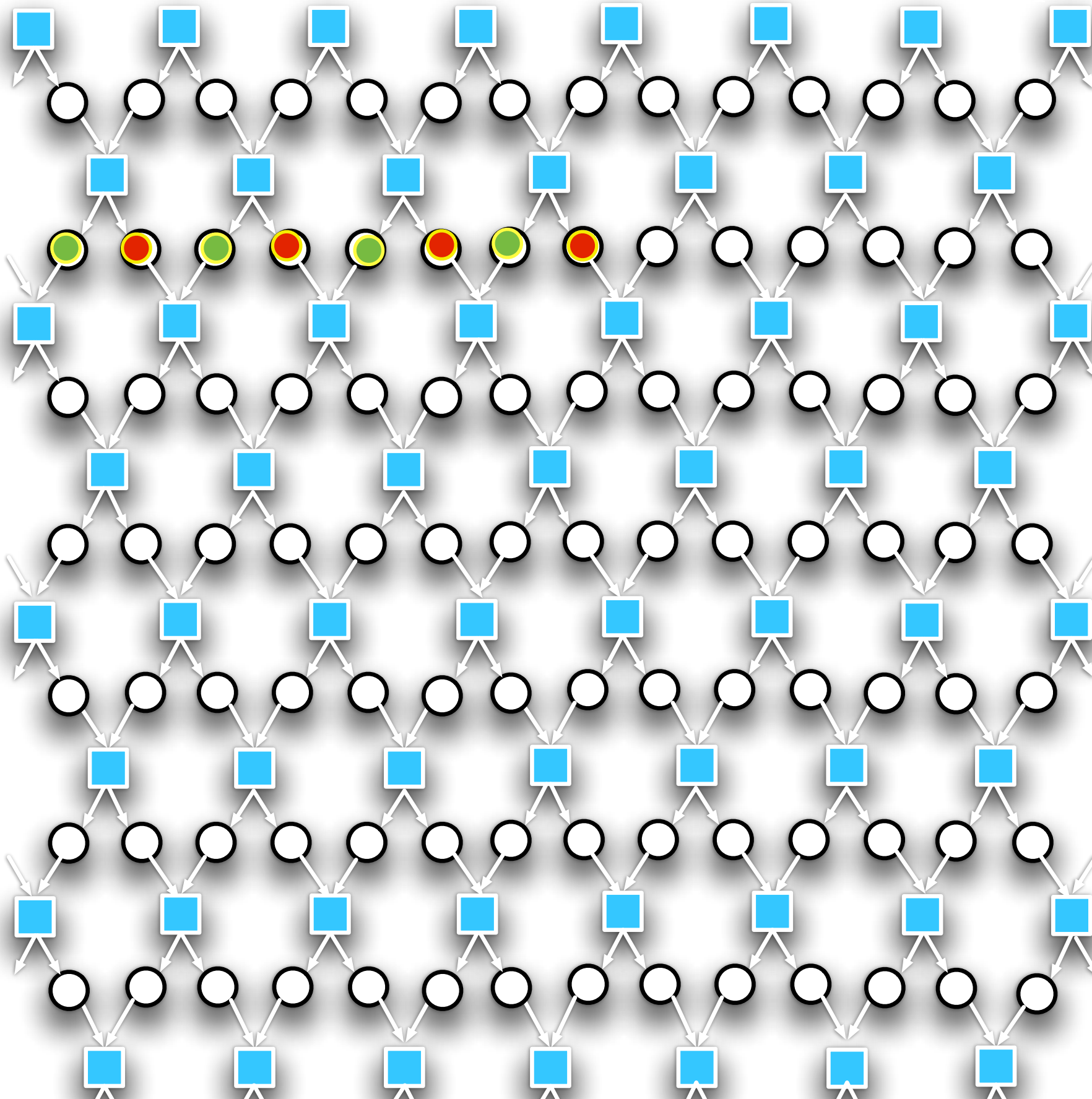




cars



gap item

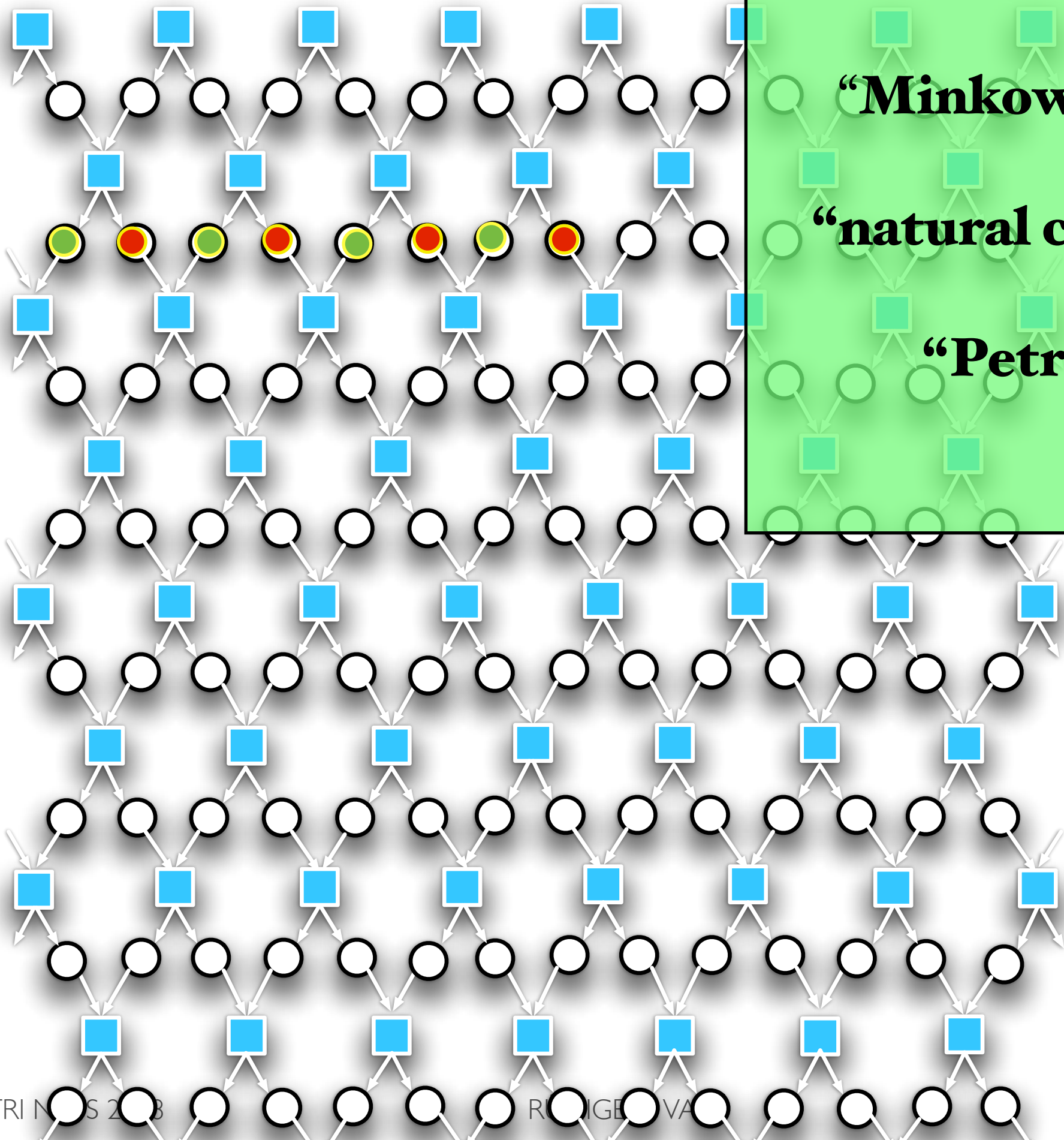




cars



gap item



“Minkowski Space”

“natural coordinates”

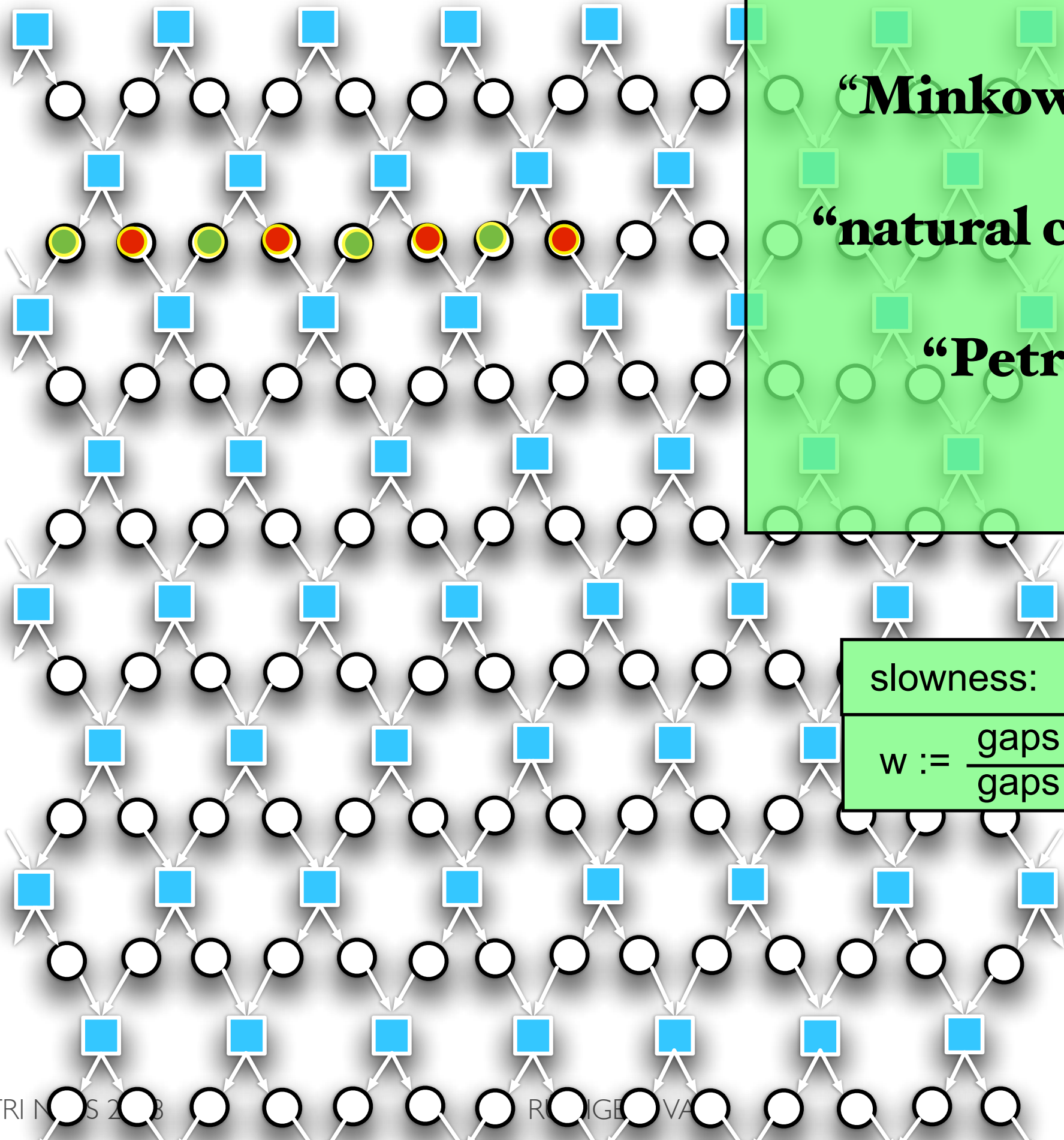
“Petri space”



cars



gap item



“Minkowski Space”

“natural coordinates”

“Petri space”

slowness:

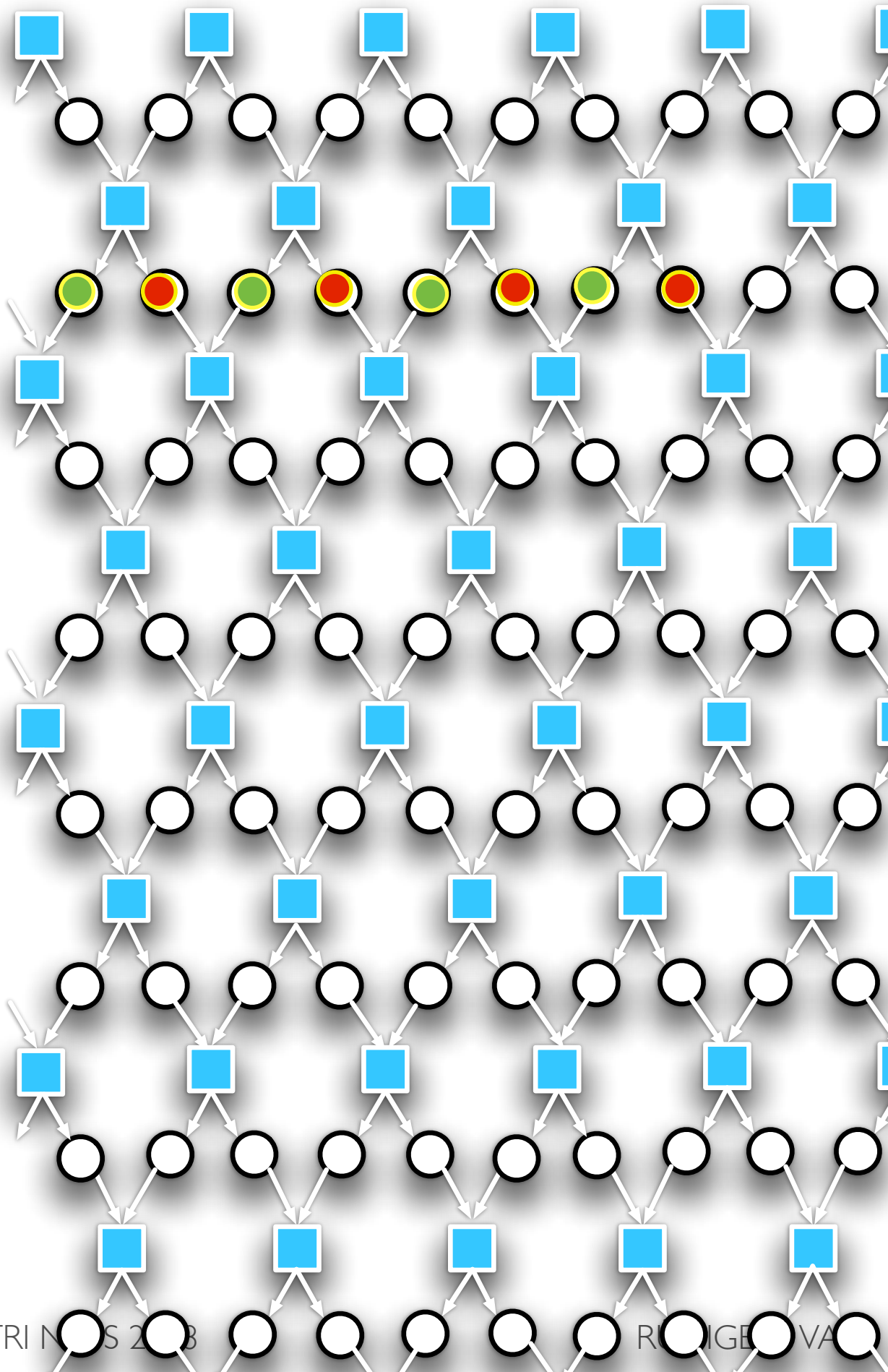
$$w := \frac{\text{gaps} - \text{cars}}{\text{gaps} + \text{cars}}$$



cars



gap item



“Minkowski Space”

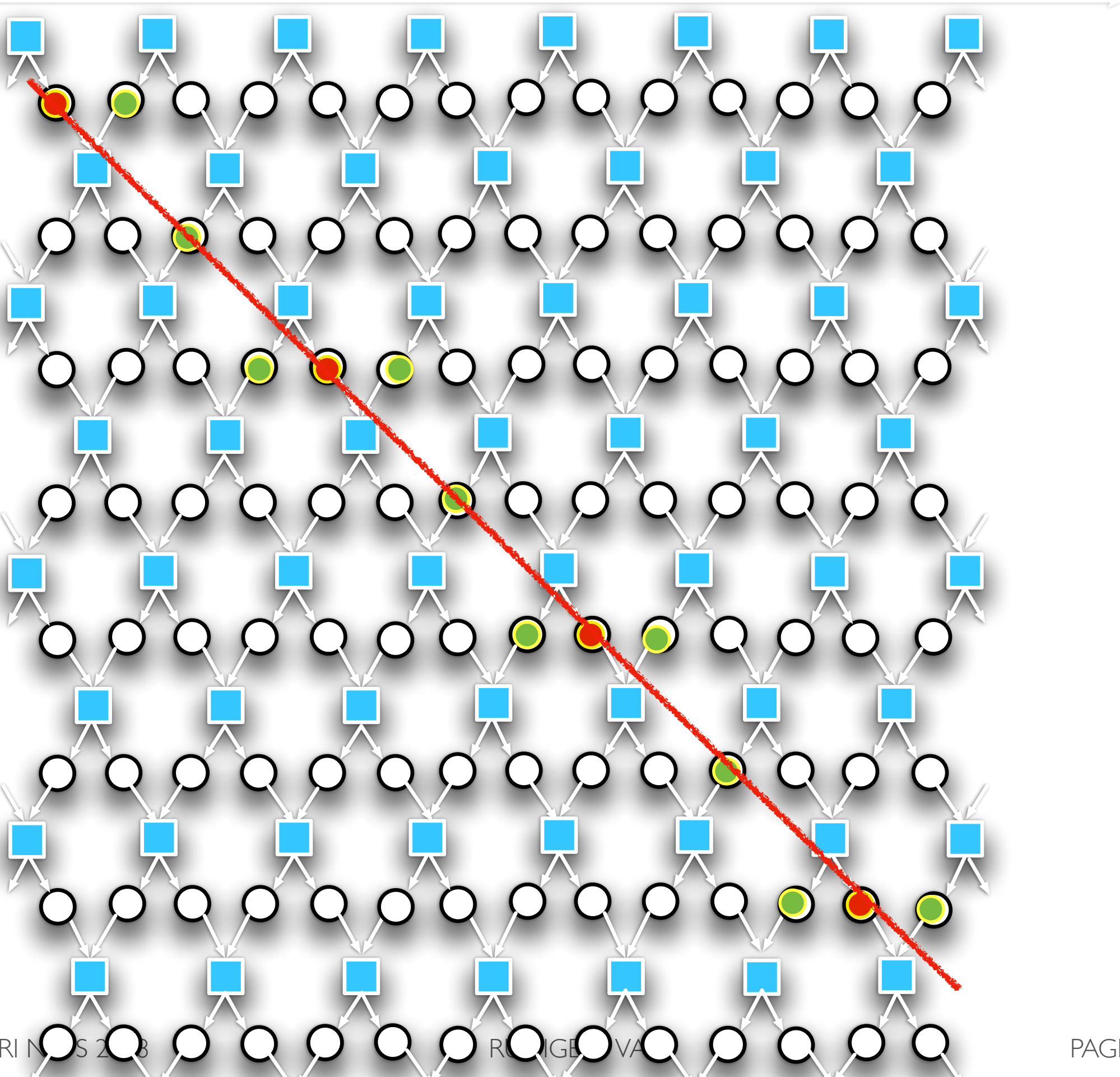
“natural coordinates”

“Petri space”

slowness:

$$w := \frac{\text{gaps} - \text{cars}}{\text{gaps} + \text{cars}}$$

$$w := \frac{4 - 4}{4 + 4} = 0 \text{ sec/m}$$

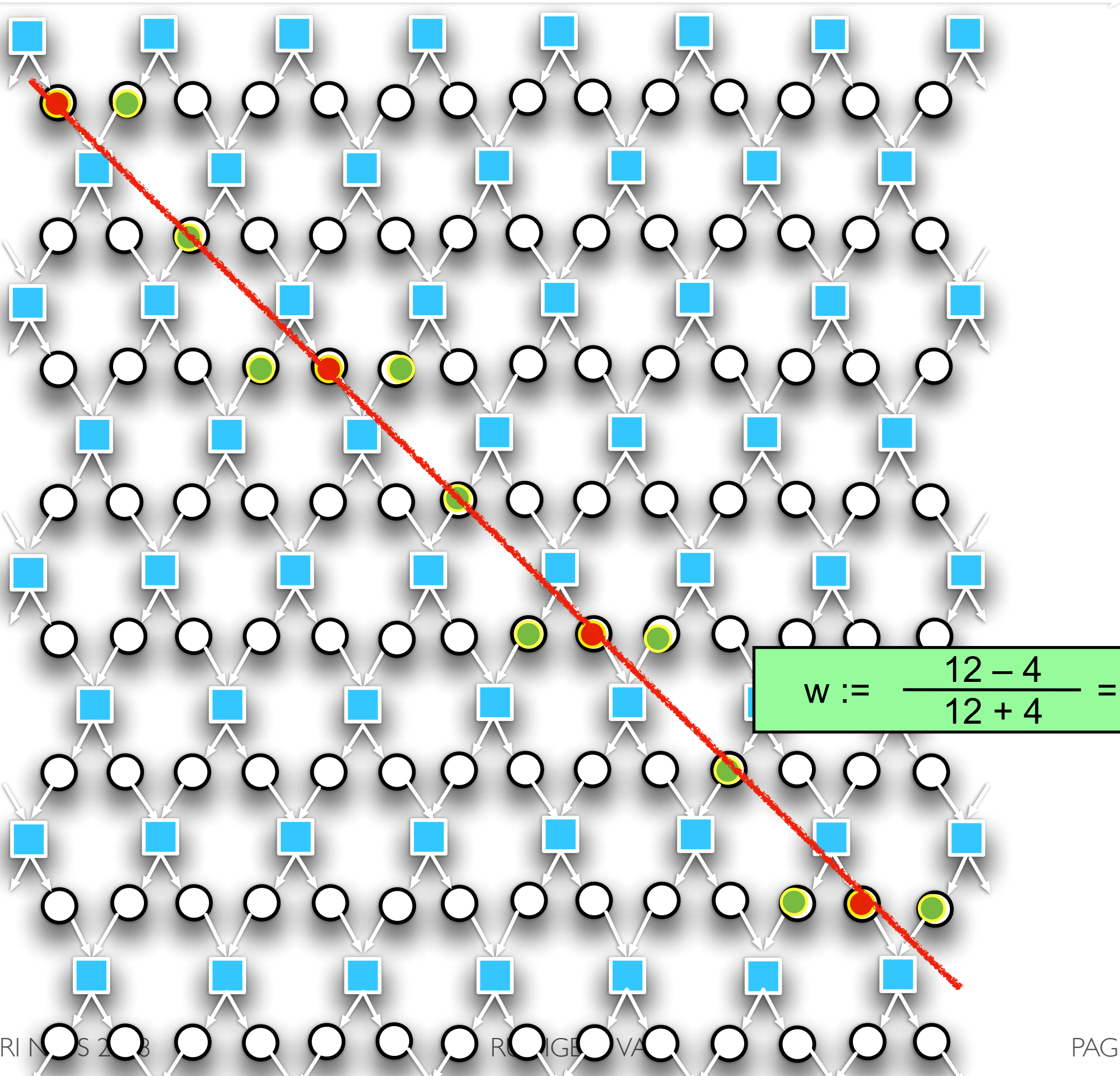


cars



gap item





cars



gap item



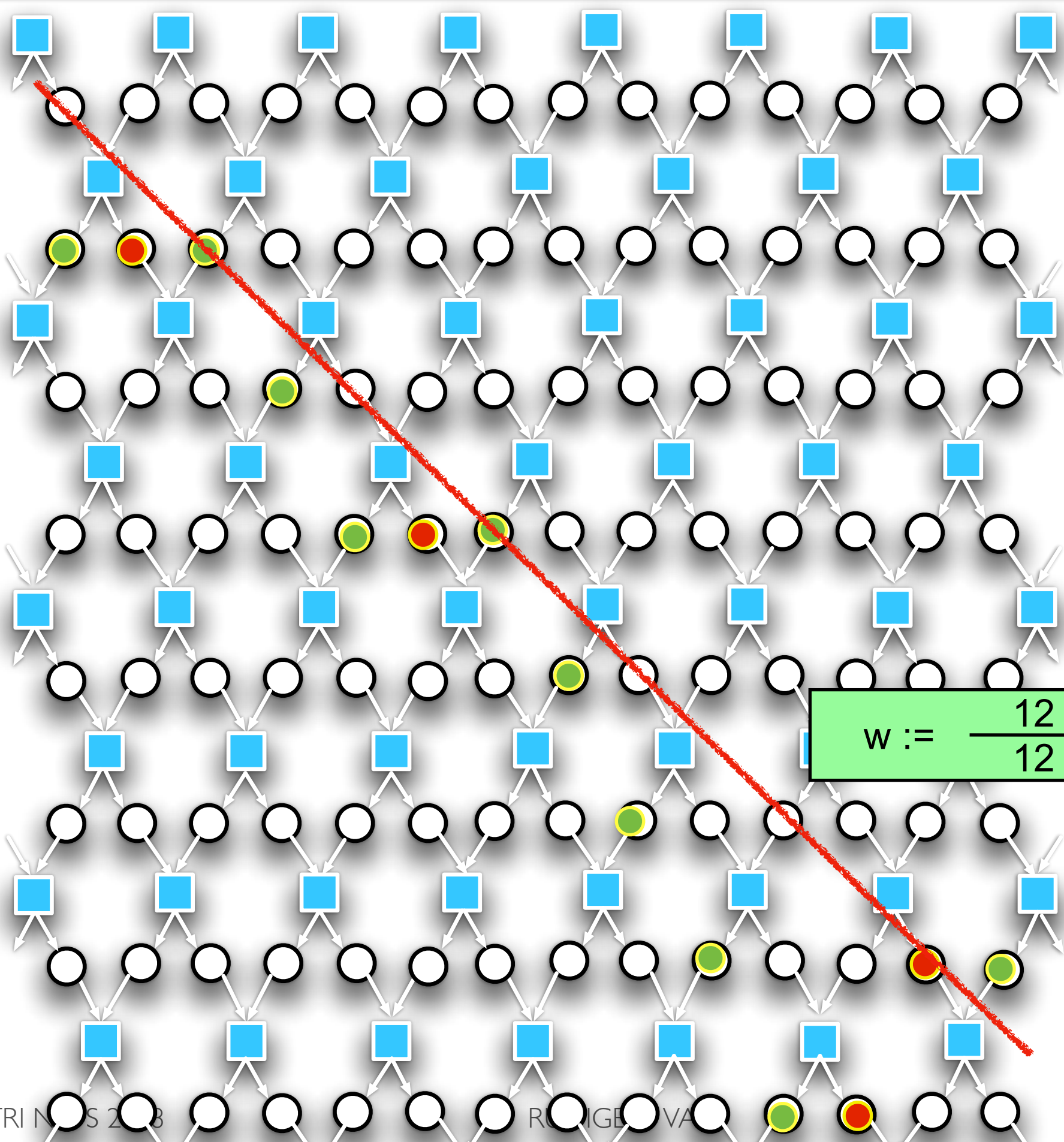
$$w := \frac{12 - 4}{12 + 4} = 1/3 \text{ sec/m}$$



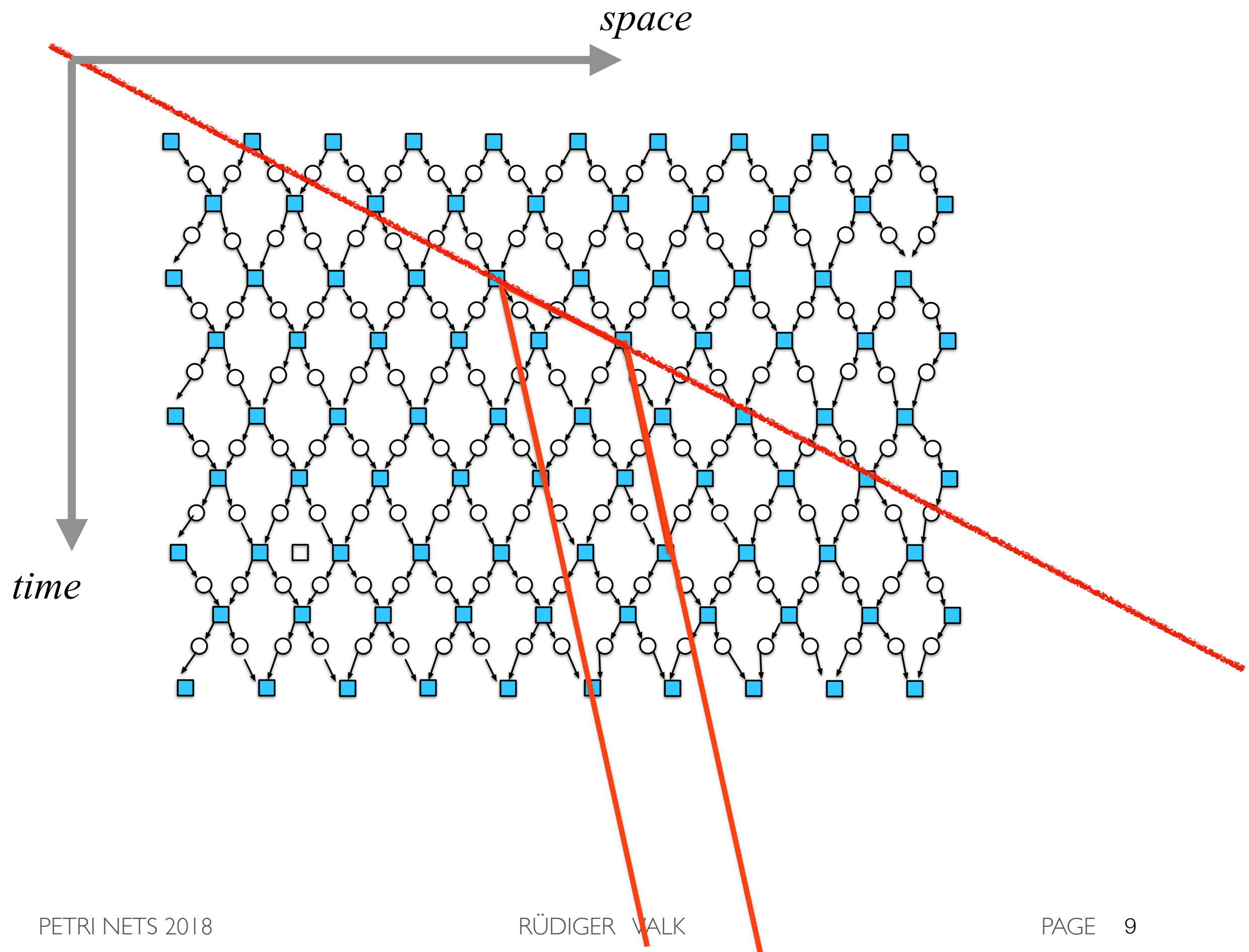
cars

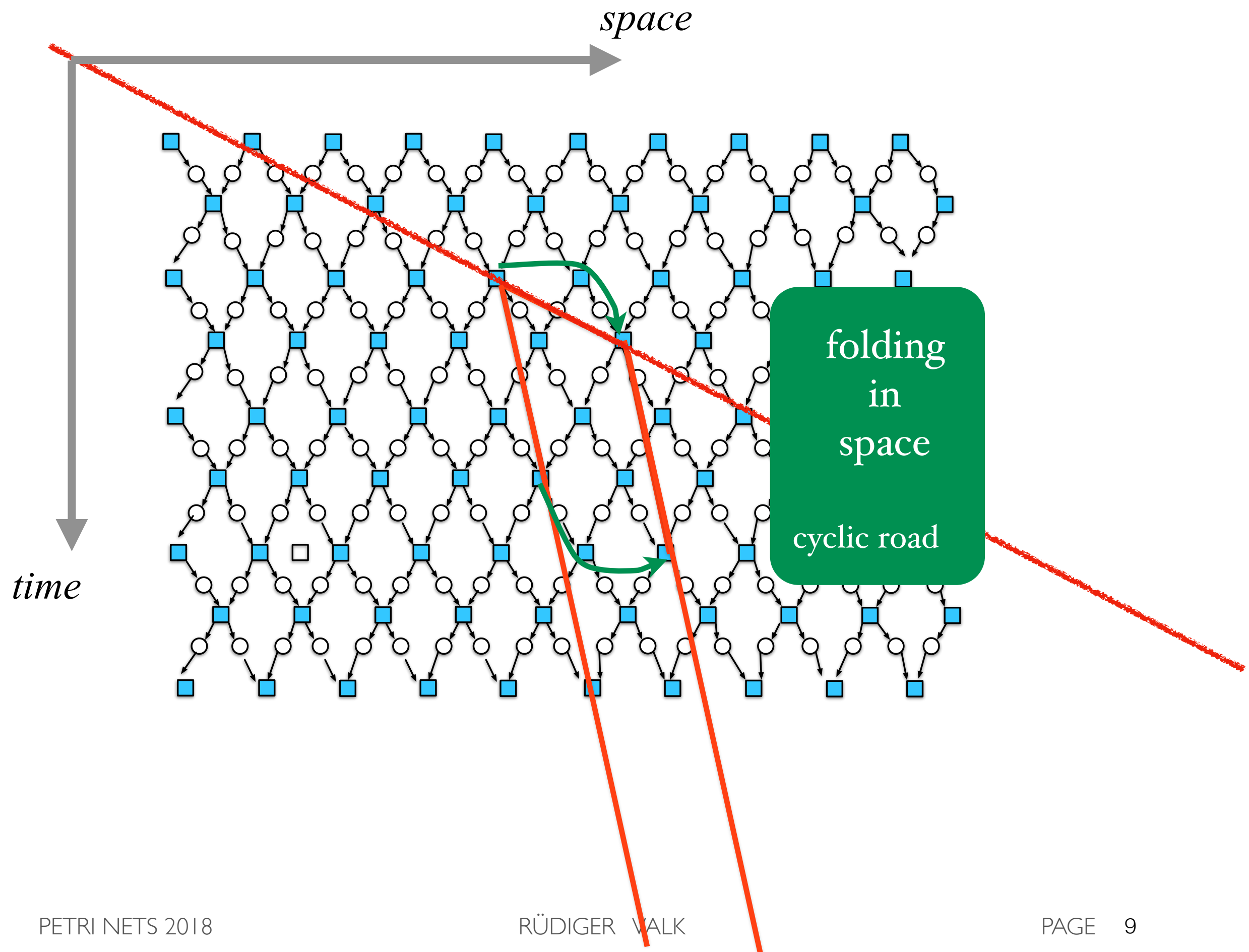


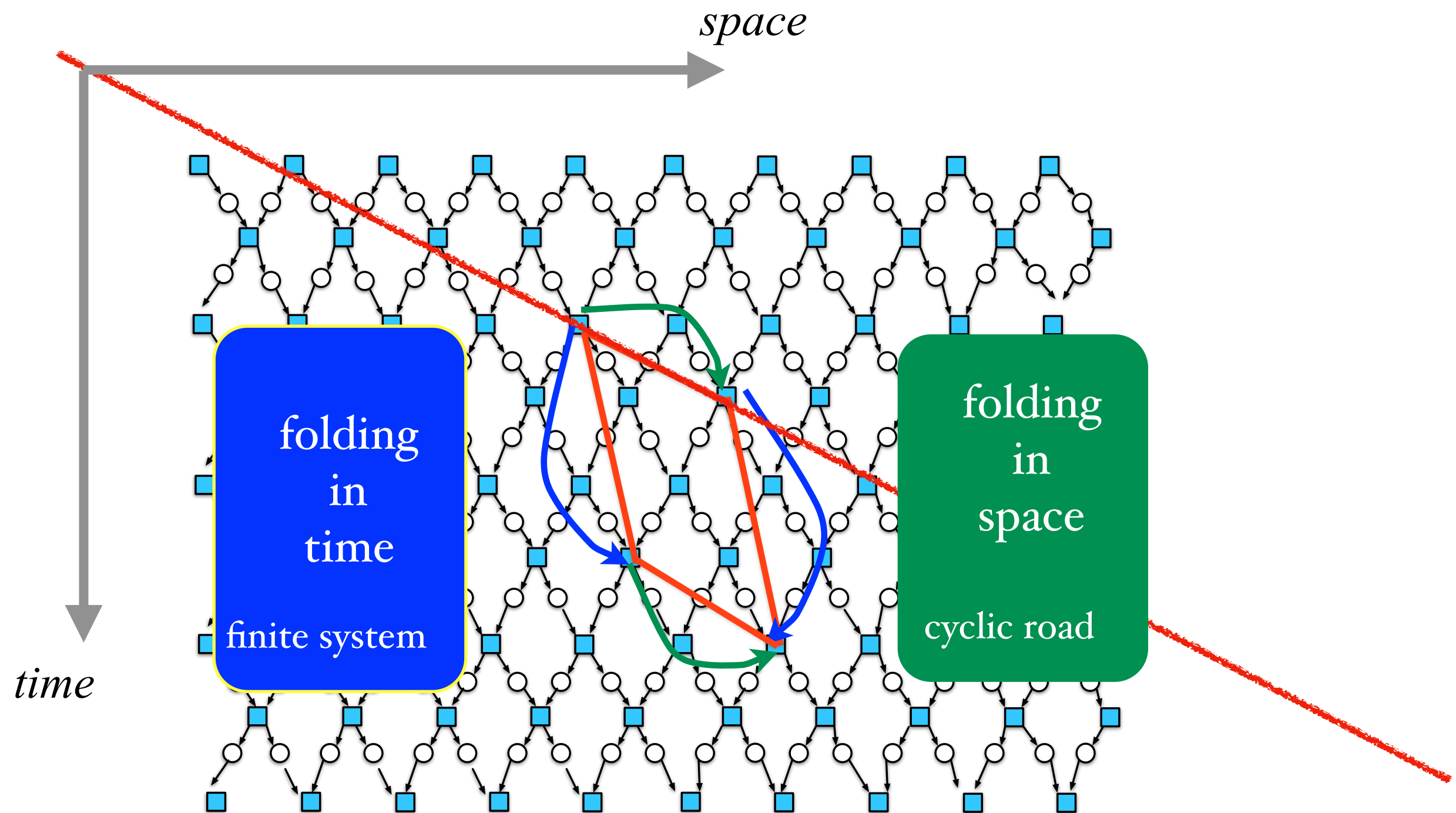
gap item

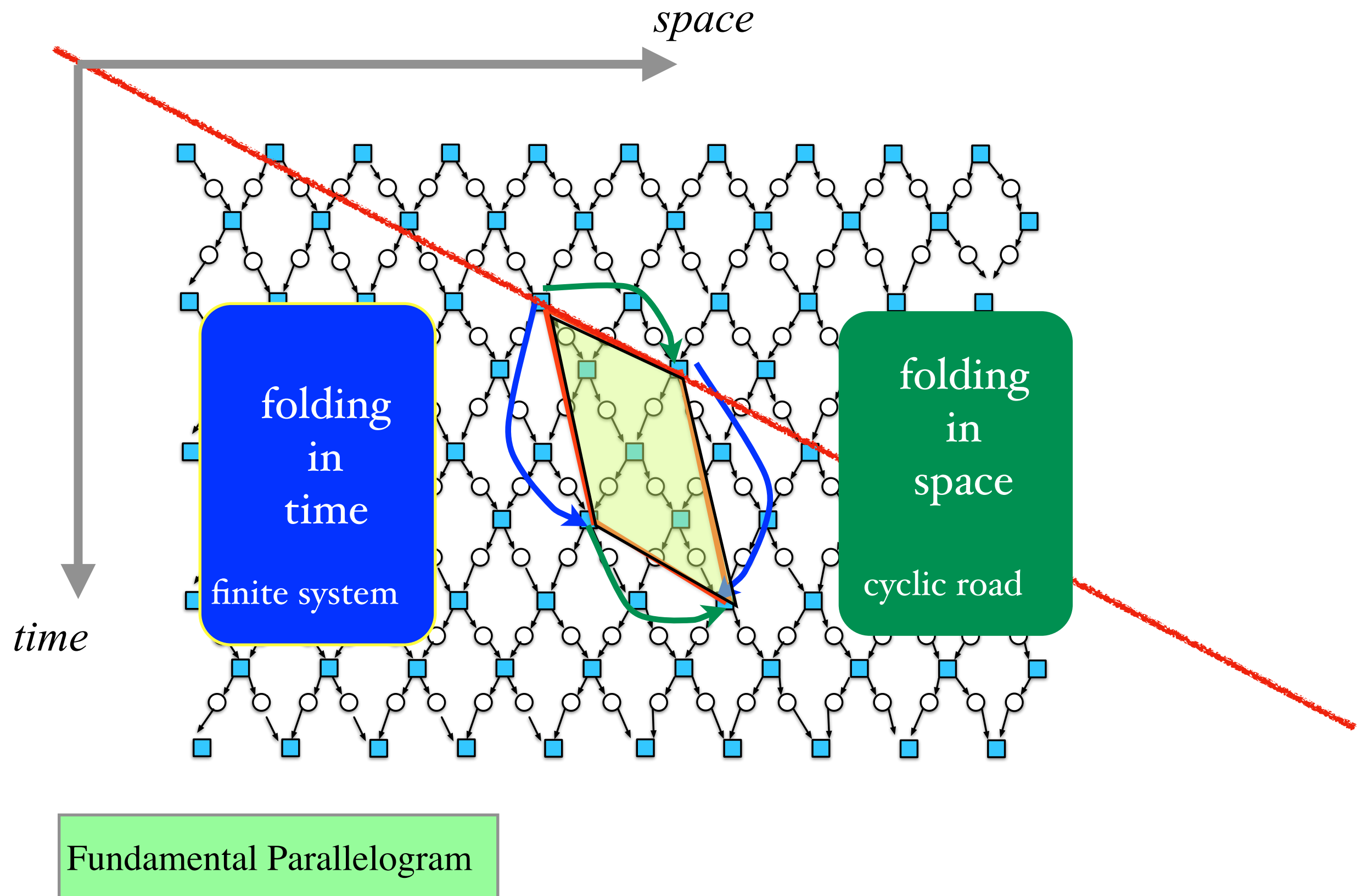


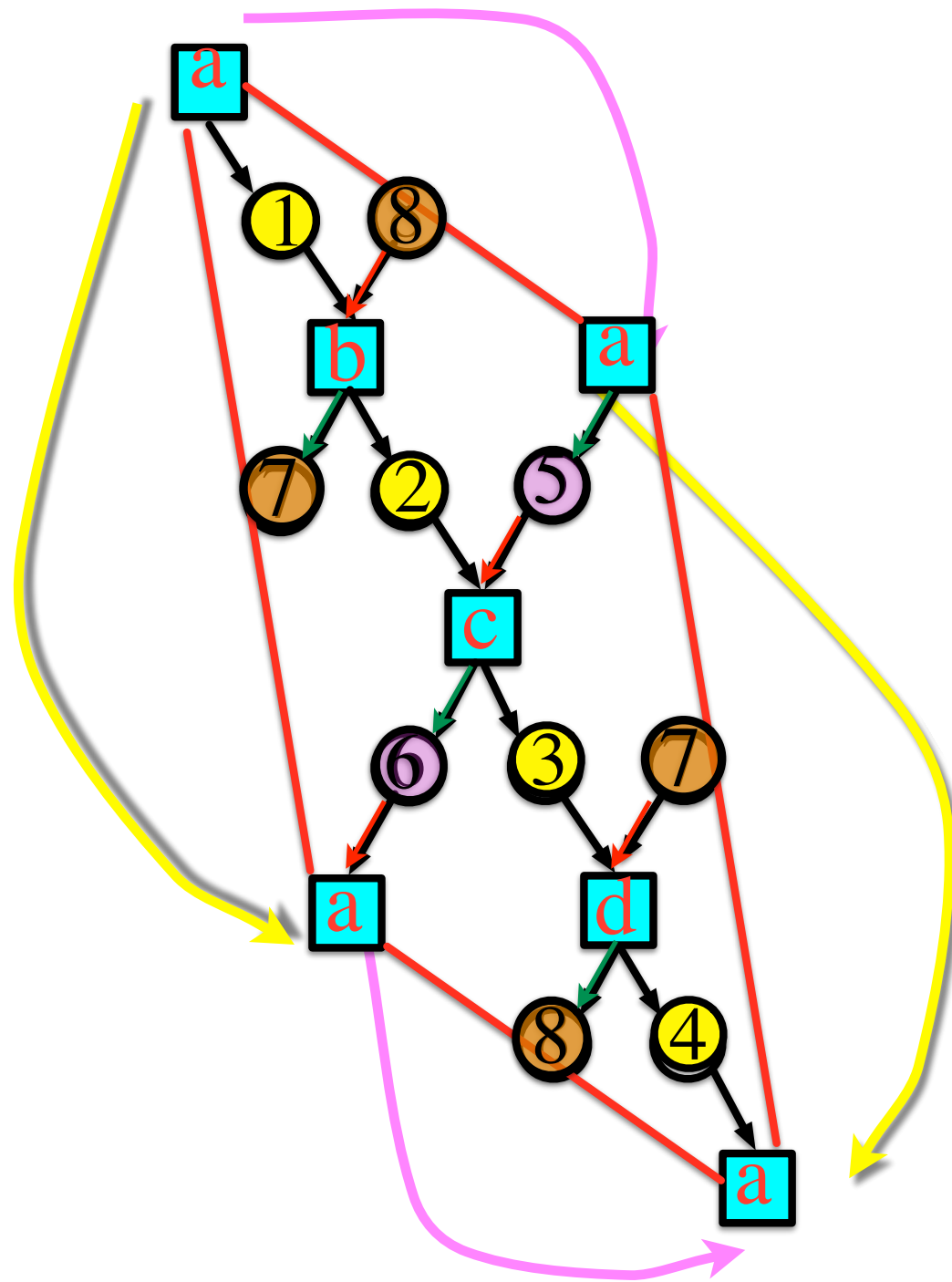
$$w := \frac{12 - 4}{12 + 4} = 1/3 \text{ sec/m}$$



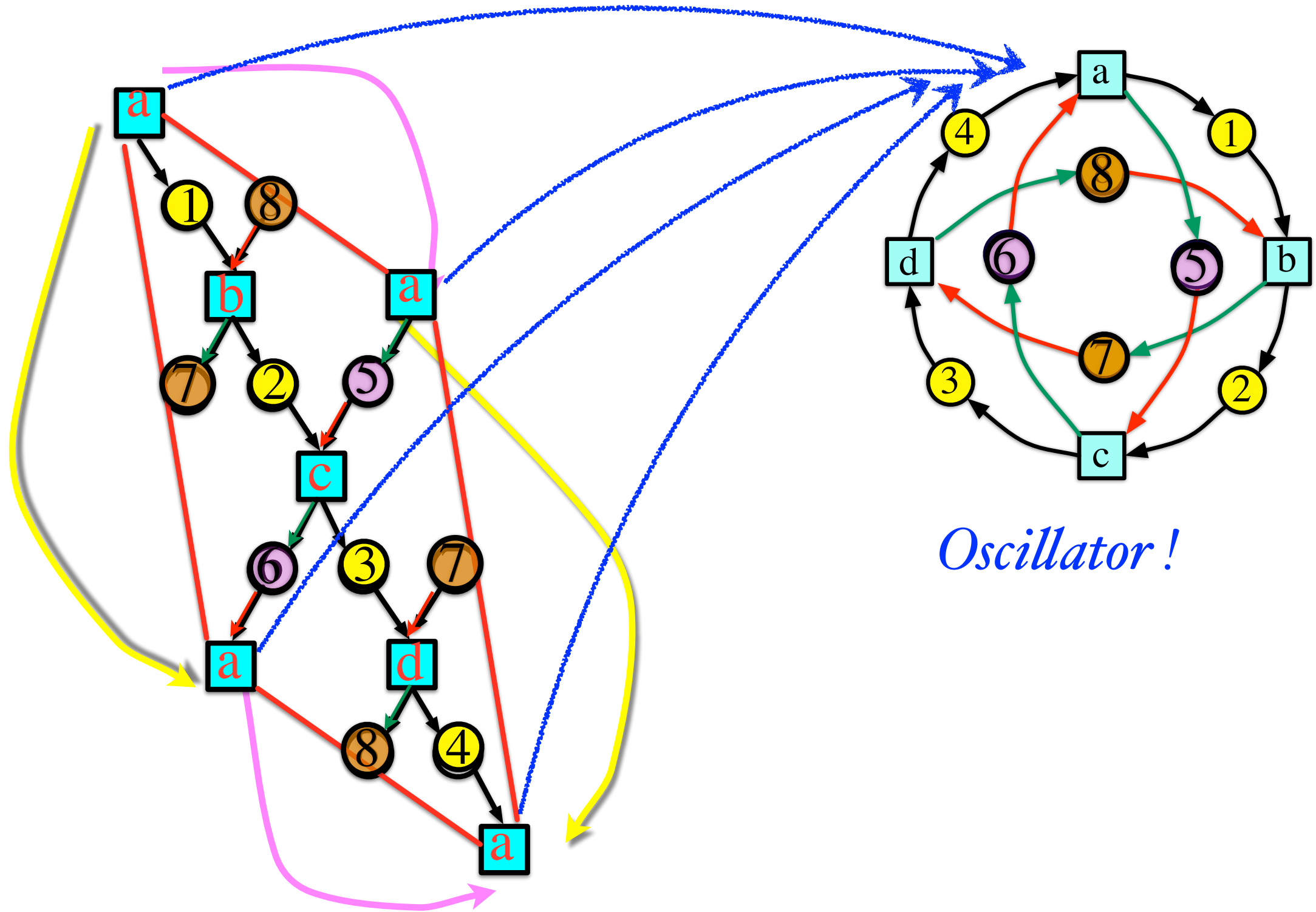




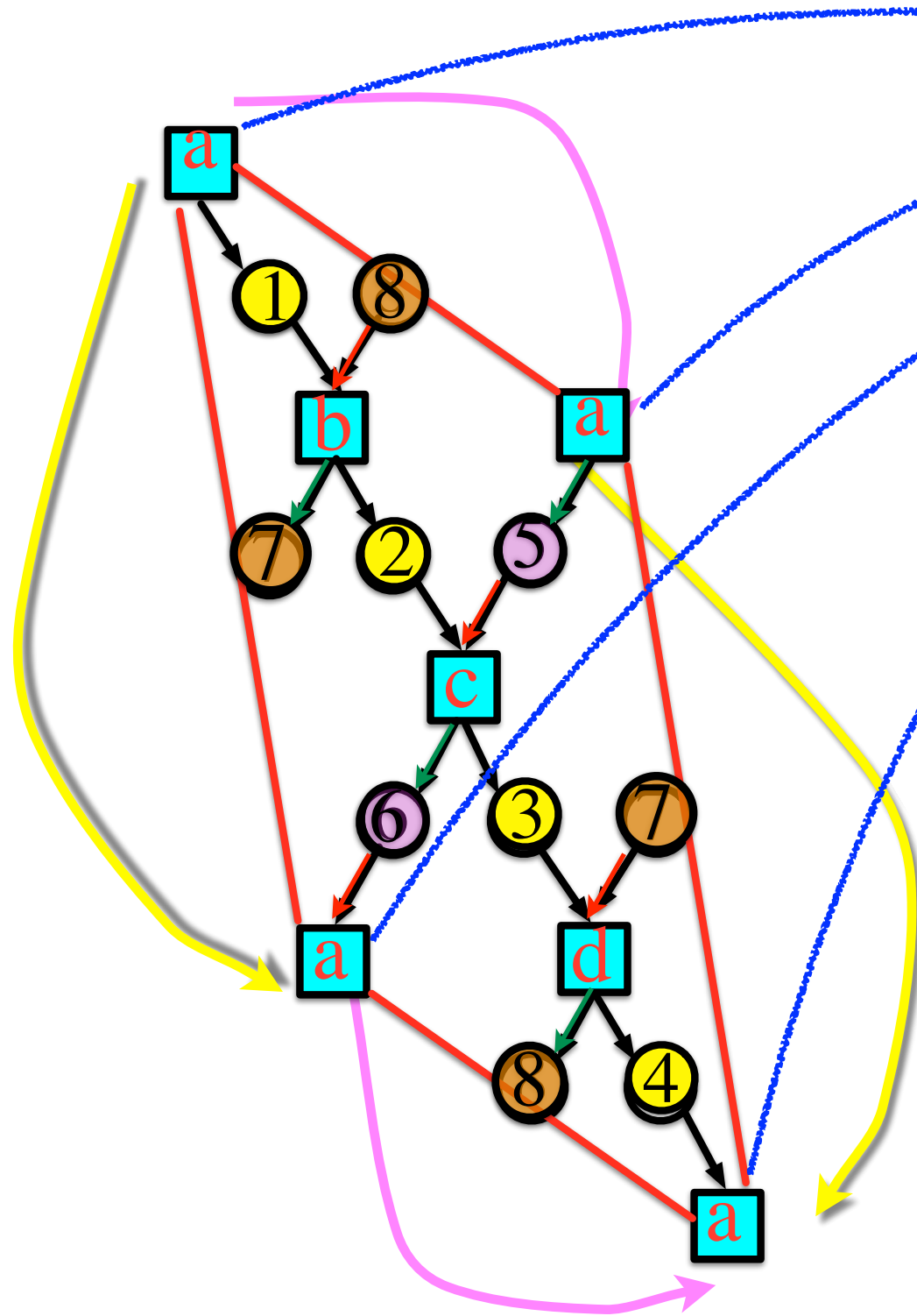




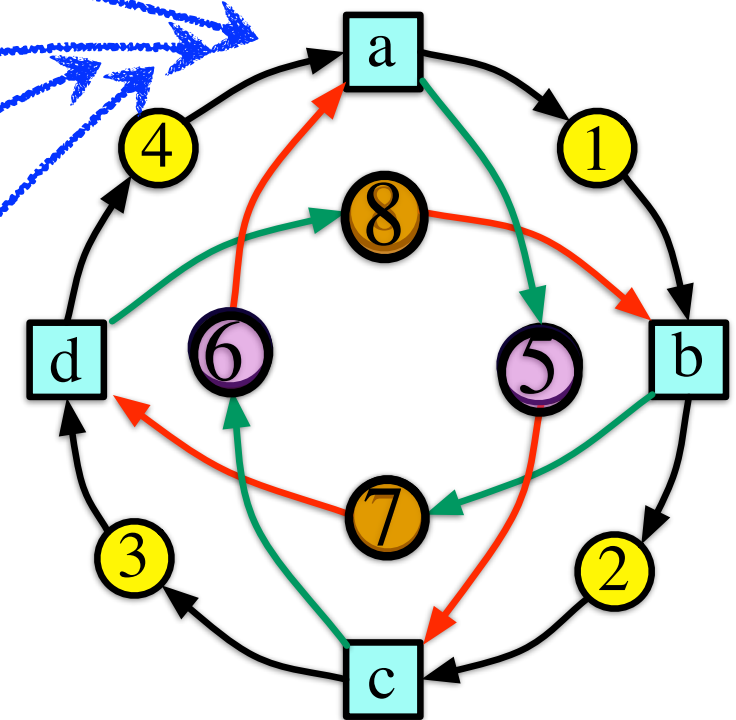
Fundamental Parallelogram



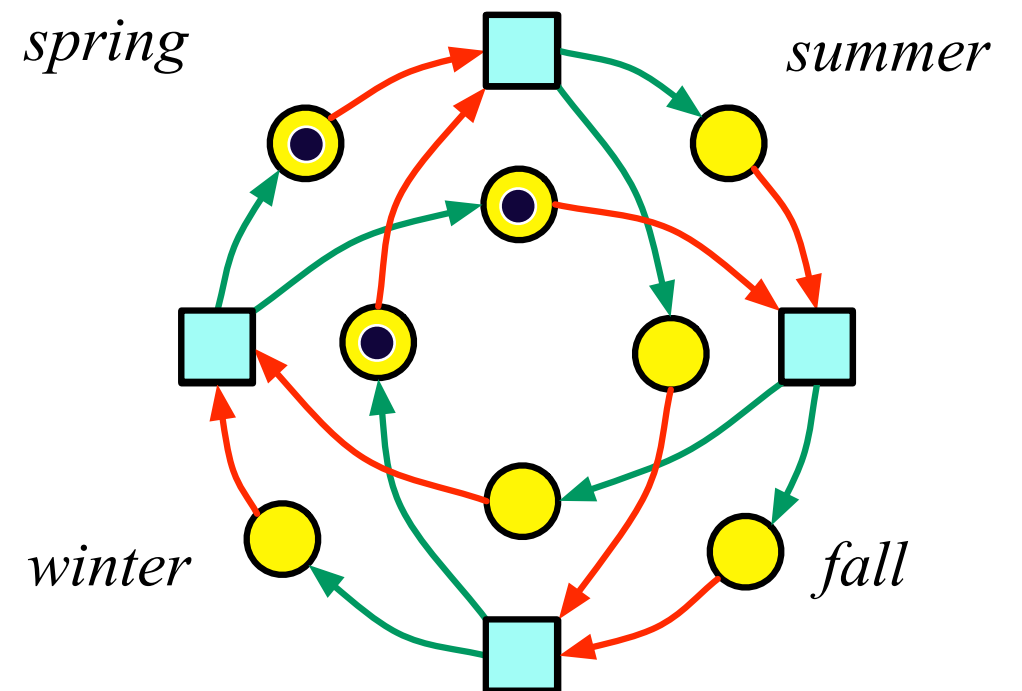
Fundamental Parallelogram



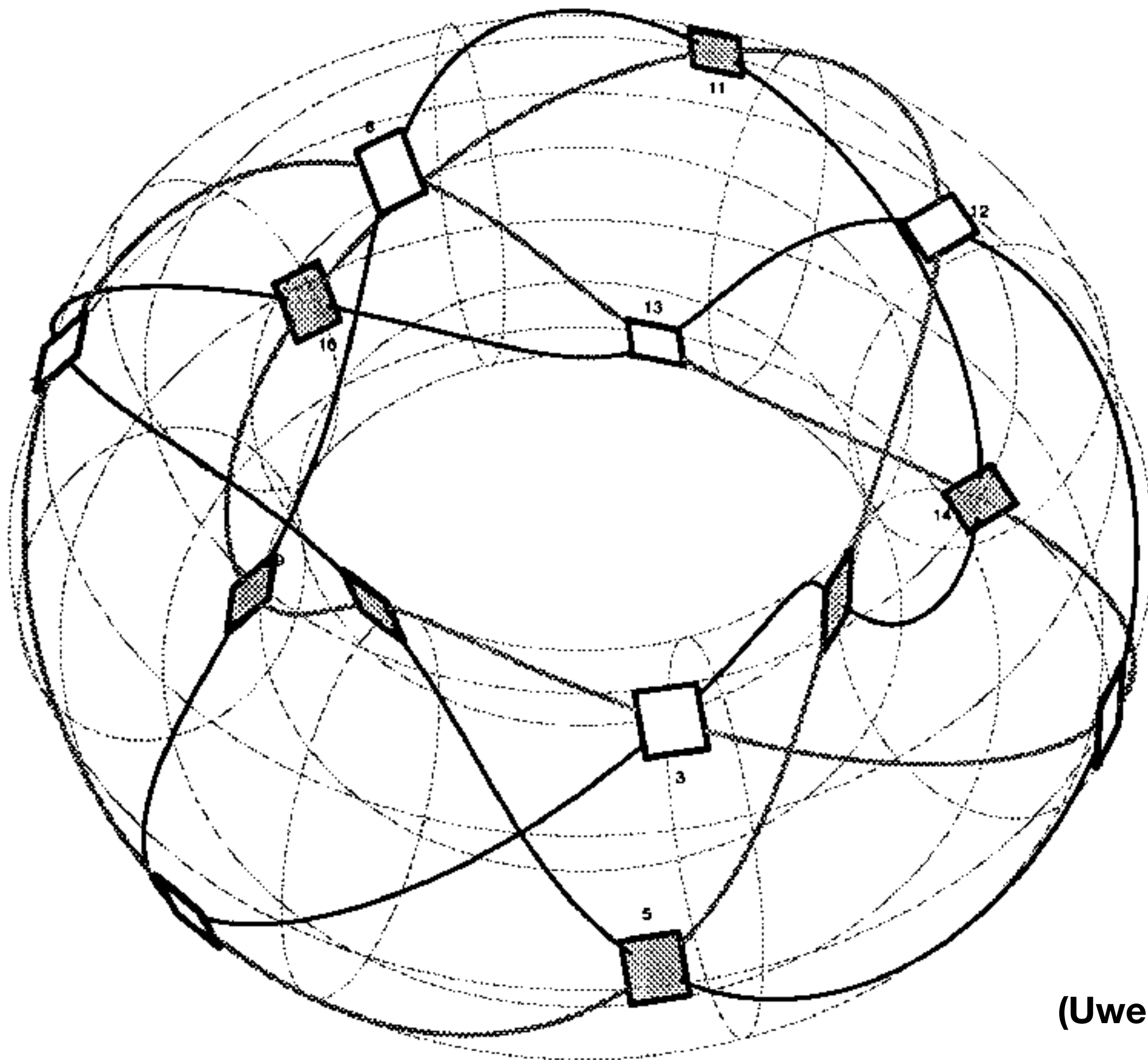
Fundamental Parallelogram



Oscillator !



4 Seasons



(Uwe Fenske)

Cycloids allow to describe the Lorentz-Transformation from classical physics to Special Relativity Theory.

see:

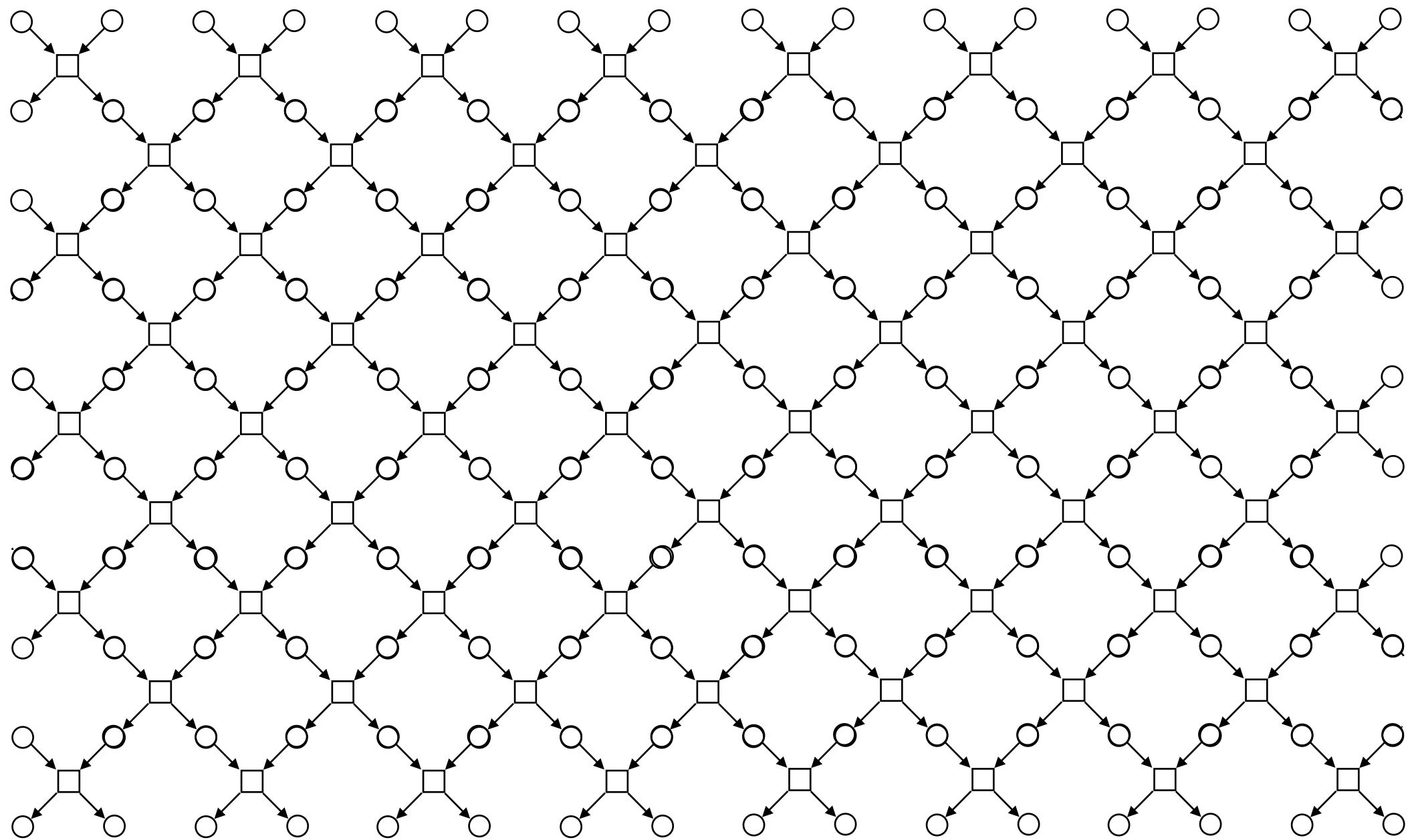
Petri, C.A.: Nets, Time and Space. Theor. Comput. Sci. 153, 3–48 (1996)

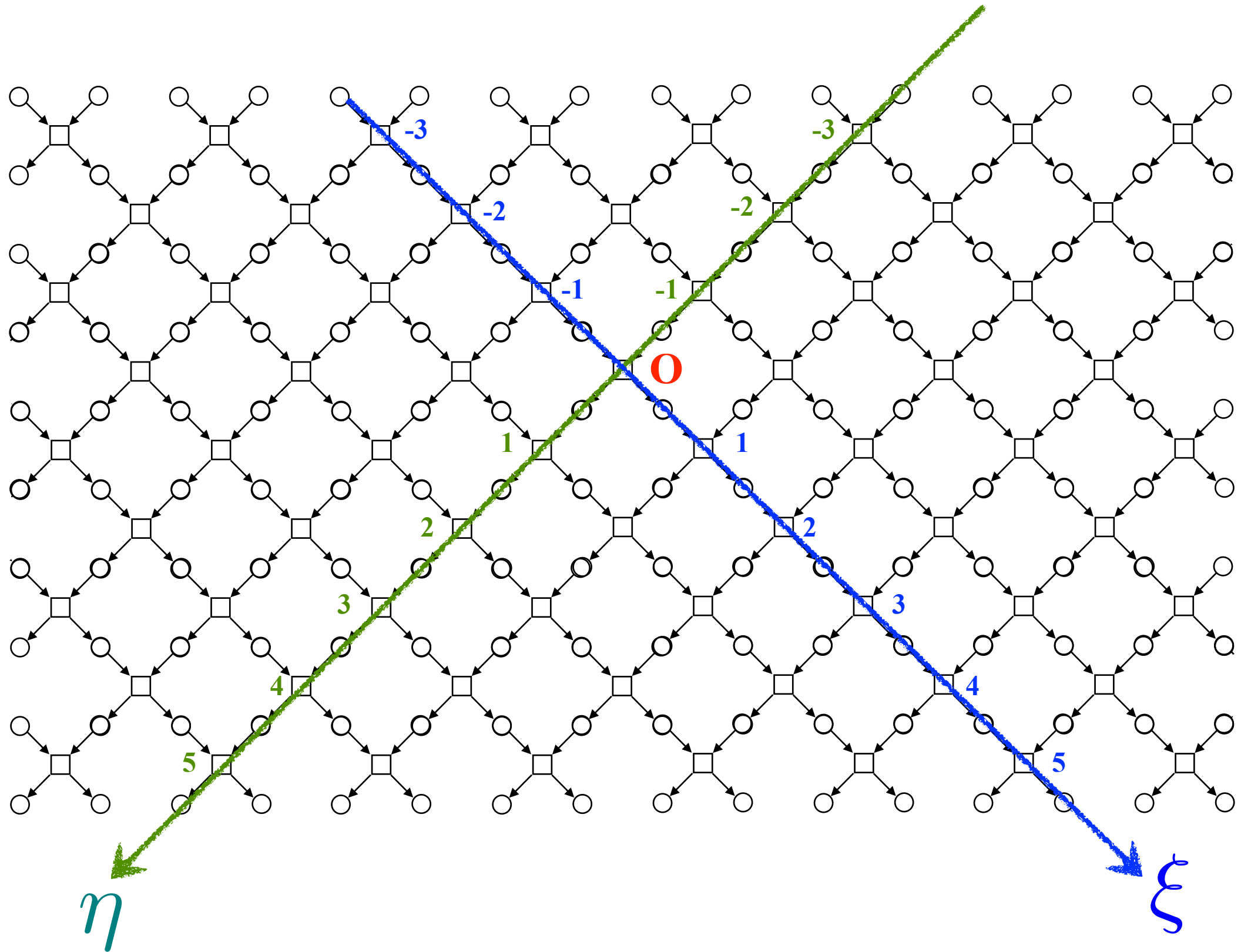
and:

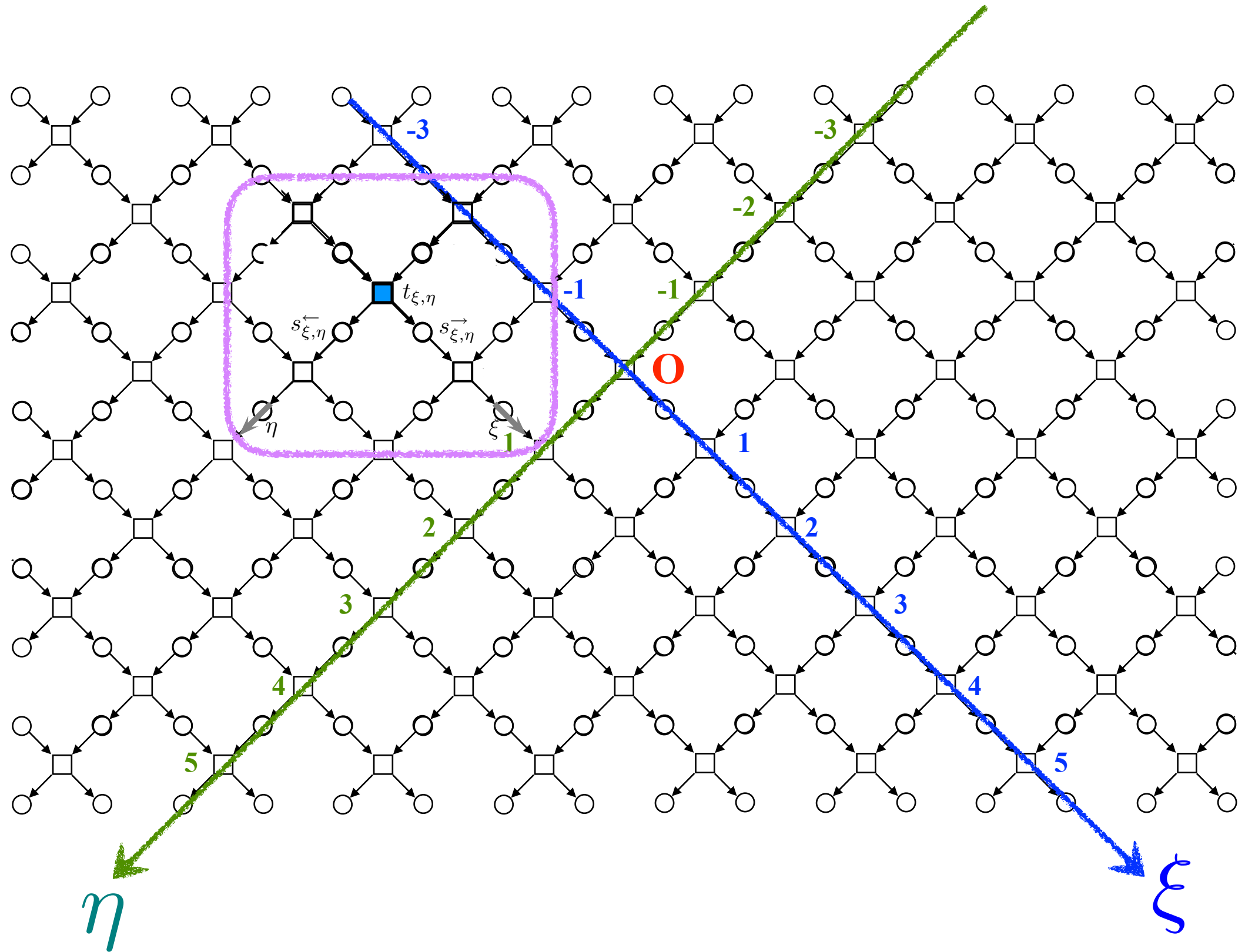
Advanced tutorial on
NETS, PHYSICS AND COORDINATION, PETRI NETS 2013, Milano
slides on my home page

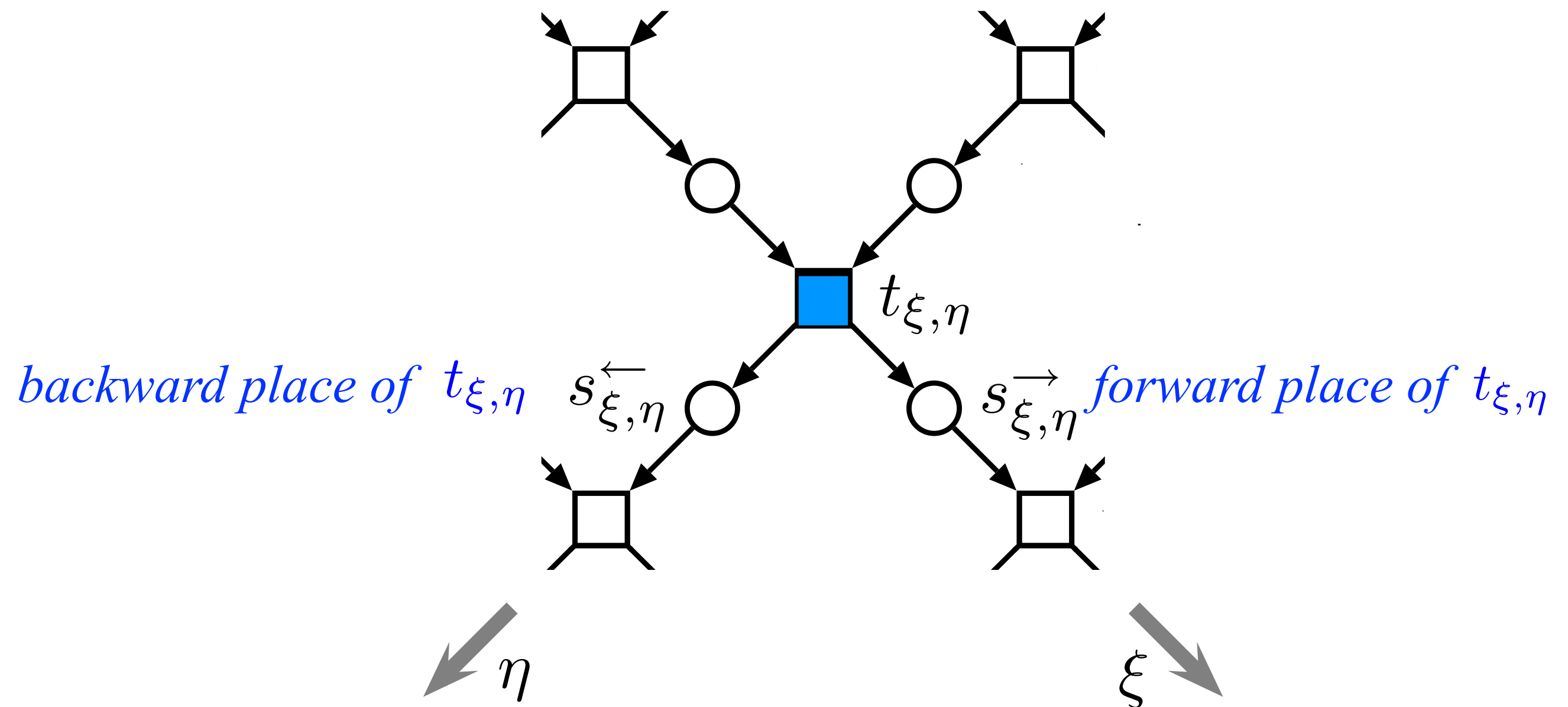
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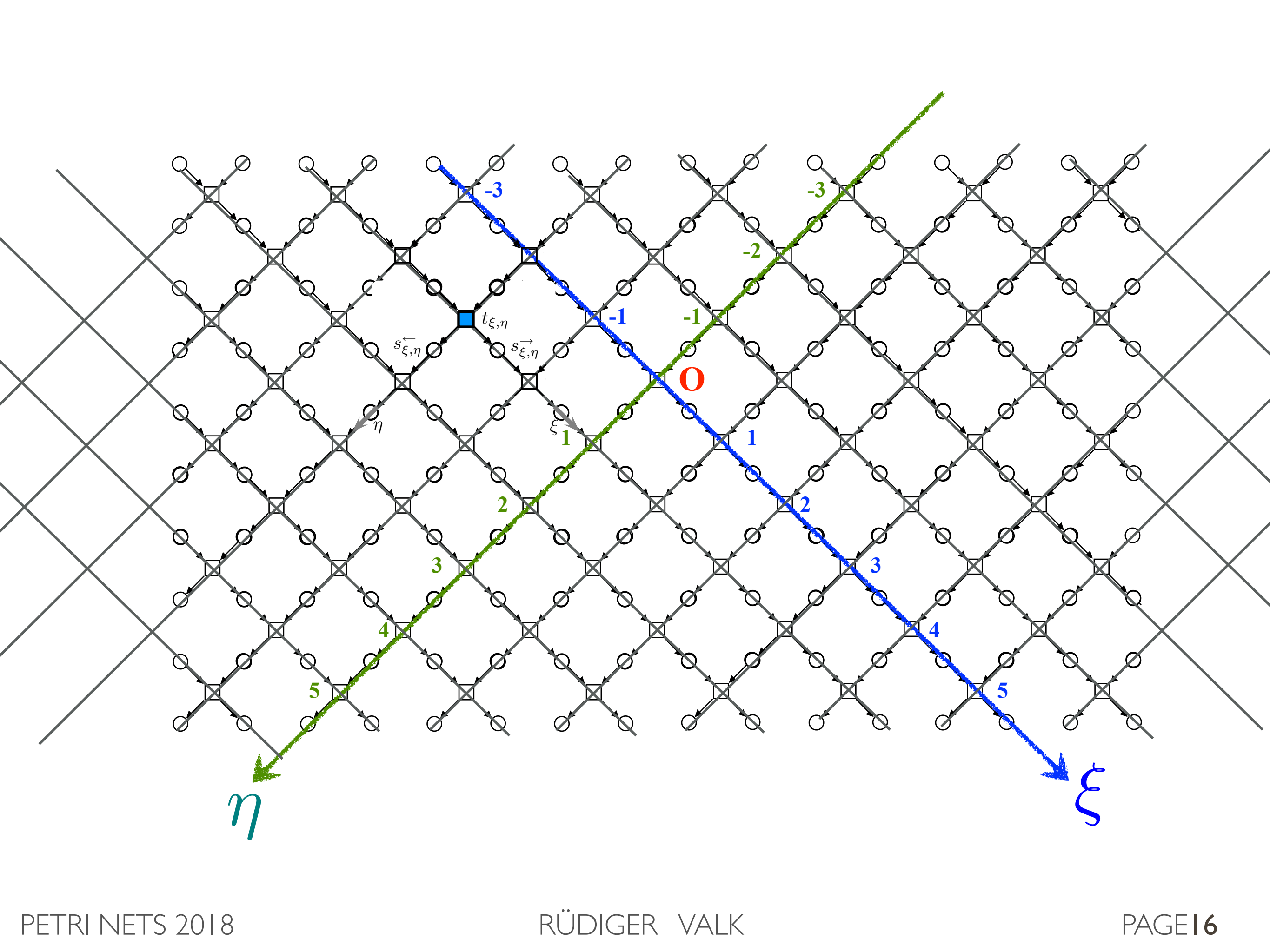
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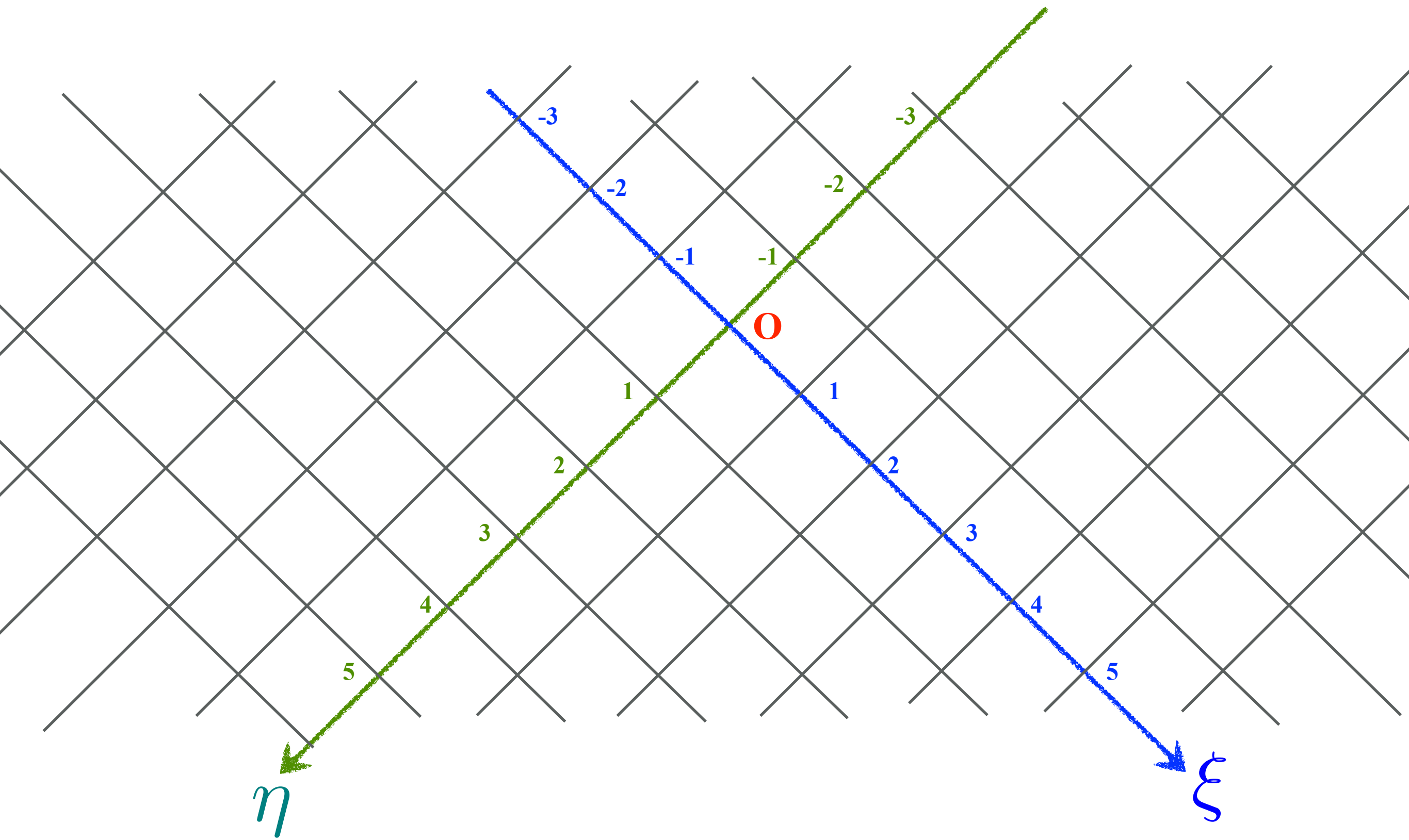




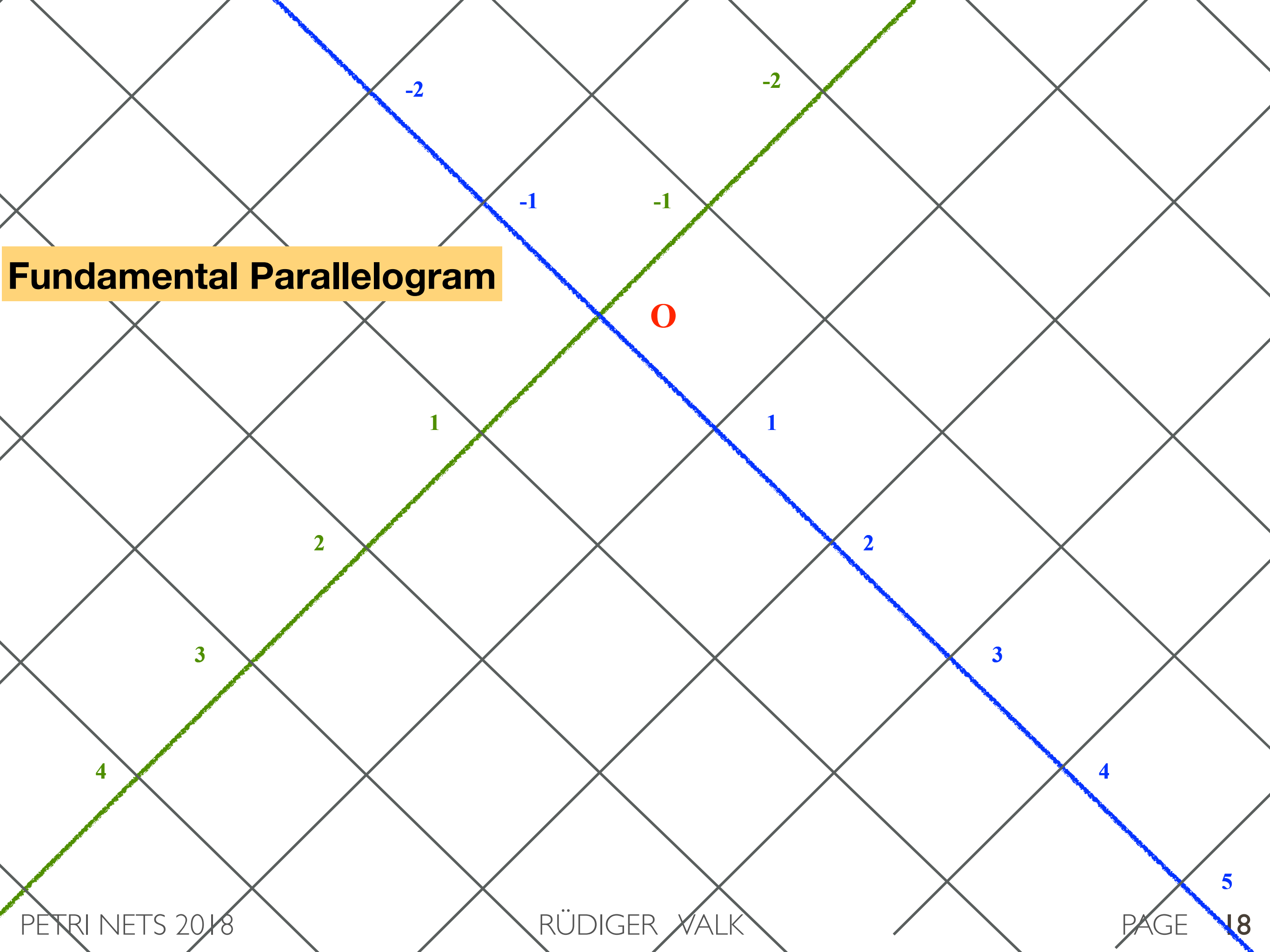




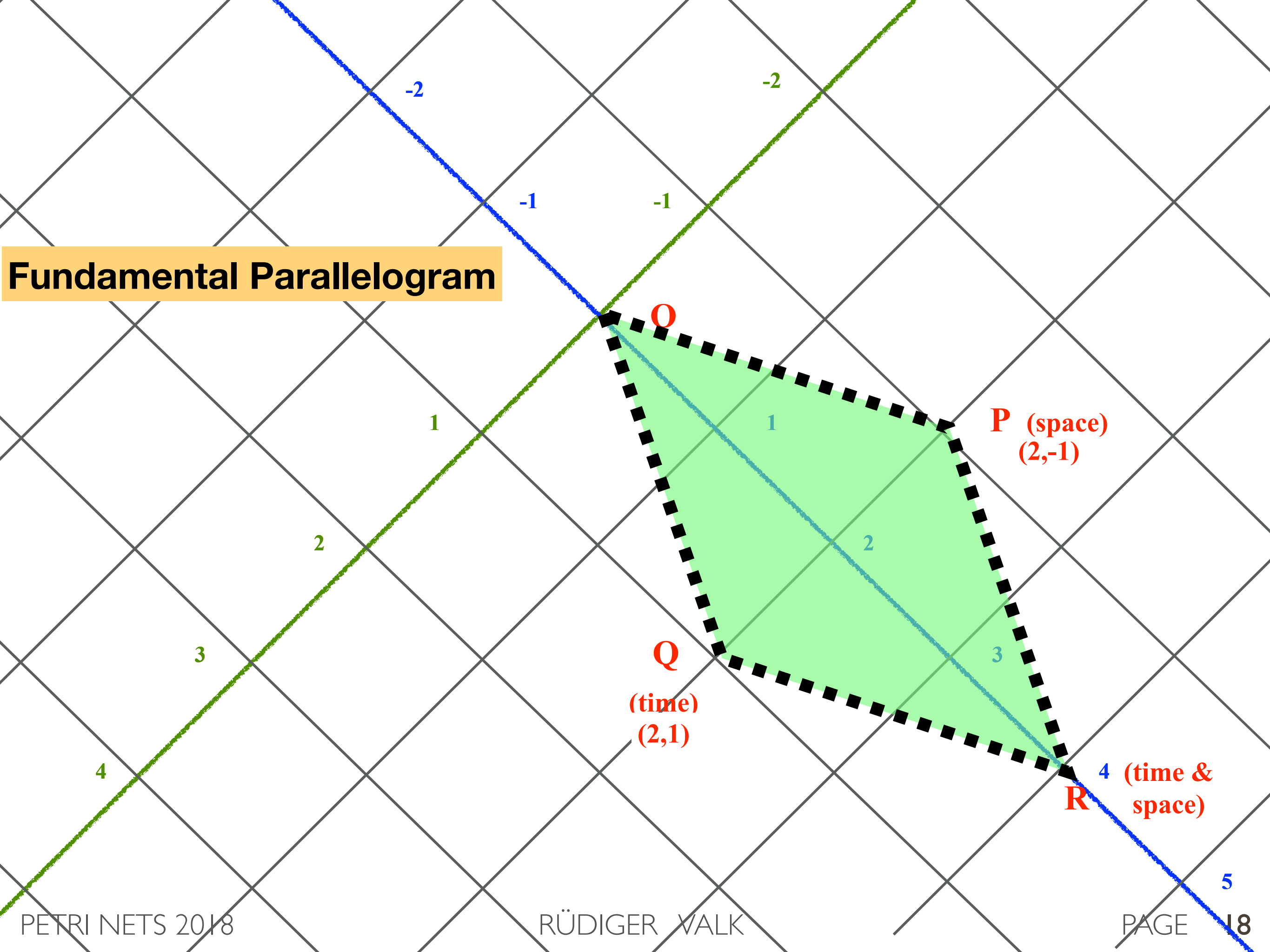




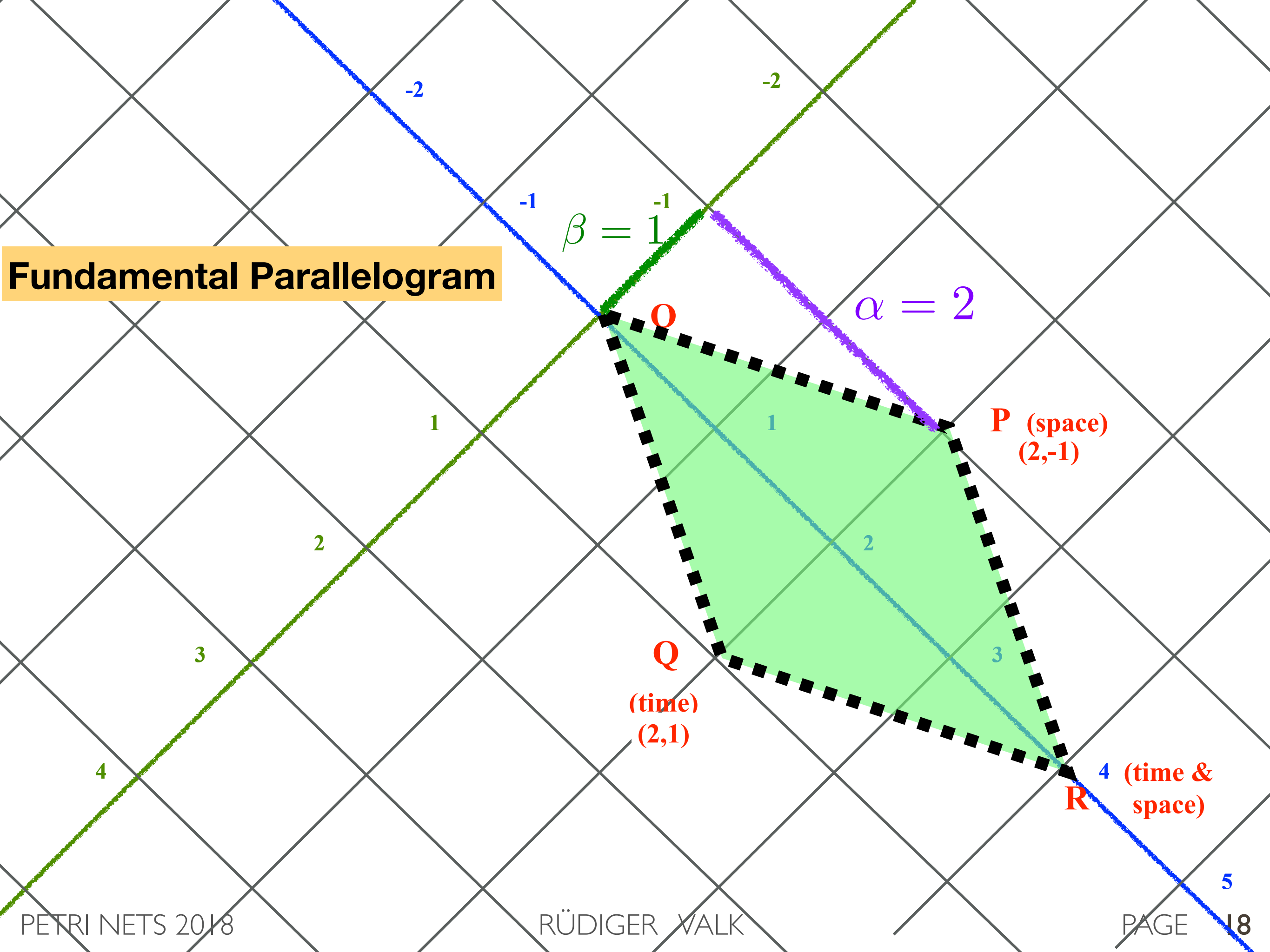
Fundamental Parallelogram



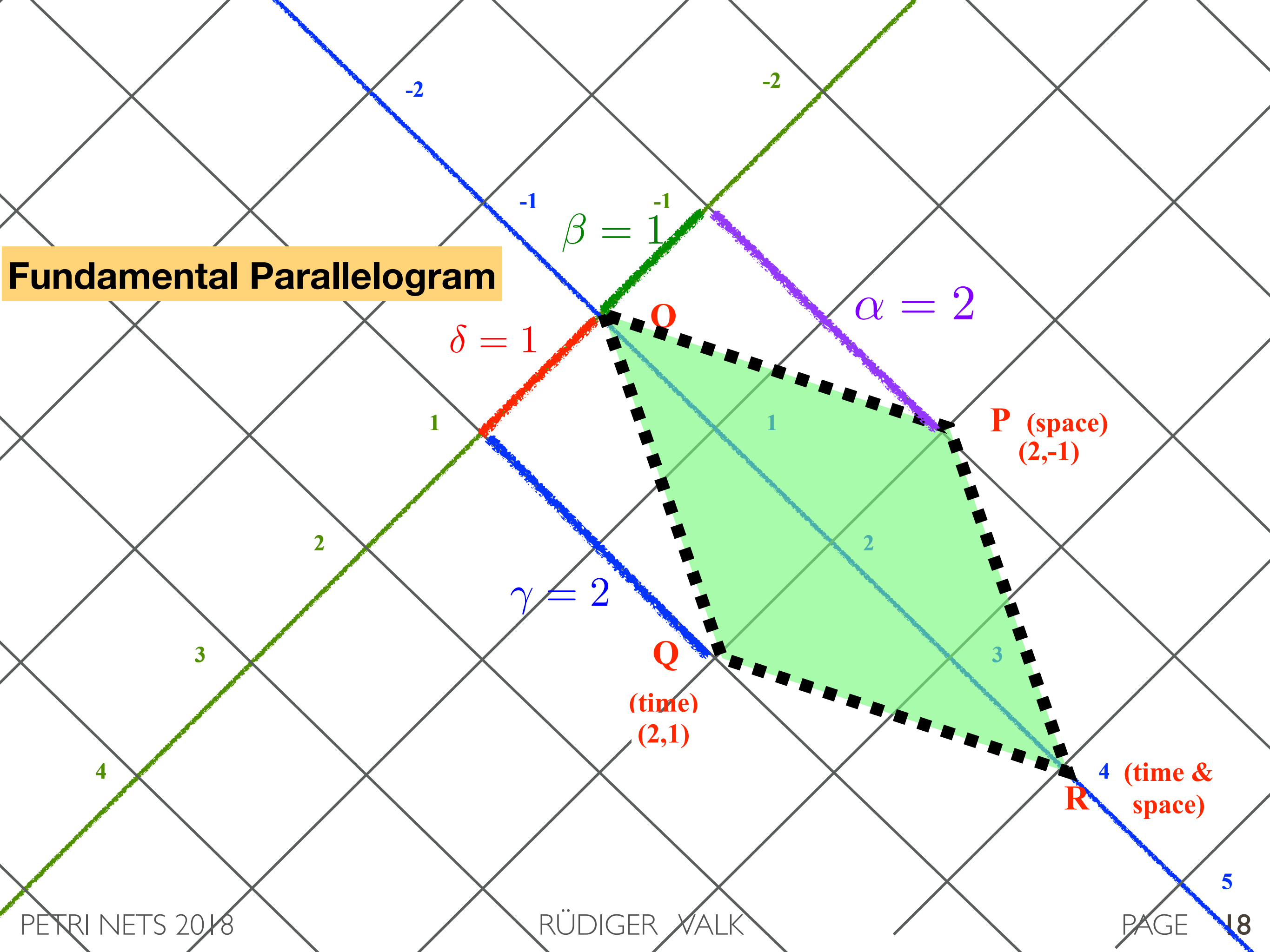
Fundamental Parallelogram



Fundamental Parallelogram



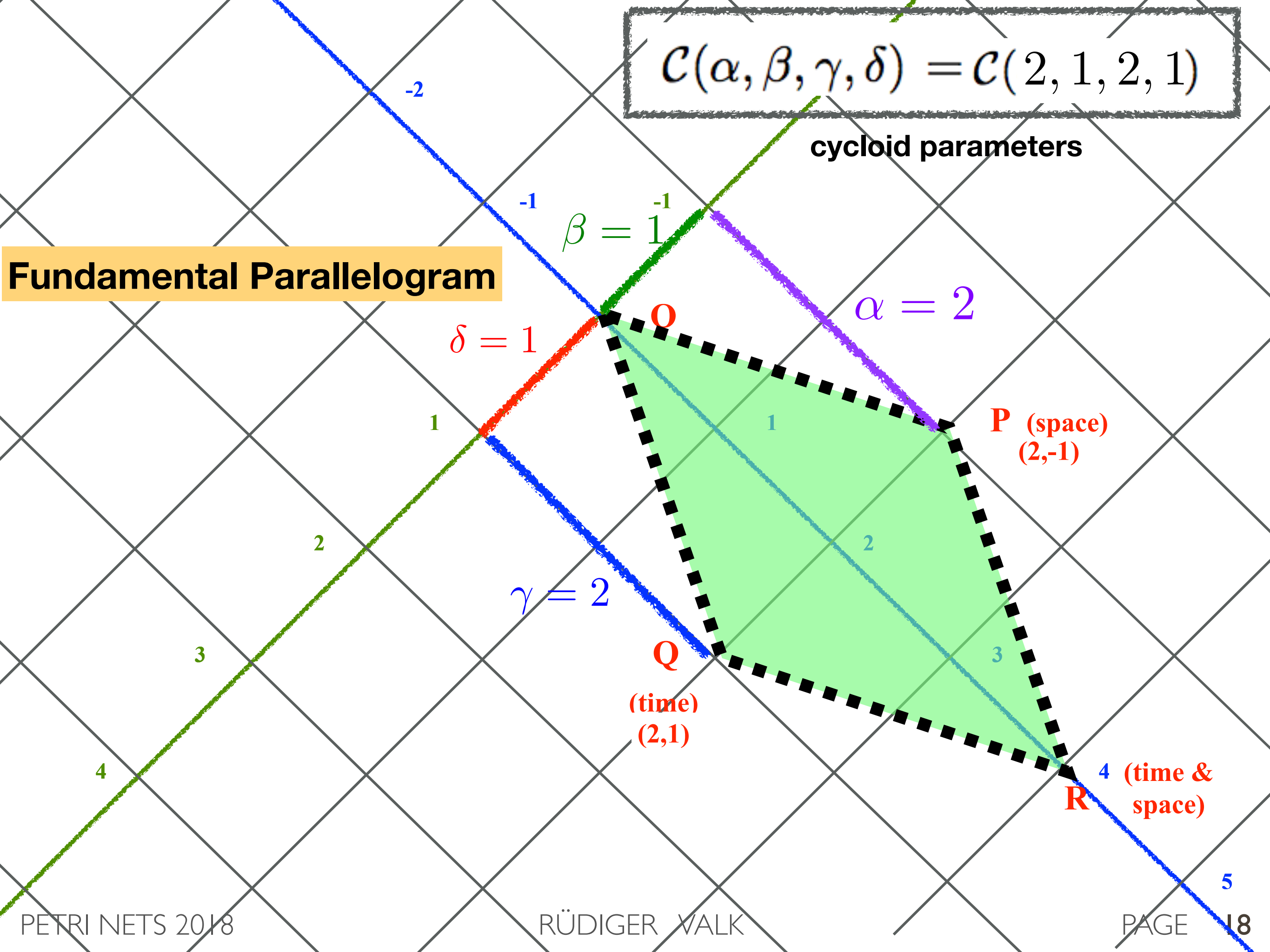
Fundamental Parallelogram



$$C(\alpha, \beta, \gamma, \delta) = C(2, 1, 2, 1)$$

cycloid parameters

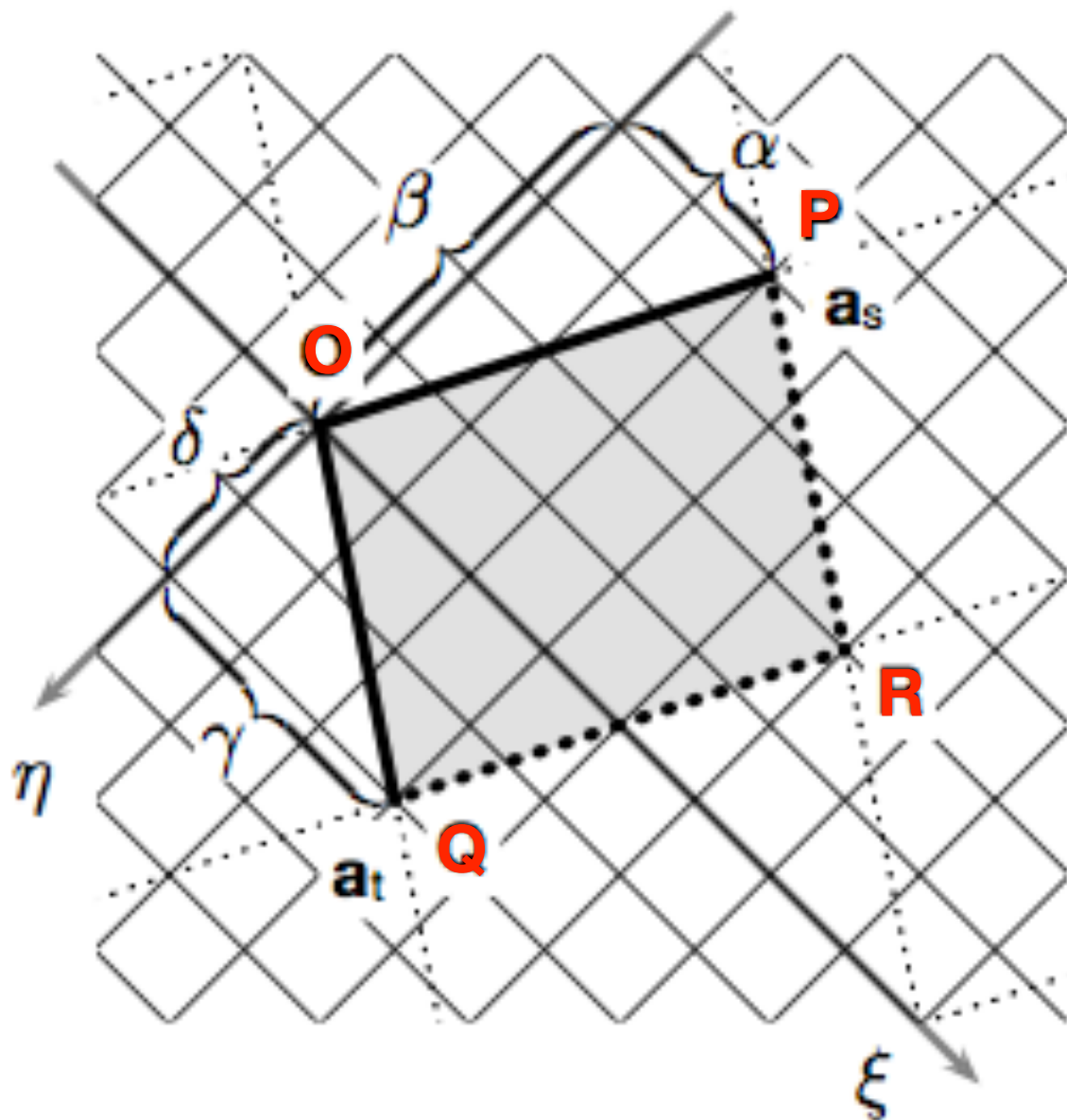
Fundamental Parallelogram



$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 4, 3, 2)$$

cycloid parameters

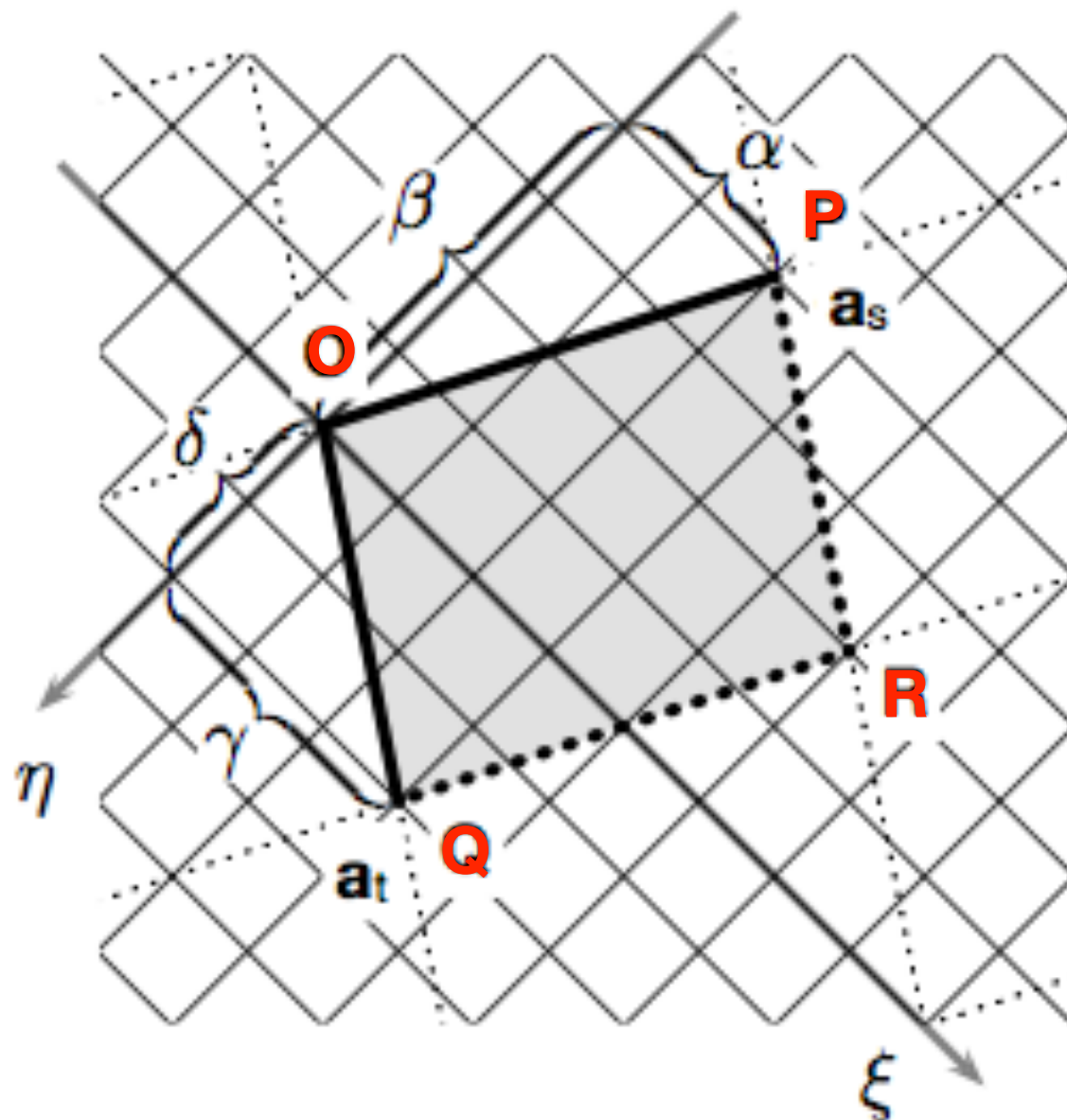
Fundamental Parallelogram



$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 4, 3, 2)$$

cycloid parameters

Fundamental Parallelogram



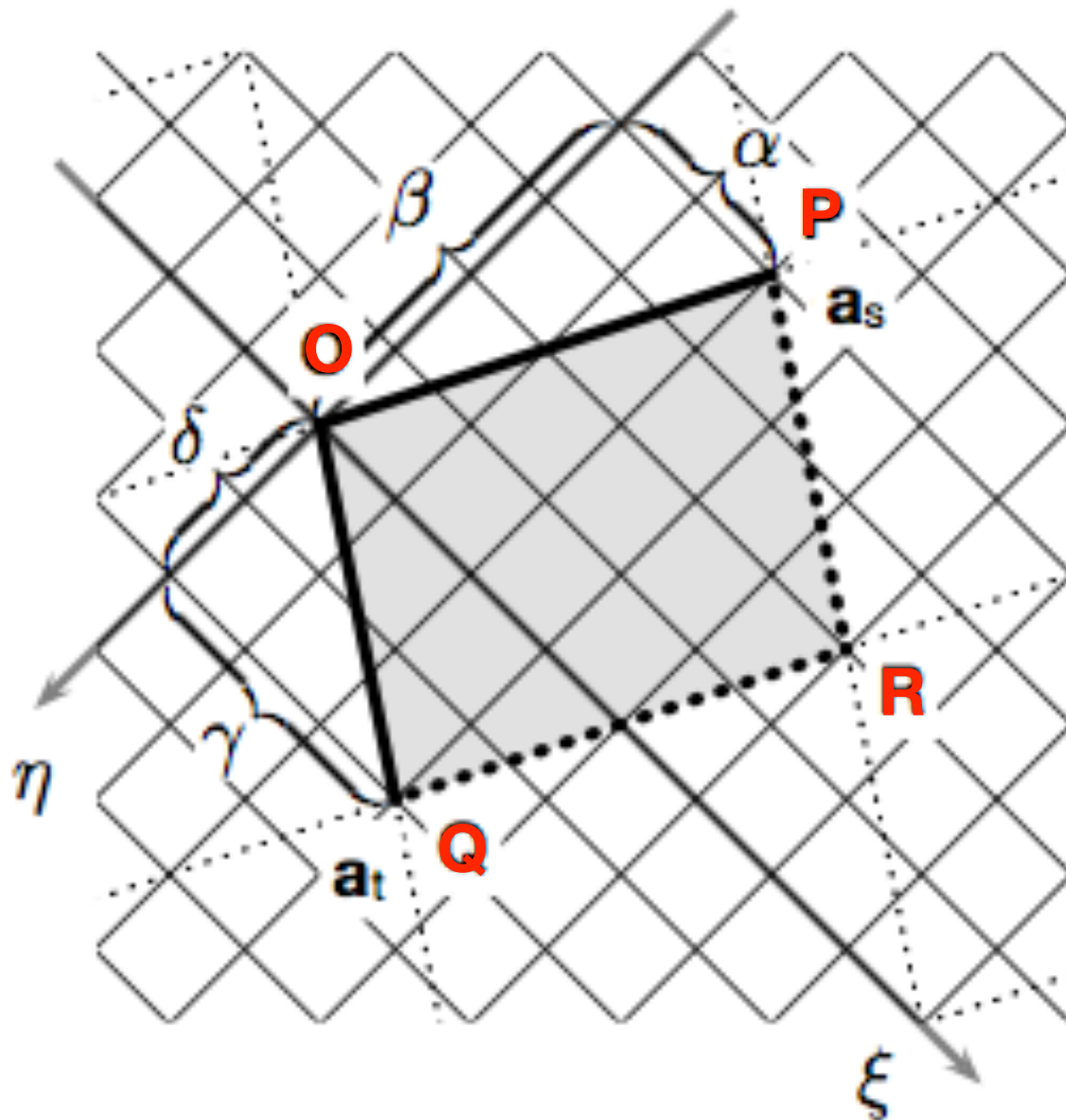
(A, B)	Equation for $\overline{AB}$	Vector $\overrightarrow{AB}$	Distance $d(A, B)$
(O, P)	$\eta = -\frac{\beta}{\alpha}\xi$	$\begin{pmatrix} \alpha \\ -\beta \end{pmatrix} = \mathbf{a}_s$	$\alpha + \beta$
(O, Q)	$\eta = \frac{\delta}{\gamma}\xi$	$\begin{pmatrix} \gamma \\ \delta \end{pmatrix} = \mathbf{a}_t$	$\gamma + \delta$
(P, R)	$\eta = \frac{\delta}{\gamma}(\xi - \alpha) - \beta$	$\begin{pmatrix} \gamma \\ \delta \end{pmatrix}$	$\gamma + \delta$
(Q, R)	$\eta = \frac{-\beta}{\alpha}(\xi - \gamma) + \delta$	$\begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$	$\alpha + \beta$
(P, Q)	$\eta = \frac{\beta + \delta}{\gamma - \alpha}(\xi - \alpha) - \beta$	$\begin{pmatrix} -\alpha + \gamma \\ \beta + \delta \end{pmatrix}$	$ \alpha - \gamma + \beta + \delta$
(O, R)	$\eta = \frac{\delta - \beta}{\alpha + \gamma}\xi$	$\begin{pmatrix} \alpha + \gamma \\ \delta - \beta \end{pmatrix}$	$\alpha + \gamma + \beta - \delta $

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 4, 3, 2)$$

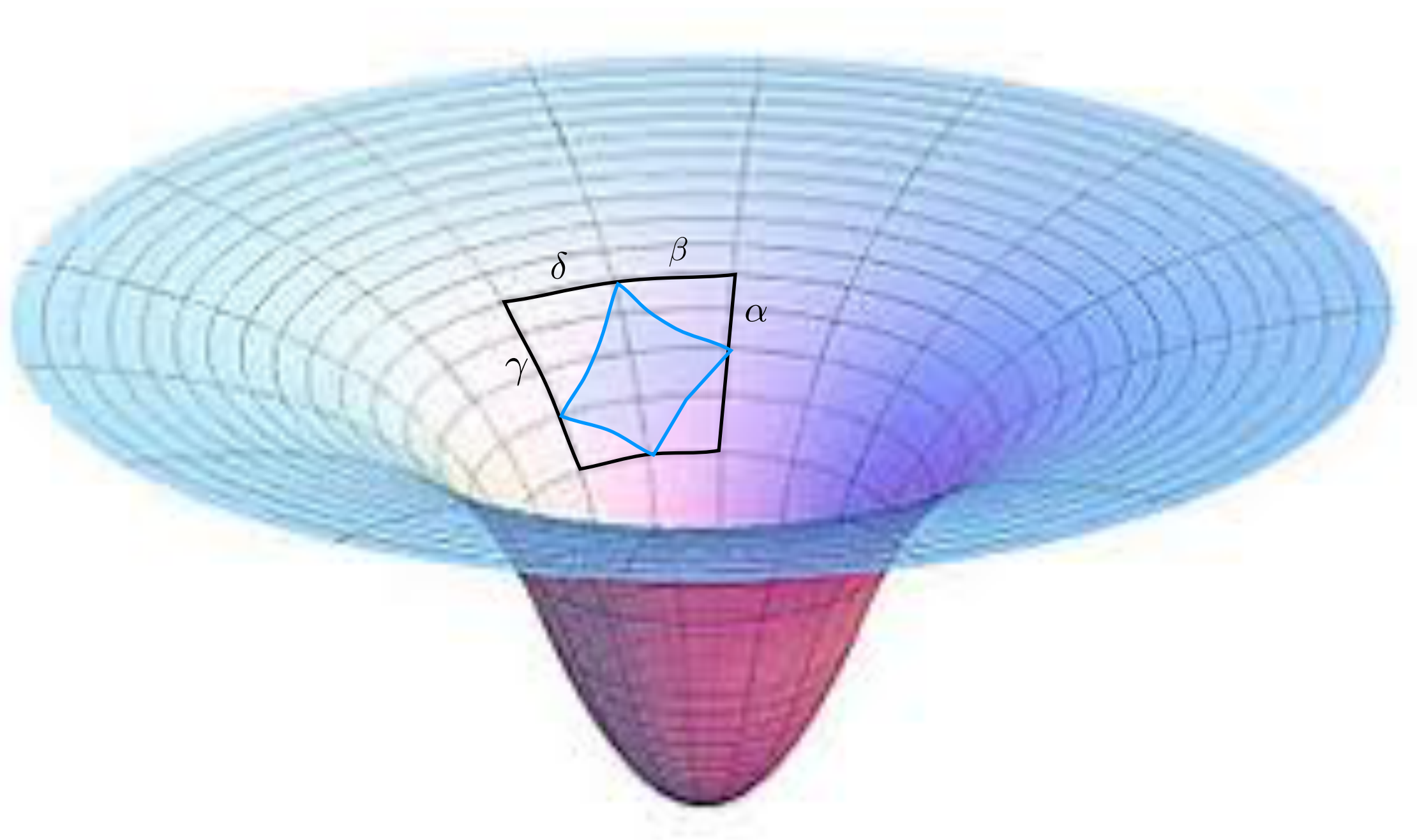
cycloid parameters

non-Euclidean
distance
number of steps
in Petri space

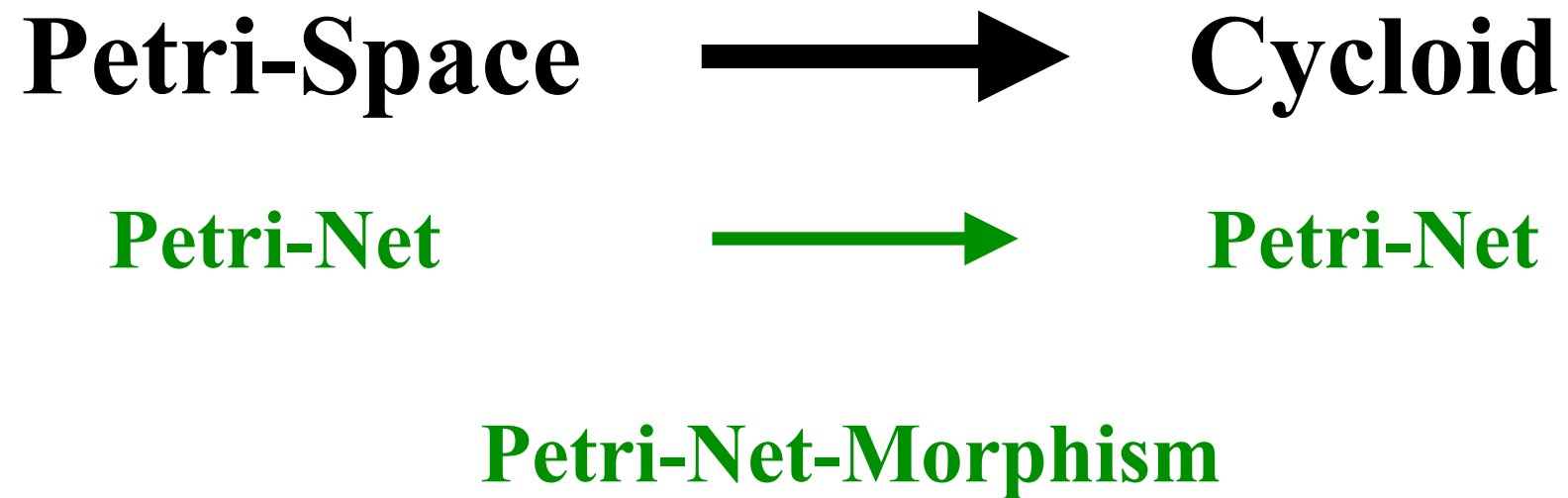
Fundamental Parallelogram



(A, B)	Equation for $\overline{AB}$	Vector $\overrightarrow{AB}$	Distance $d(A, B)$
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(O, R)	$\eta = \frac{\delta - \beta}{\alpha + \gamma}\xi$	$\begin{pmatrix} \alpha + \gamma \\ \delta - \beta \end{pmatrix}$	$\alpha + \gamma + \beta - \delta $



A cycloid is a folding of Petri space



(Uwe Fenske)

A cycloid is a folding of Petri space



Petri-Net-Morphism

$$f(x_{\xi,\eta}) = f(x_{\xi+m\alpha+n\gamma}, \eta-m\beta+n\delta)$$

(Uwe Fenske)

A cycloid is a folding of Petri space



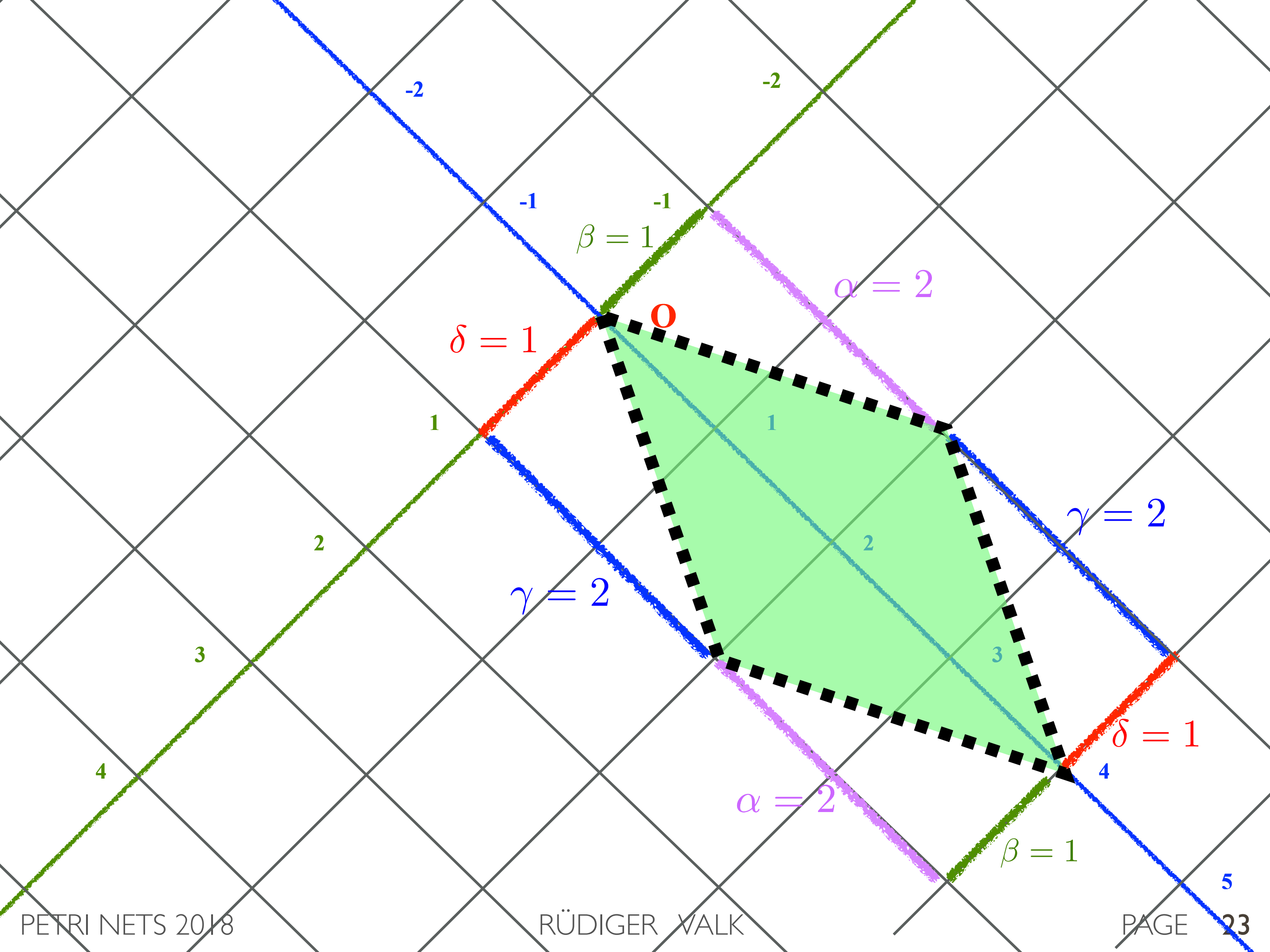
Petri-Net-Morphism

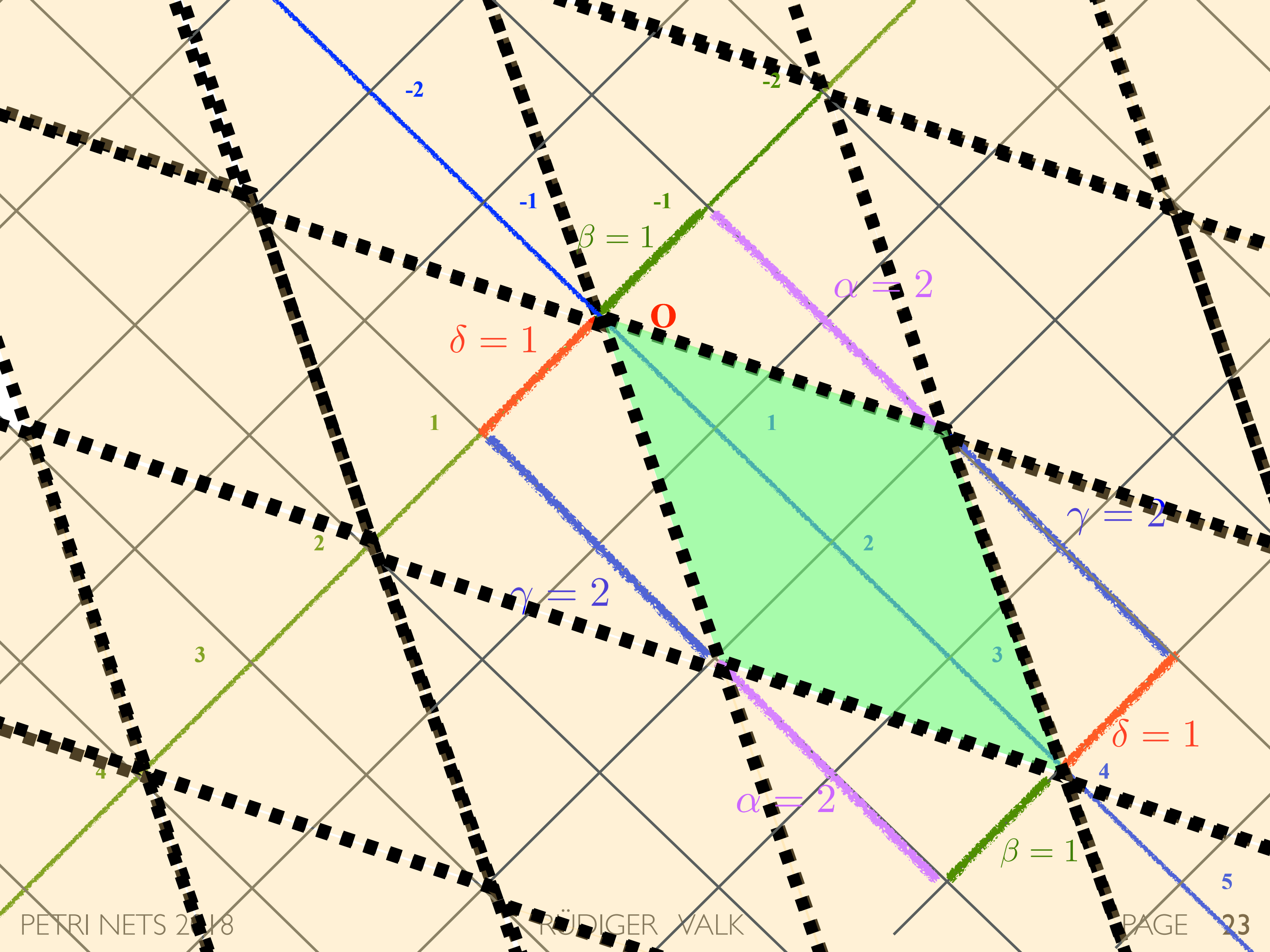
$$f(x_{\xi,\eta}) = f(x_{\xi+m\alpha+n\gamma}, \eta-m\beta+n\delta)$$

$$x_{\xi,\eta} \equiv \underbrace{x_{\xi} + m\alpha + n\gamma}_{\xi} \underbrace{\eta - m\beta + n\delta}_{\eta} \quad \text{for all } \xi, \eta, m, n \in \mathbb{N}$$

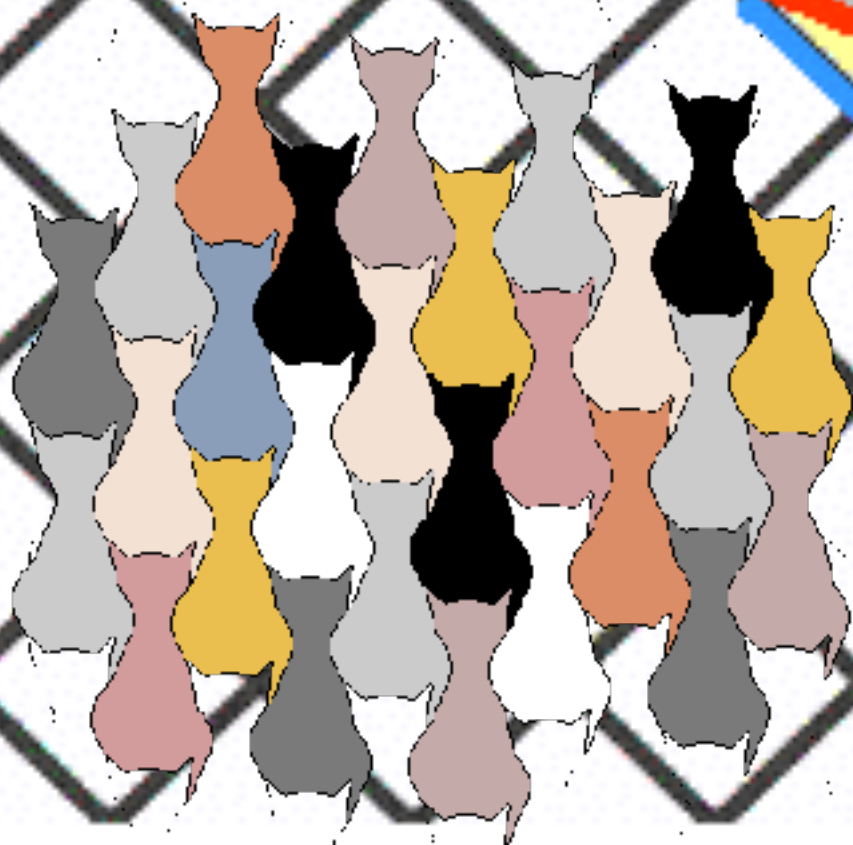
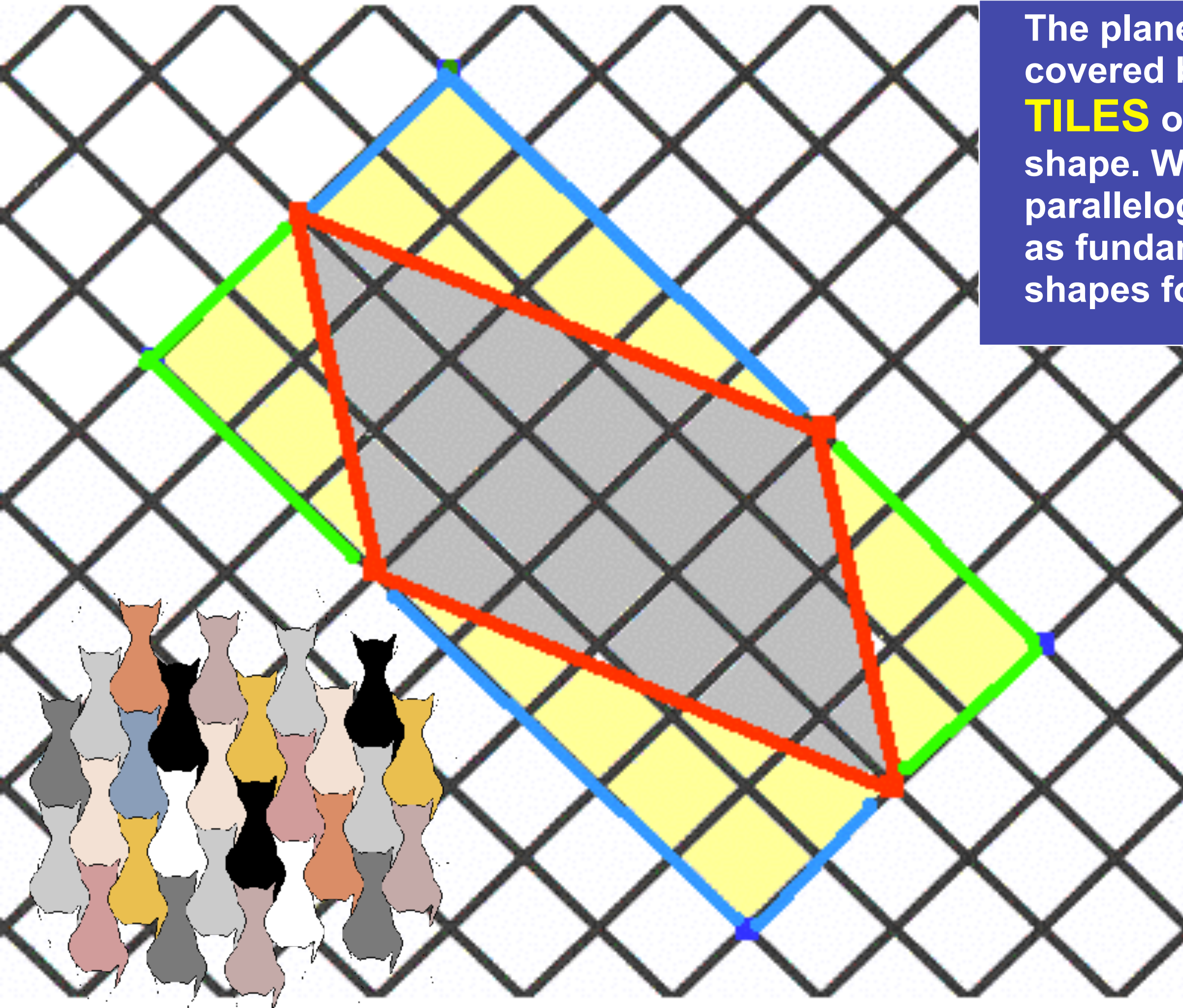
folding in direction ξ η
time *space*

(Uwe Fenske)



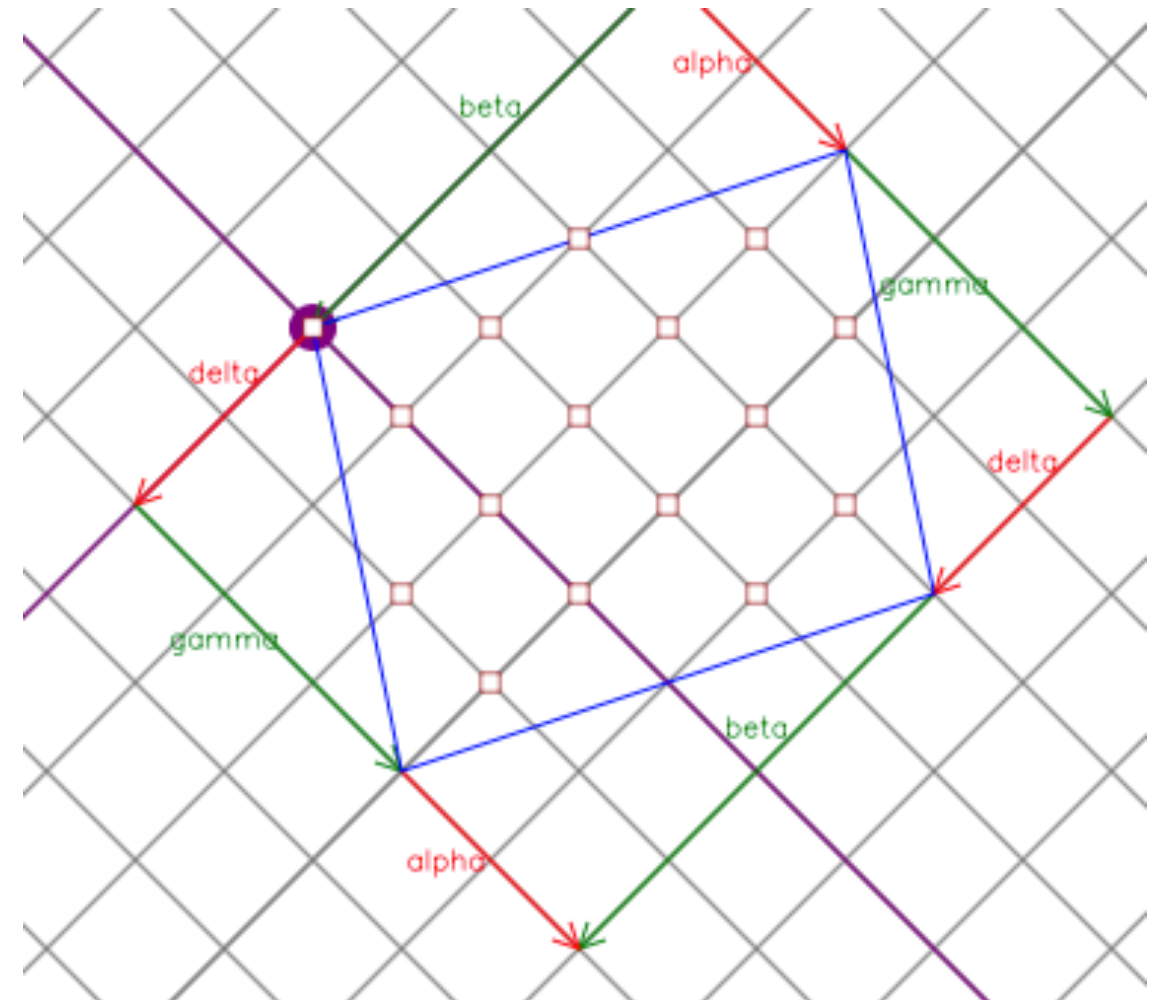
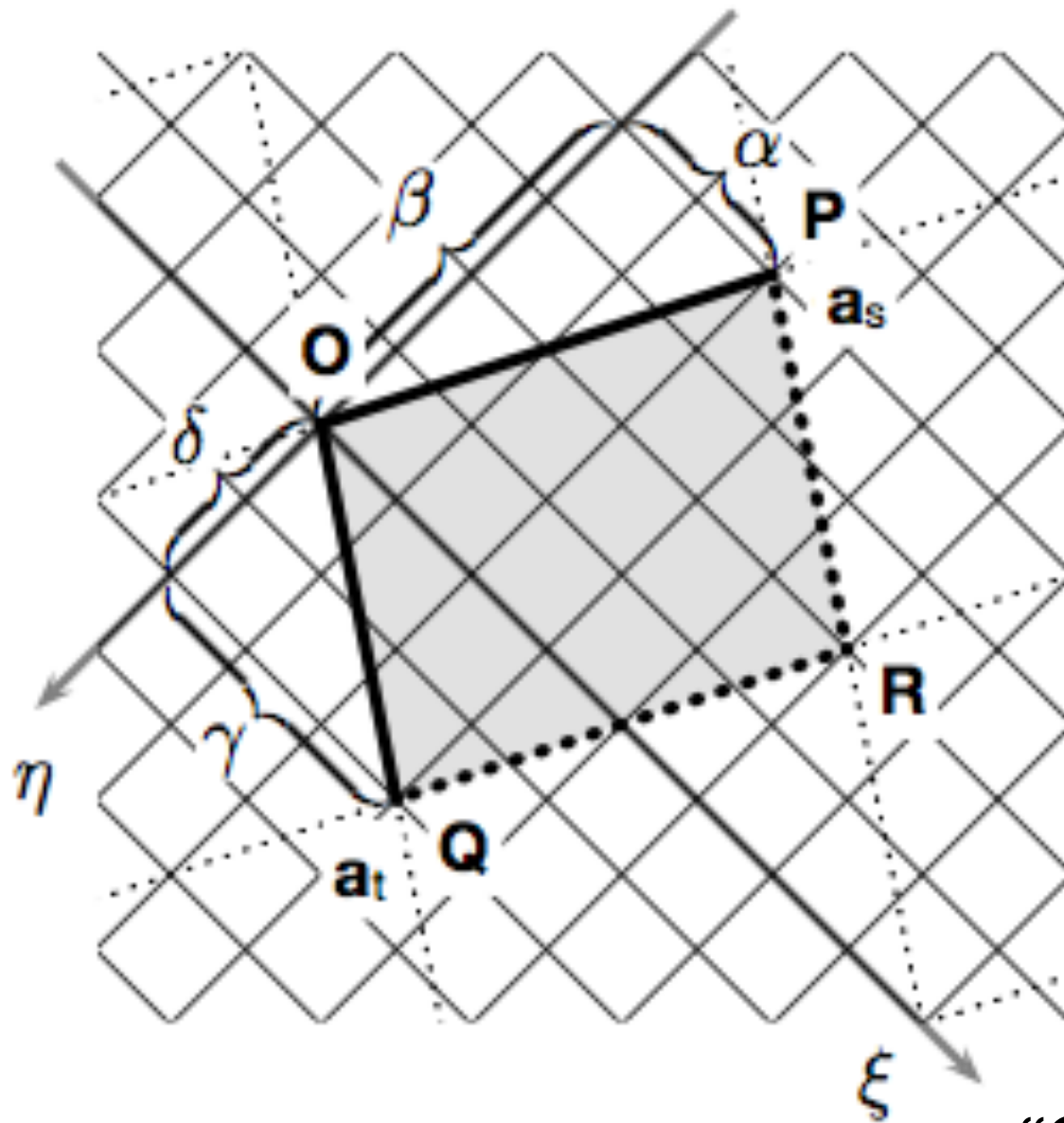


The plane can be covered by **TILES** of equal shape. We use parallelograms as fundamental shapes for tiling.



(C.A.Petri)

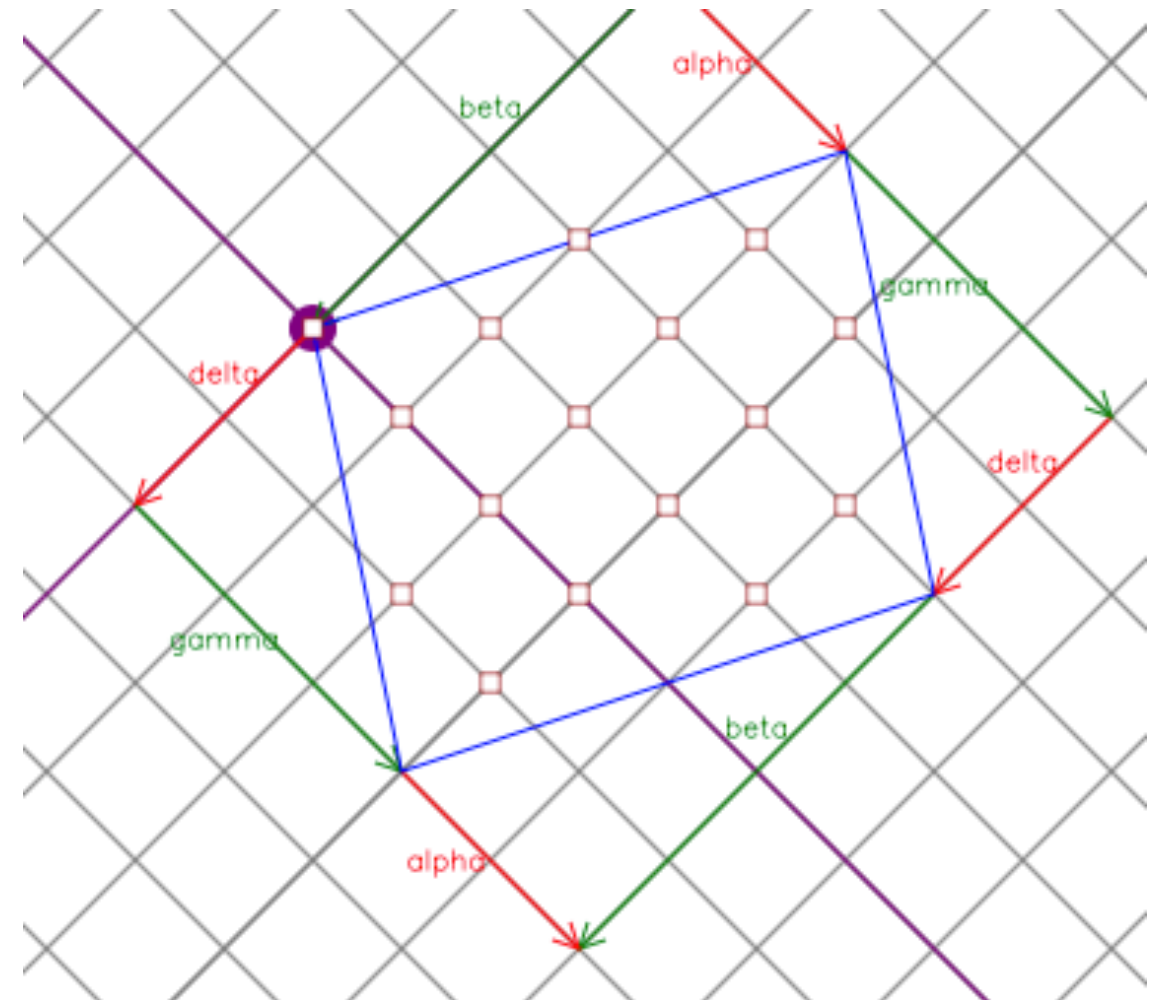
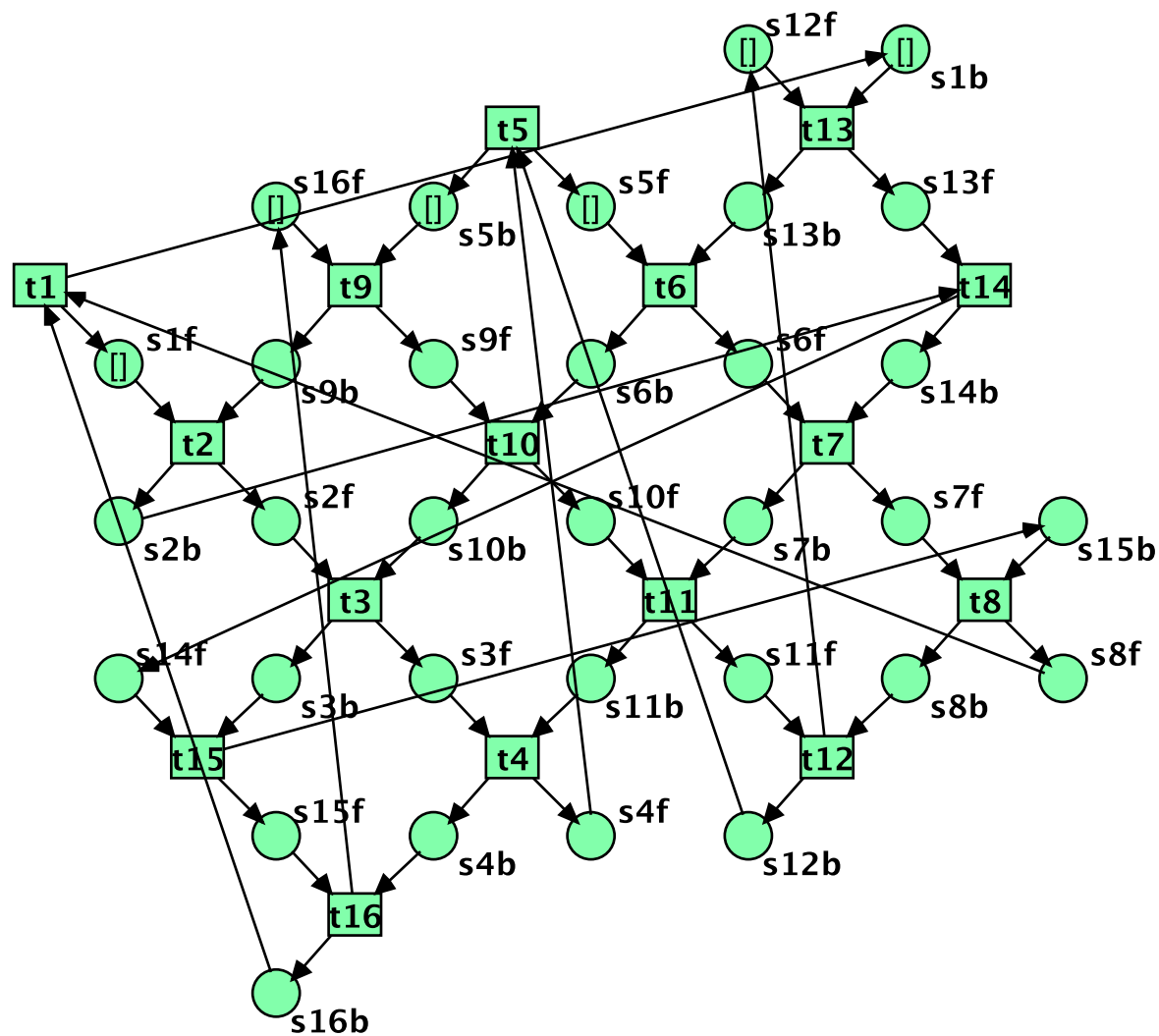
$$C(\alpha, \beta, \gamma, \delta) = C(2, 4, 3, 2)$$



“Cycloids' Characteristics” by Peter Langner

<http://cycloids.adventas.de>

$$\mathcal{C}(\alpha, \beta, \gamma, \delta) = \mathcal{C}(2, 4, 3, 2)$$



**Fenske's cycloid-generator-tool
with output in RENEW
<http://renew.de>**

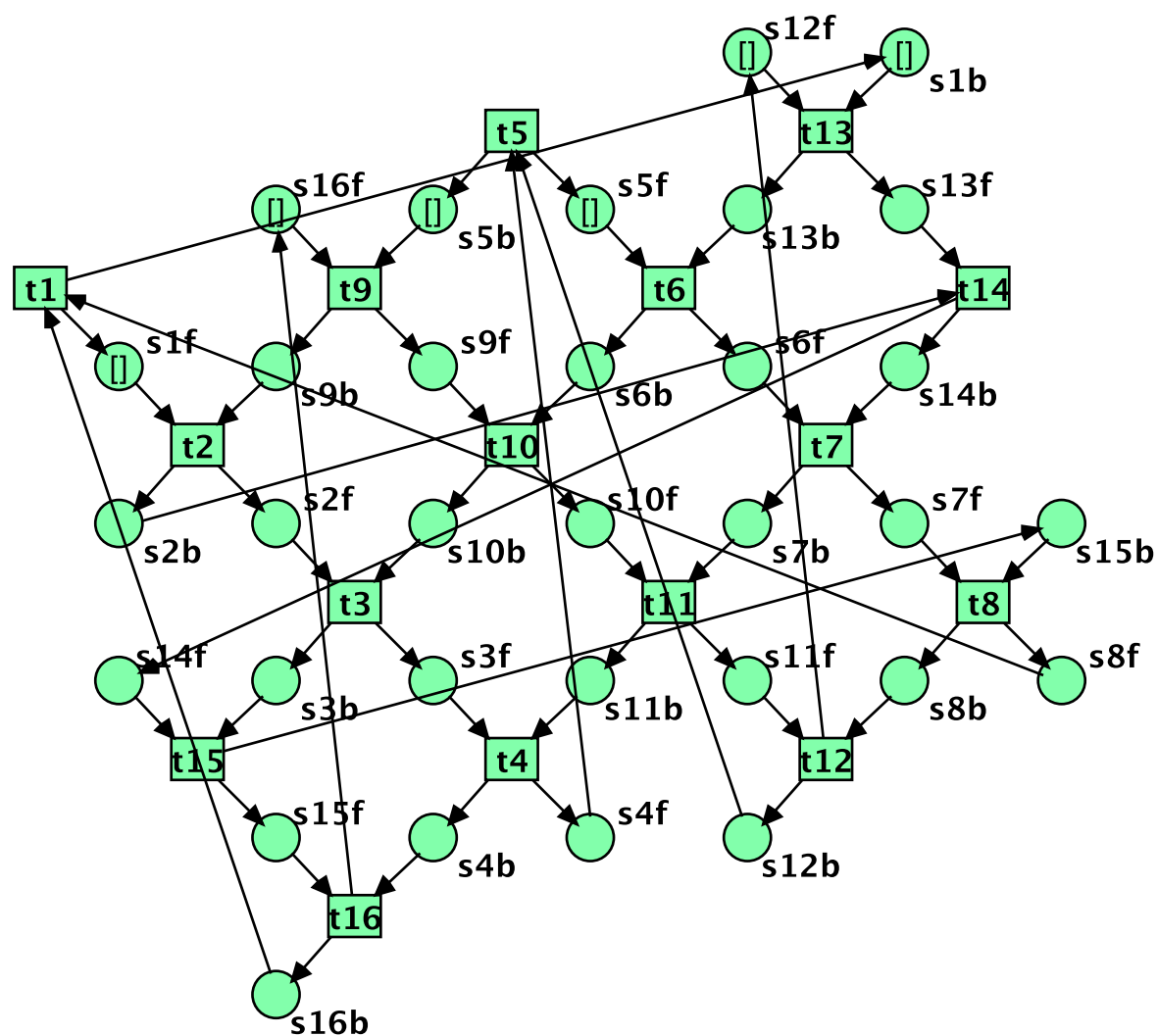
<http://cycloids.adventas.de>

Cycloids $\mathcal{C}(\alpha, \beta, \gamma, \delta)$ and $\mathcal{C}(\alpha, \beta, \gamma - \alpha, \delta + \beta)$

are isomorphic if $\gamma > \alpha$.

(Olaf Kummer)

$$\mathcal{C}(2, 4, 3, 2) \longrightarrow \mathcal{C}(2, 4, 3 - 2, 2 + 4) = \mathcal{C}(2, 4, 1, 6)$$

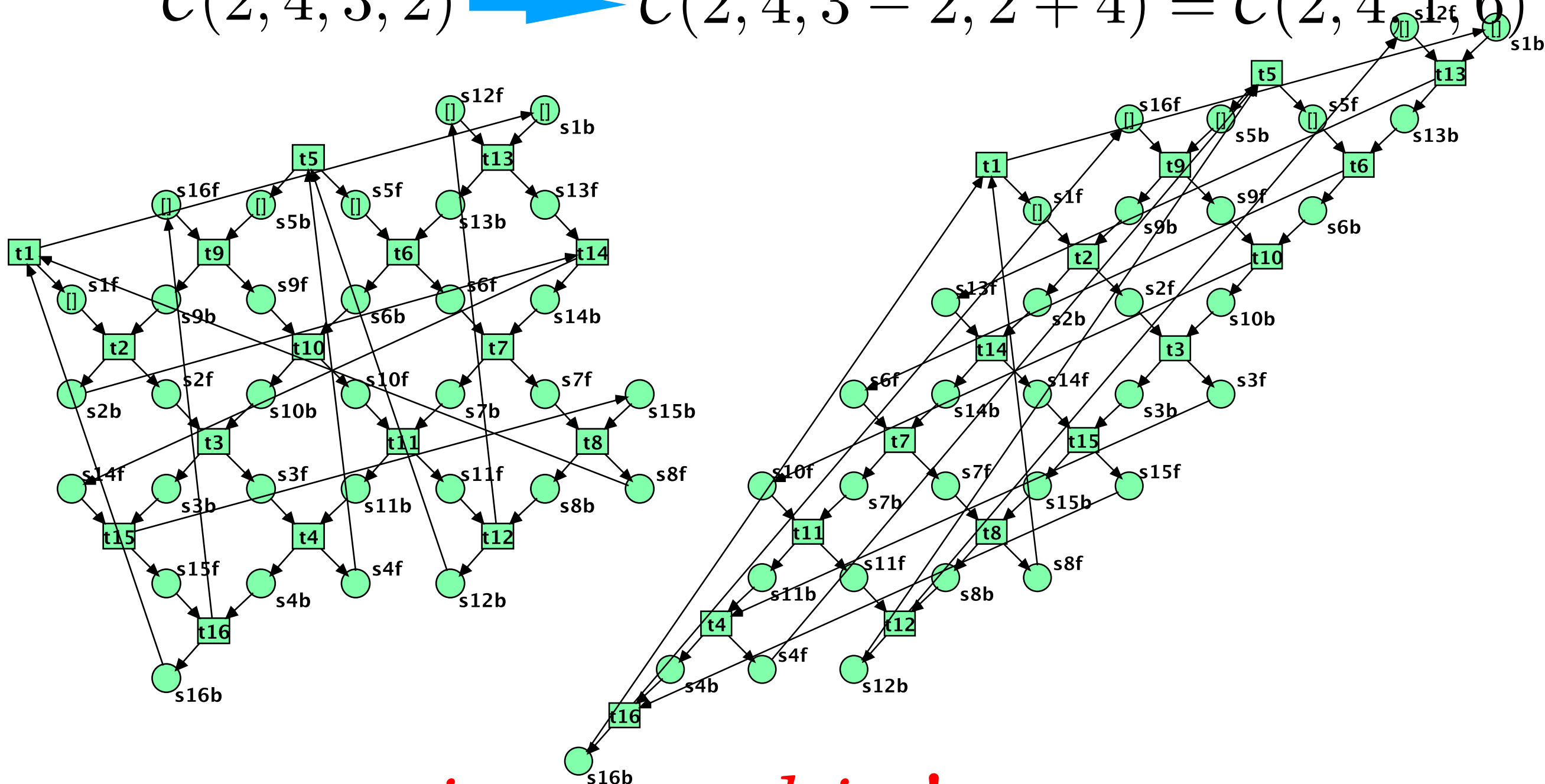


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isomorphic!

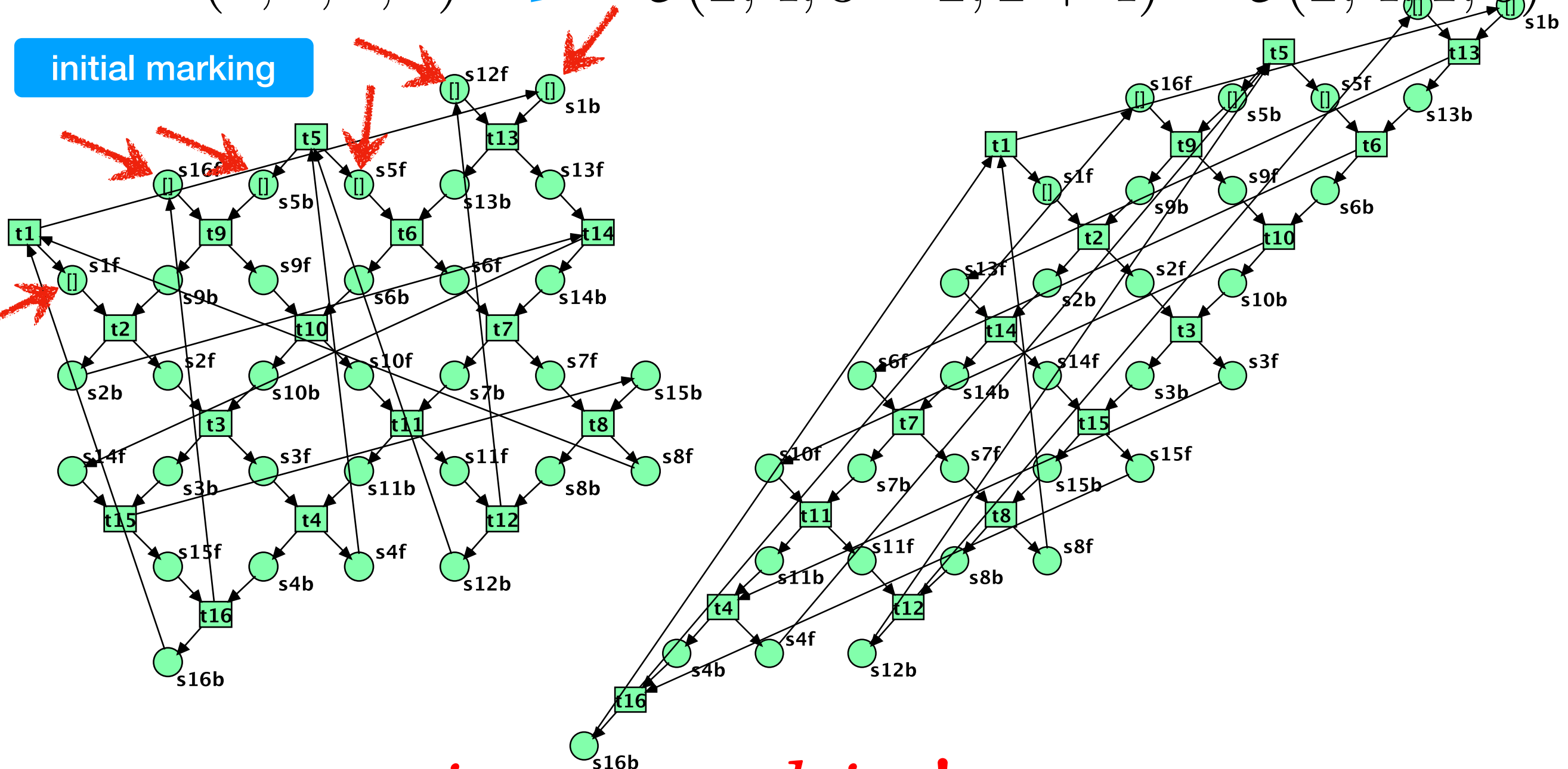
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initial marking



isomorphic!

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Standard Marking

$$M_0 = \{s_{\xi,\eta}^{\rightarrow} \in S_1^{\rightarrow} \mid \eta \leq -\frac{\beta}{\alpha}\xi \wedge \eta > -\frac{\beta}{\alpha}(\xi + 1)\} / \equiv \cup \{s_{\xi,\eta}^{\leftarrow} \in S_1^{\leftarrow} \mid \eta \leq -\frac{\beta}{\alpha}\xi \wedge \eta > -\frac{\beta}{\alpha}\xi - 1\} / \equiv$$

tokens in

forward places

backward places

(Uwe Fenske)

Cycloid-System

a life, safe and secure net

$$M_0 = \{s_{\xi,\eta}^{\rightarrow} \in S_1^{\rightarrow} \mid \eta \leq -\frac{\beta}{\alpha}\xi \wedge \eta > -\frac{\beta}{\alpha}(\xi + 1)\} / \equiv \cup$$

$$\{s_{\xi,\eta}^{\leftarrow} \in S_1^{\leftarrow} \mid \eta \leq -\frac{\beta}{\alpha}\xi \wedge \eta > -\frac{\beta}{\alpha}\xi - 1\} / \equiv$$

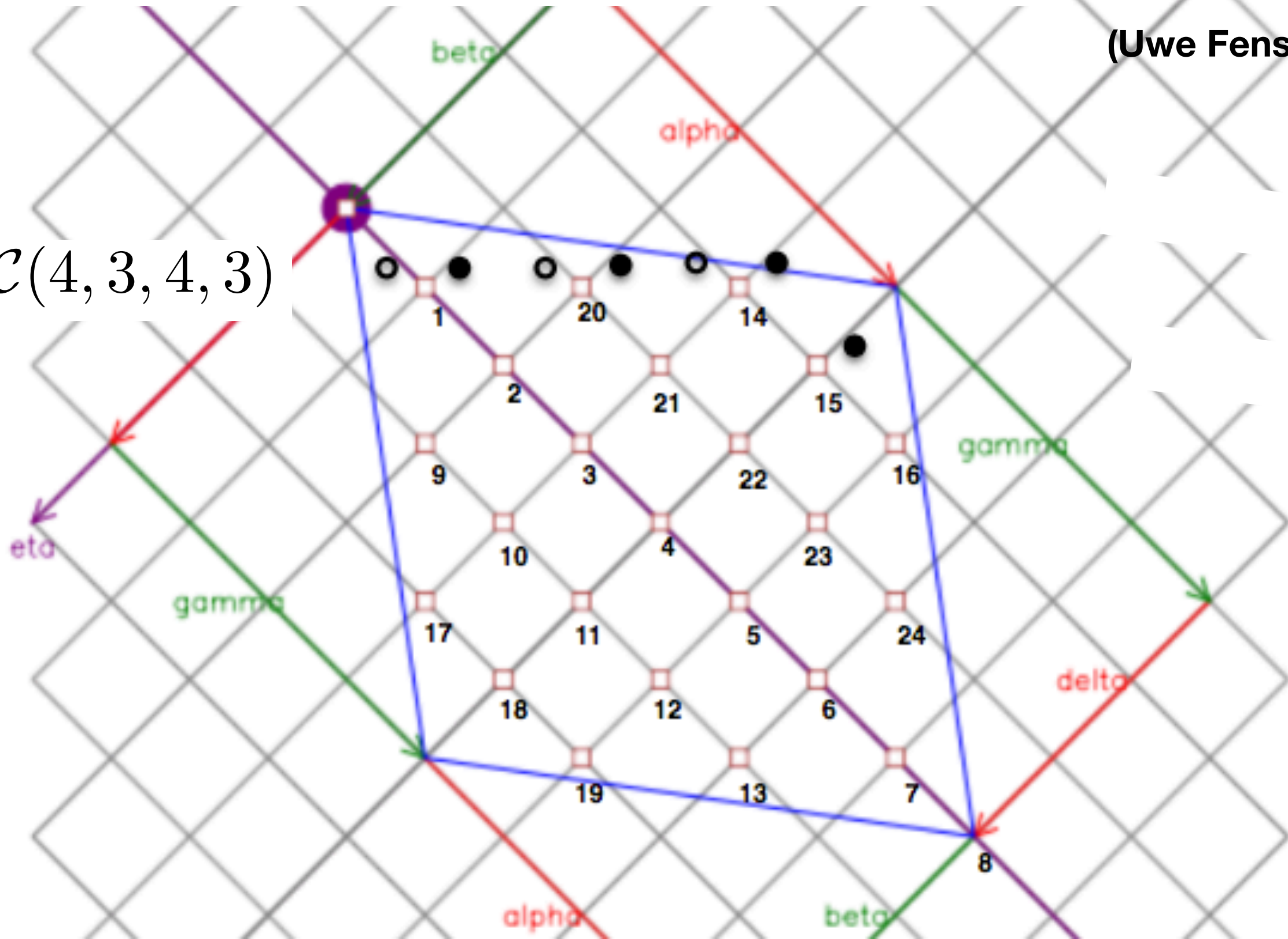
tokens in

forward places

backward places

(Uwe Fenske)

$C(4, 3, 4, 3)$



$$M_0 = \{s_{\xi,\eta}^{\rightarrow} \in S_1^{\rightarrow} \mid \eta \leq -\frac{\beta}{\alpha}\xi \wedge \eta > -\frac{\beta}{\alpha}(\xi + 1)\} / \equiv \cup$$

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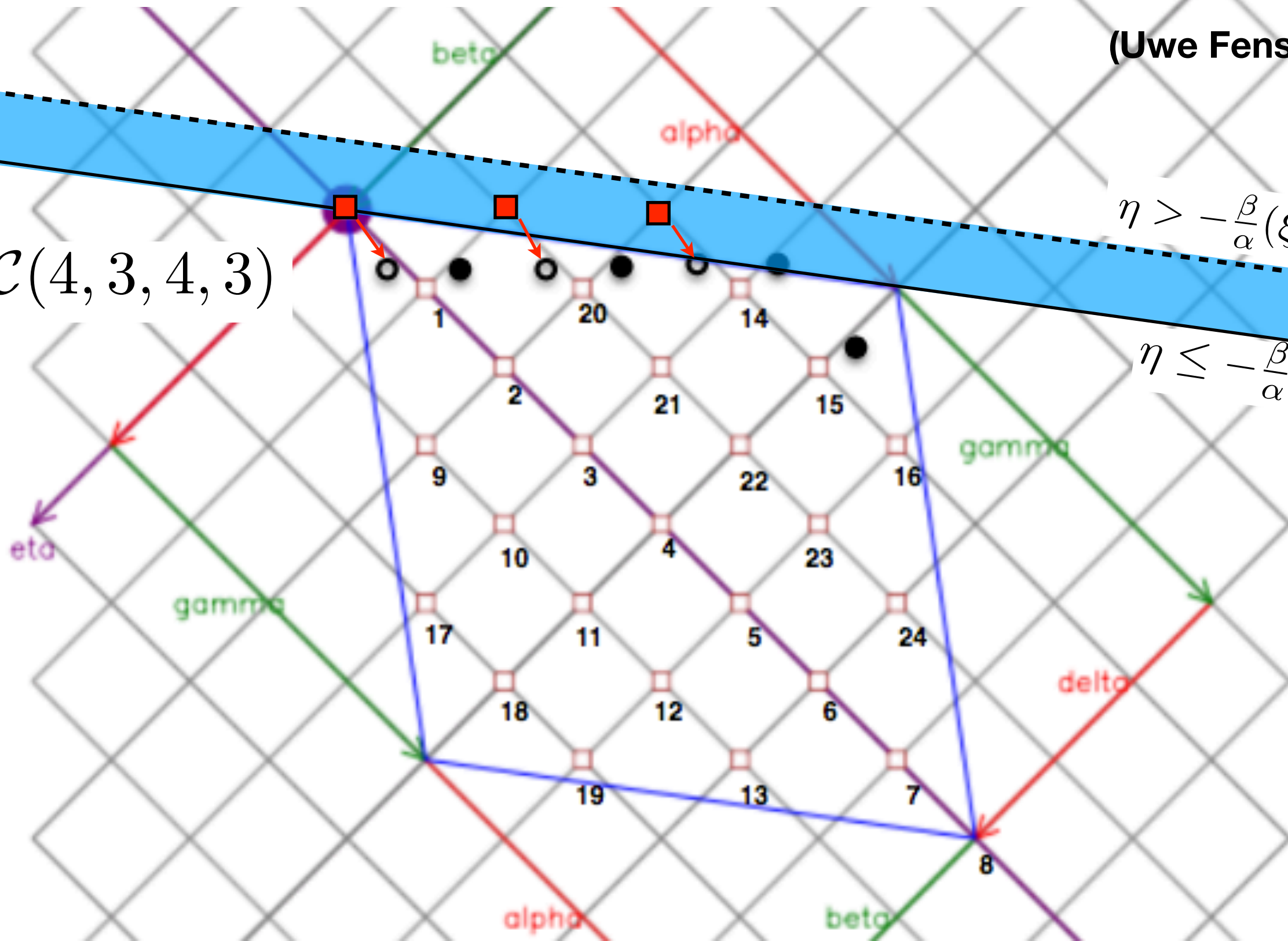
tokens in
 forward places
 backward places

(Uwe Fenske)

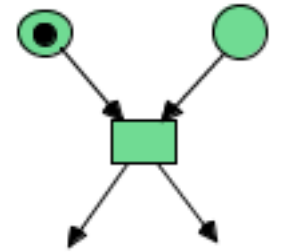
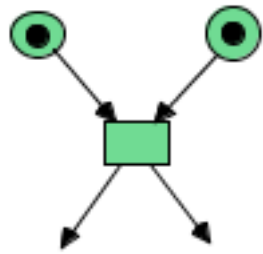
$\mathcal{C}(4, 3, 4, 3)$

$$\eta > -\frac{\beta}{\alpha}(\xi + 1)$$

$$\eta \leq -\frac{\beta}{\alpha}\xi$$

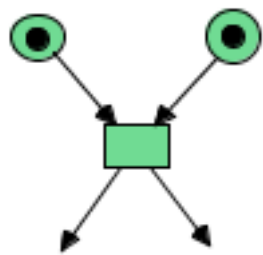


properties of the initial marking

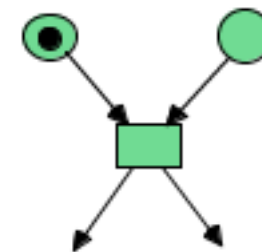


transitions

tokens



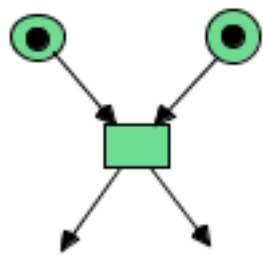
properties of the initial marking



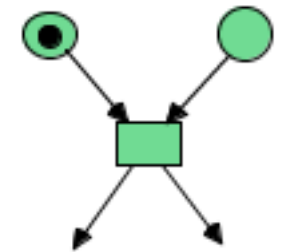
$\tau_0 := |\{t \mid |t \cap M_0| \geq 1\}|$
 is the number of marked transitions.

transitions

tokens



properties of the initial marking

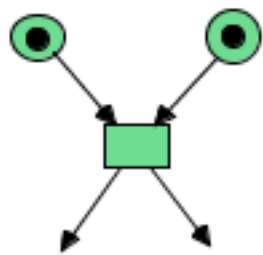


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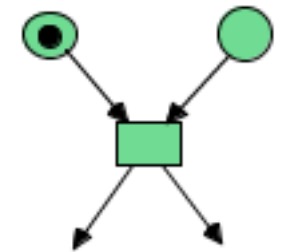
$\tau_a := |\{t \mid |t \cap M_0| = 2 \}|$
is the number of active transitions.

transitions

tokens



properties of the initial marking



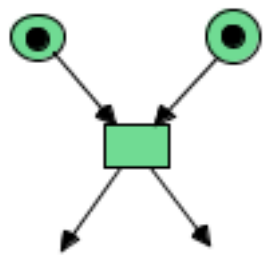
transitions

$\tau_0 := |\{t \mid |t \cap M_0| \geq 1 \}|$
is the number of **marked transitions**.

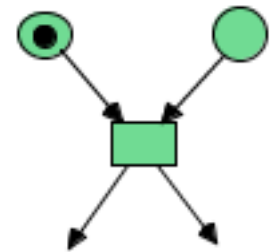
$\tau_a := |\{t \mid |t \cap M_0| = 2 \}|$
is the number of **active transitions**.

$\tau := |\{t \mid |t \cap M_0| = 1 \}|$
is the number of **semi-active transitions**.

tokens



properties of the initial marking



transitions

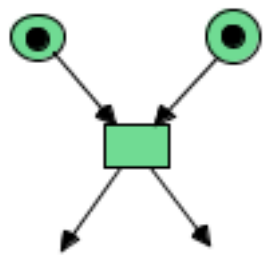
$\tau_0 := |\{t \mid |t \cap M_0| \geq 1\}|$
is the number of **marked transitions**.

$\tau_a := |\{t \mid |t \cap M_0| = 2\}|$
is the number of **active transitions**.

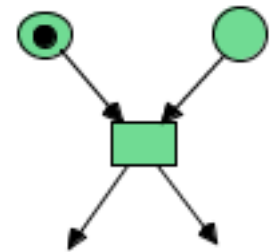
$\tau := |\{t \mid |t \cap M_0| = 1\}|$
is the number of **semi-active transitions**.

tokens

$\mu_0 := |M_0|$ is the **total number of tokens**.



properties of the initial marking



transitions

$\tau_0 := |\{t \mid |t \cap M_0| \geq 1\}|$
is the number of **marked transitions**.

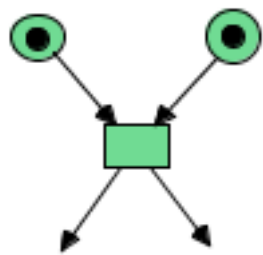
$\tau_a := |\{t \mid |t \cap M_0| = 2\}|$
is the number of **active transitions**.

$\tau := |\{t \mid |t \cap M_0| = 1\}|$
is the number of **semi-active transitions**.

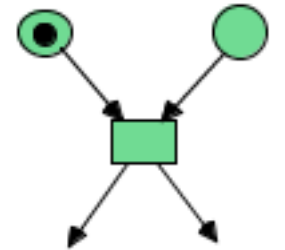
tokens

$\mu_0 := |M_0|$ is the **total number of tokens**.

μ_a is number of tokens
marking active transitions.



properties of the initial marking



transitions

$\tau_0 := |\{t \mid |t \cap M_0| \geq 1\}|$
is the number of **marked transitions**.

$\tau_a := |\{t \mid |t \cap M_0| = 2\}|$
is the number of **active transitions**.

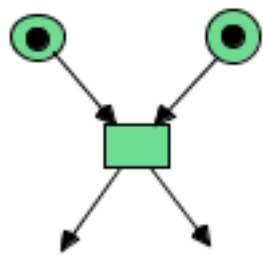
$\tau := |\{t \mid |t \cap M_0| = 1\}|$
is the number of **semi-active transitions**.

tokens

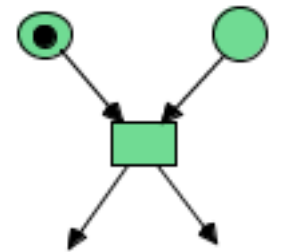
$\mu_0 := |M_0|$ is the **total number of tokens**.

μ_a is number of tokens
marking active transitions.

μ is number of tokens
marking semi-active transitions.



properties of the initial marking



transitions

$\tau_0 := |\{t \mid |t \cap M_0| \geq 1\}|$
is the number of marked transitions.

$$\tau_0 = \max\{\alpha, \beta\}$$

$\tau_a := |\{t \mid |t \cap M_0| = 2\}|$
is the number of active transitions.

$$\tau_a = \min\{\alpha, \beta\}$$

$\tau := |\{t \mid |t \cap M_0| = 1\}|$
is the number of semi-active transitions.

$$\tau = |\alpha - \beta|$$

tokens

$\mu_0 := |M_0|$ is the total number of tokens.

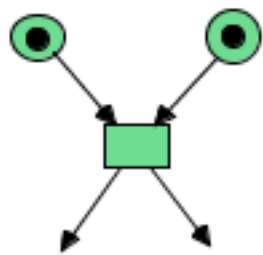
$$\mu_0 = \alpha + \beta$$

μ_a is number of tokens
marking active transitions.

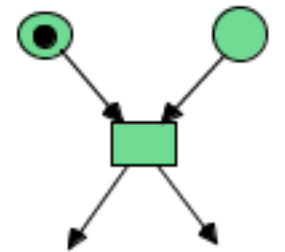
$$\mu_a = 2 \cdot \min\{\alpha, \beta\}$$

μ is number of tokens
marking semi-active transitions.

$$\mu = |\alpha - \beta|$$



properties of the initial marking



transitions

$\tau_0 := |\{t \mid |t \cap M_0| \geq 1\}|$
is the number of marked transitions.

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$$\tau = |\alpha - \beta|$$

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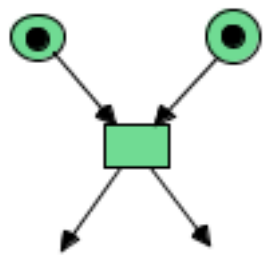
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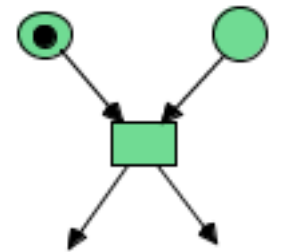
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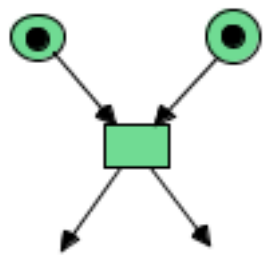
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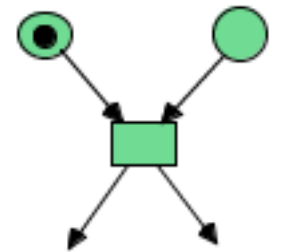
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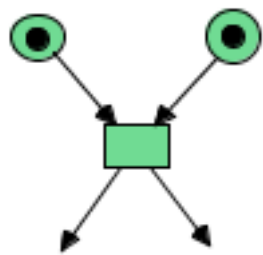
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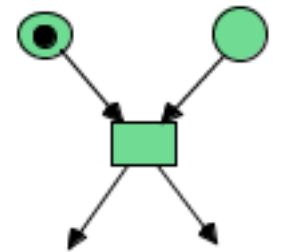
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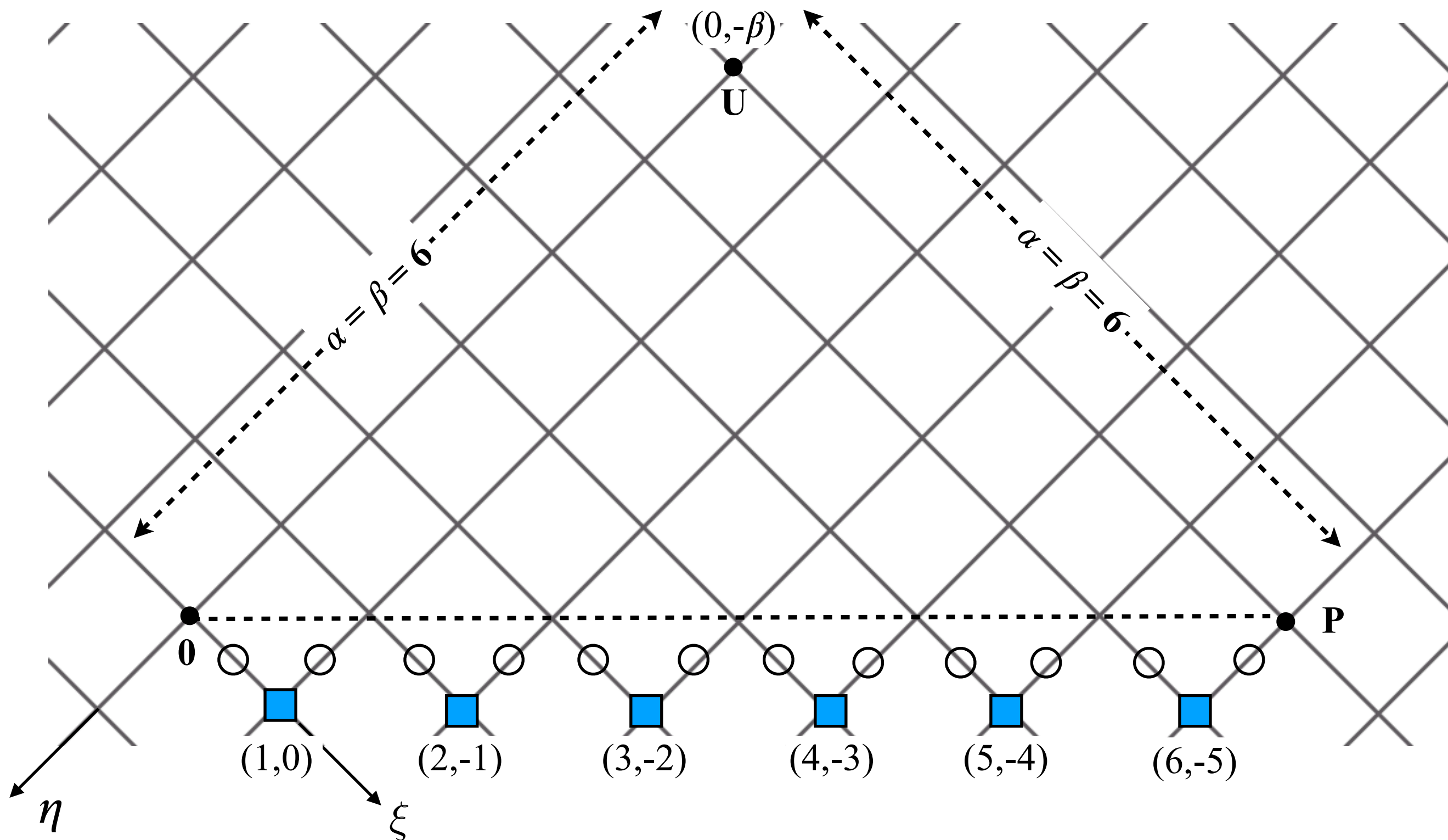
μ is number of tokens
marking semi-active transitions.

$$\mu = |\alpha - \beta|$$

μ is number of tokens

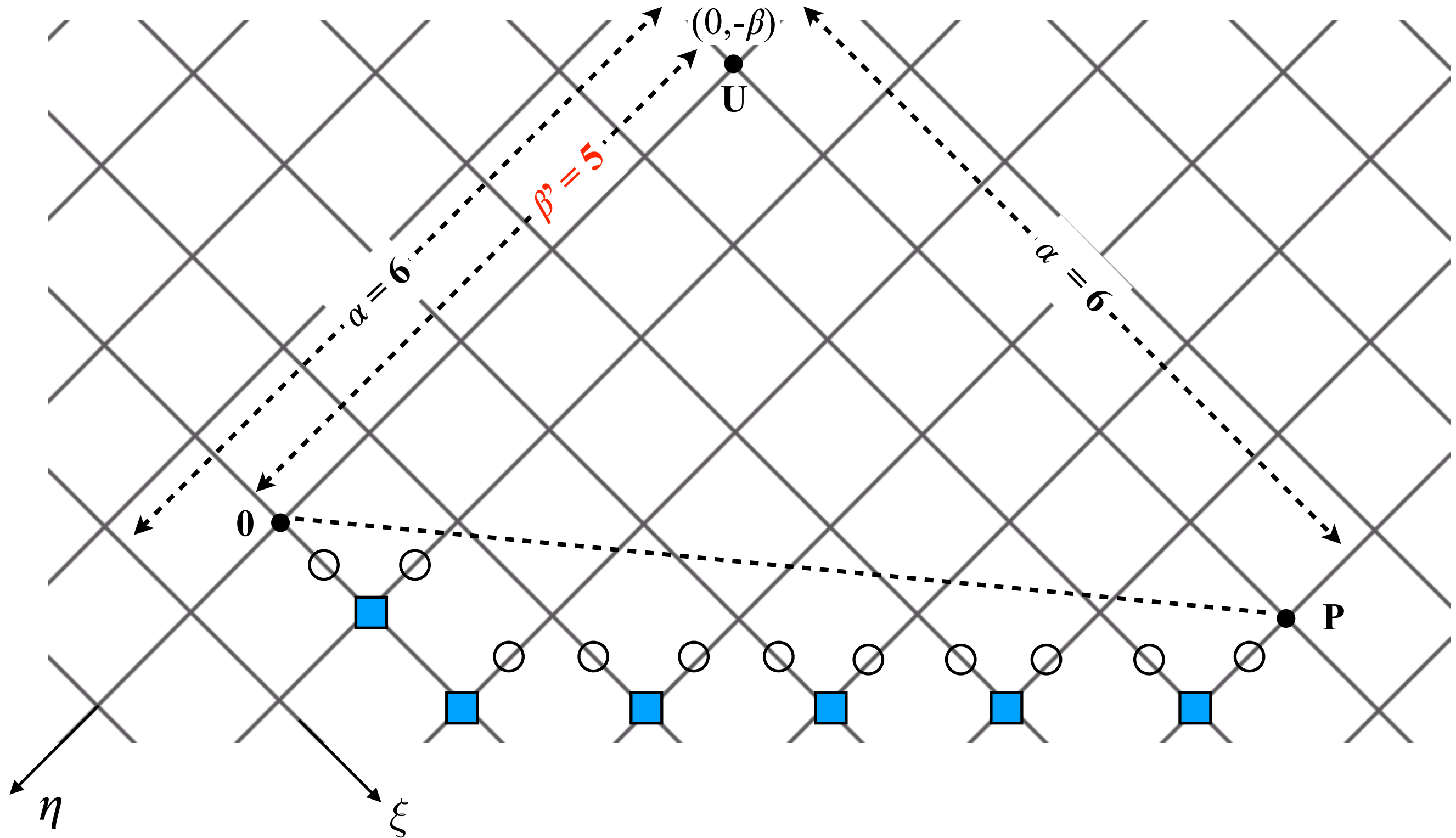
activating semi-active transitions.

$$\mu = |\alpha - \beta|$$



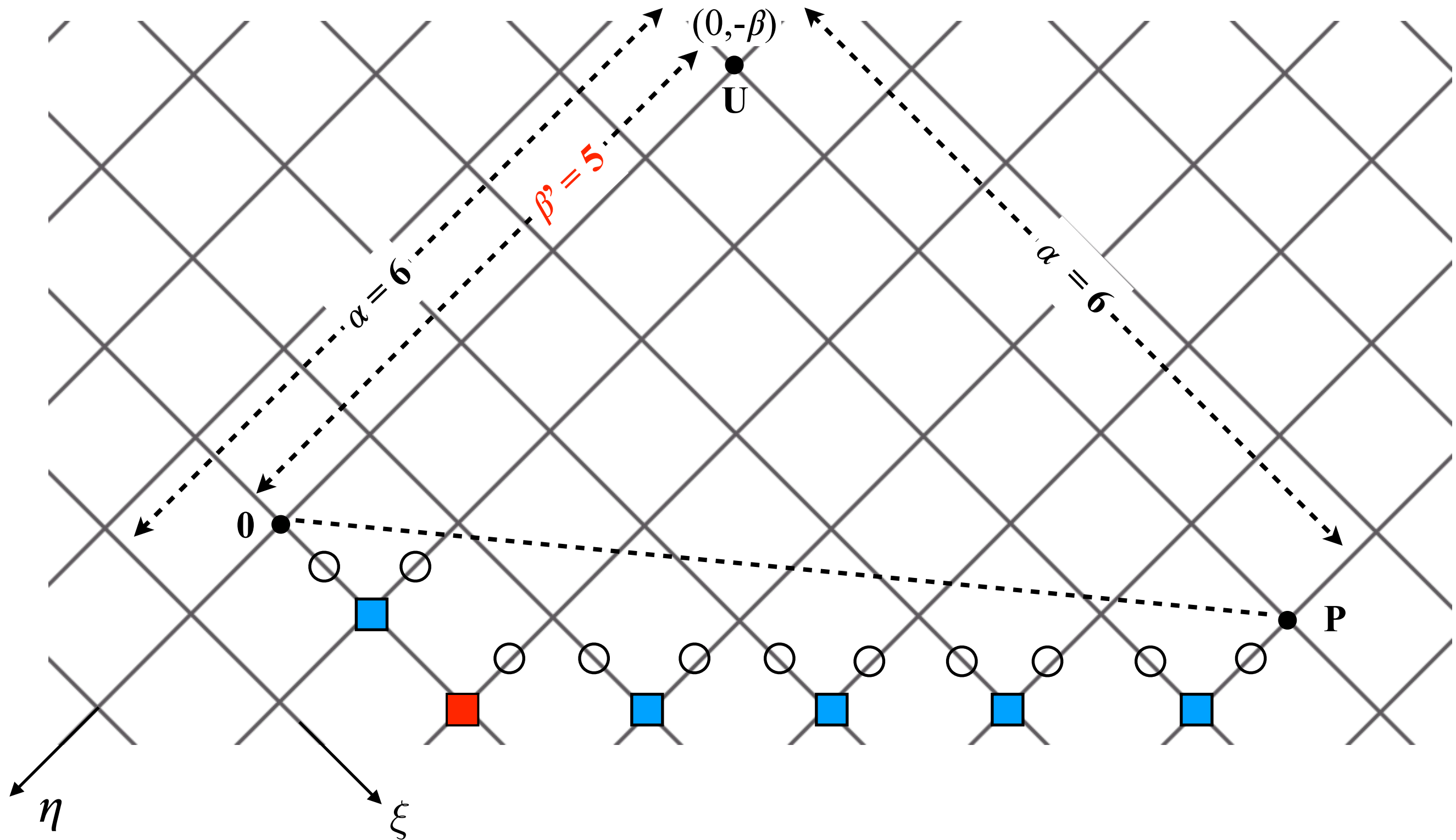
μ is number of tokens activating initially semi-active transitions.

$$\mu = |\alpha - \beta|$$



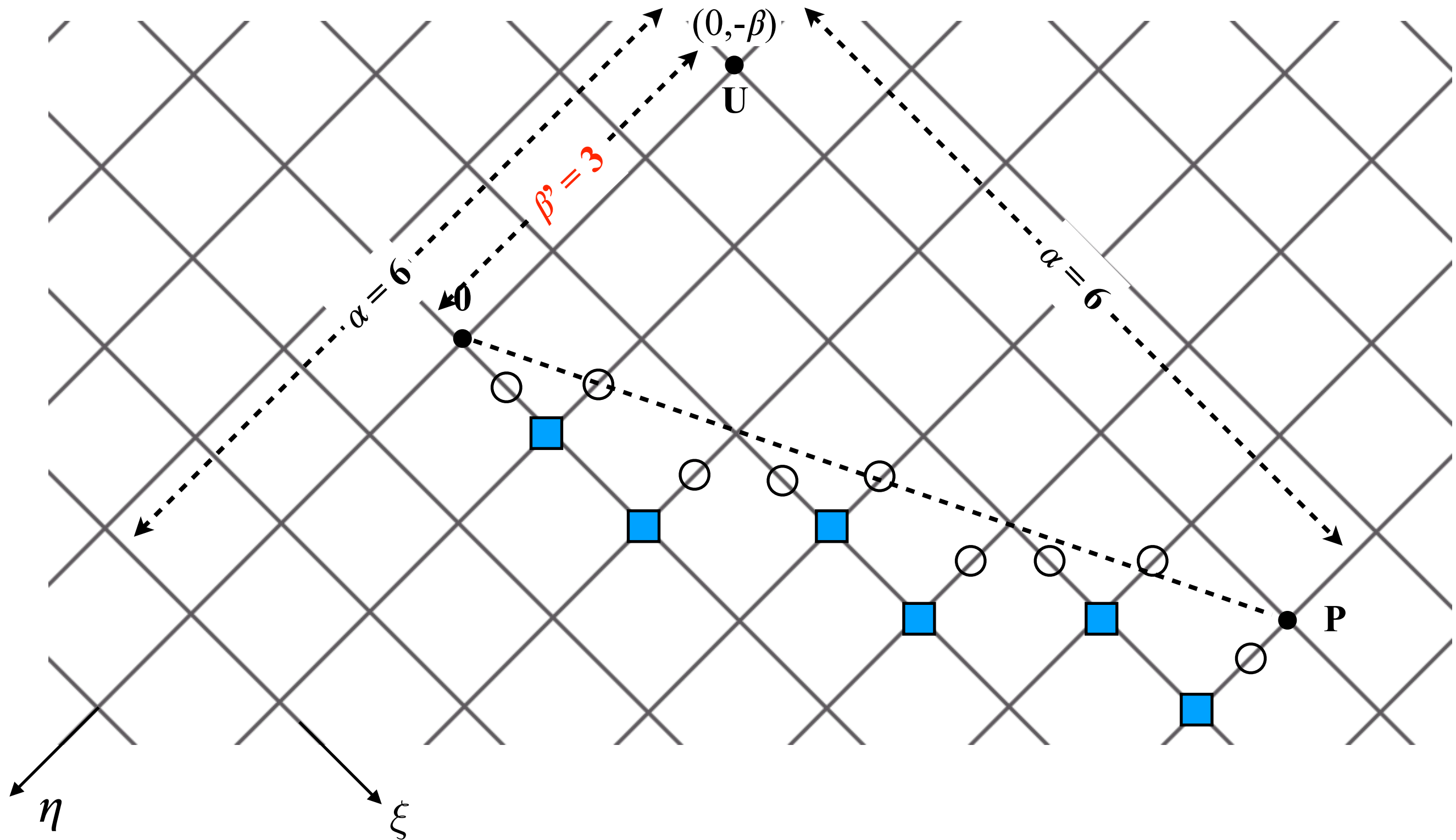
μ is number of tokens activating initially semi-active transitions.

$$\mu = |\alpha - \beta|$$



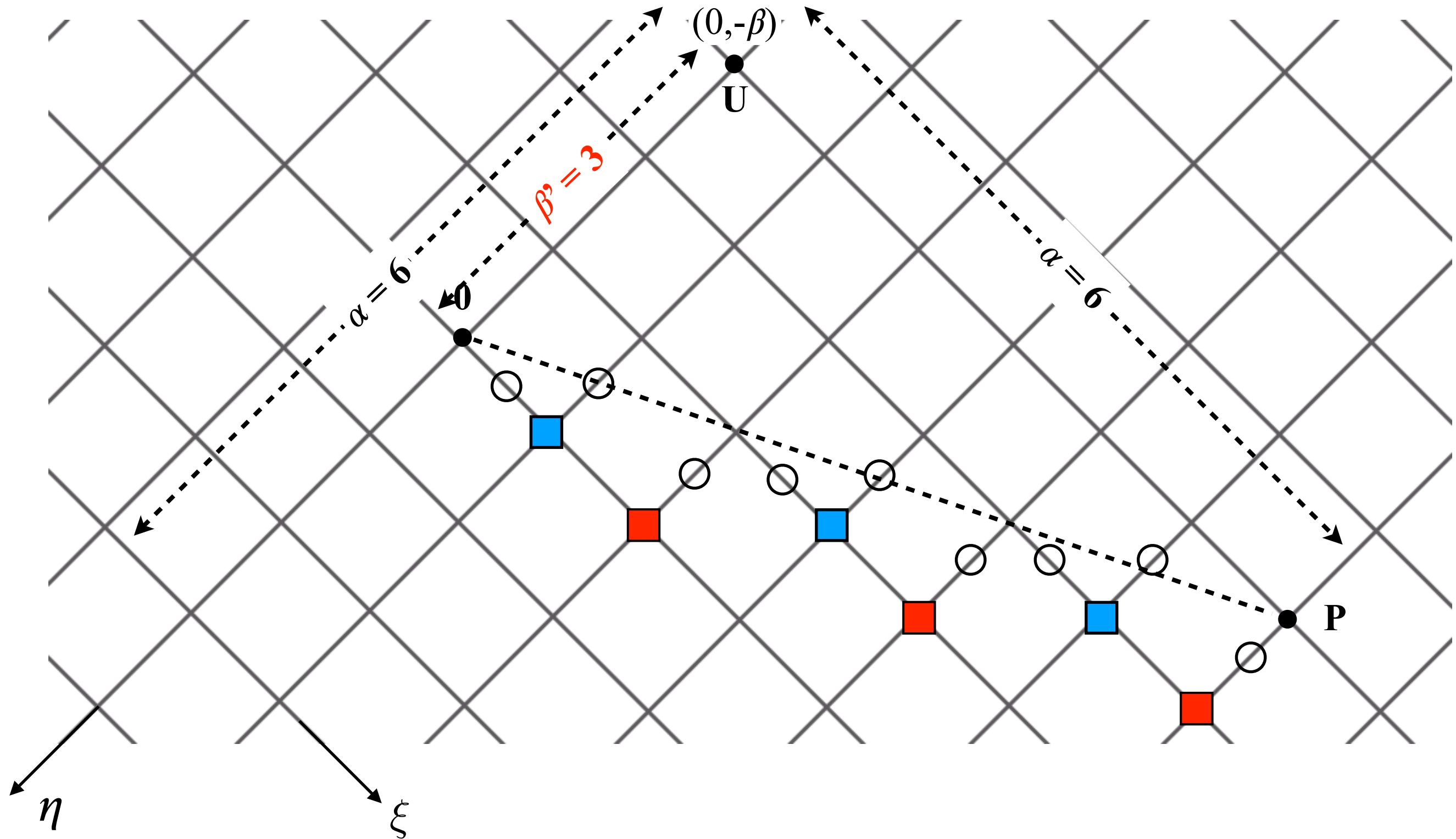
μ is number of tokens activating initially semi-active transitions.

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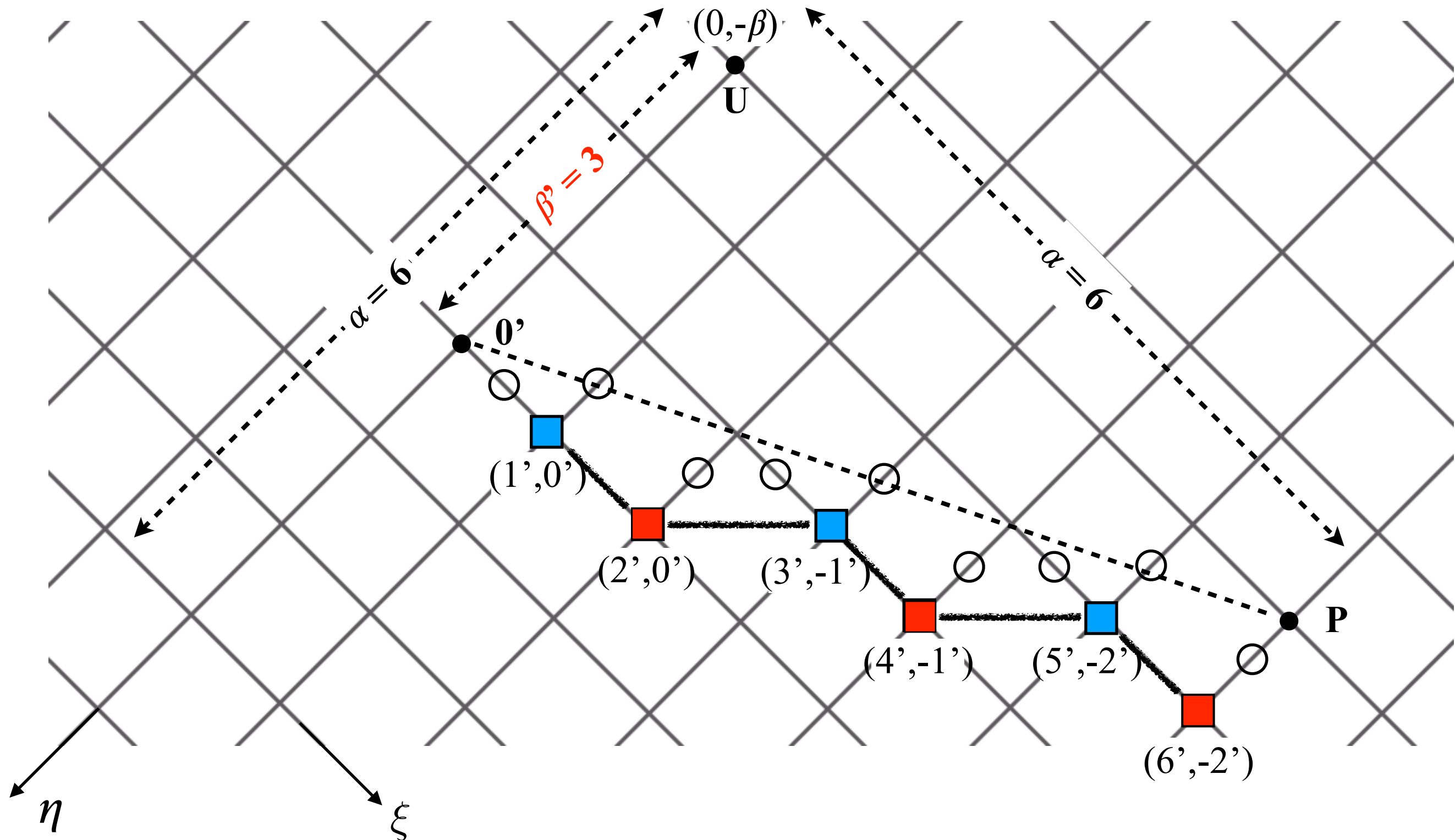
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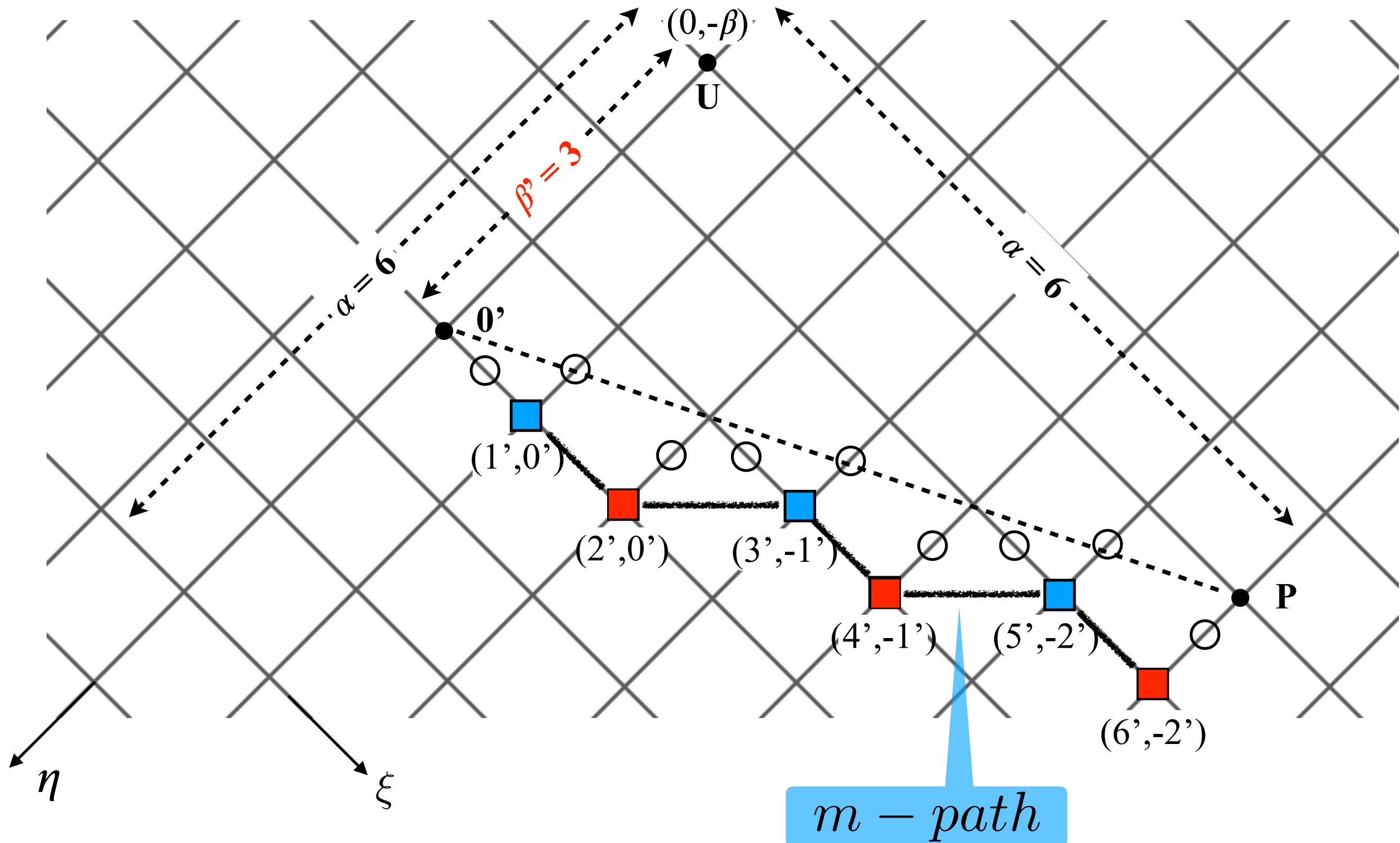
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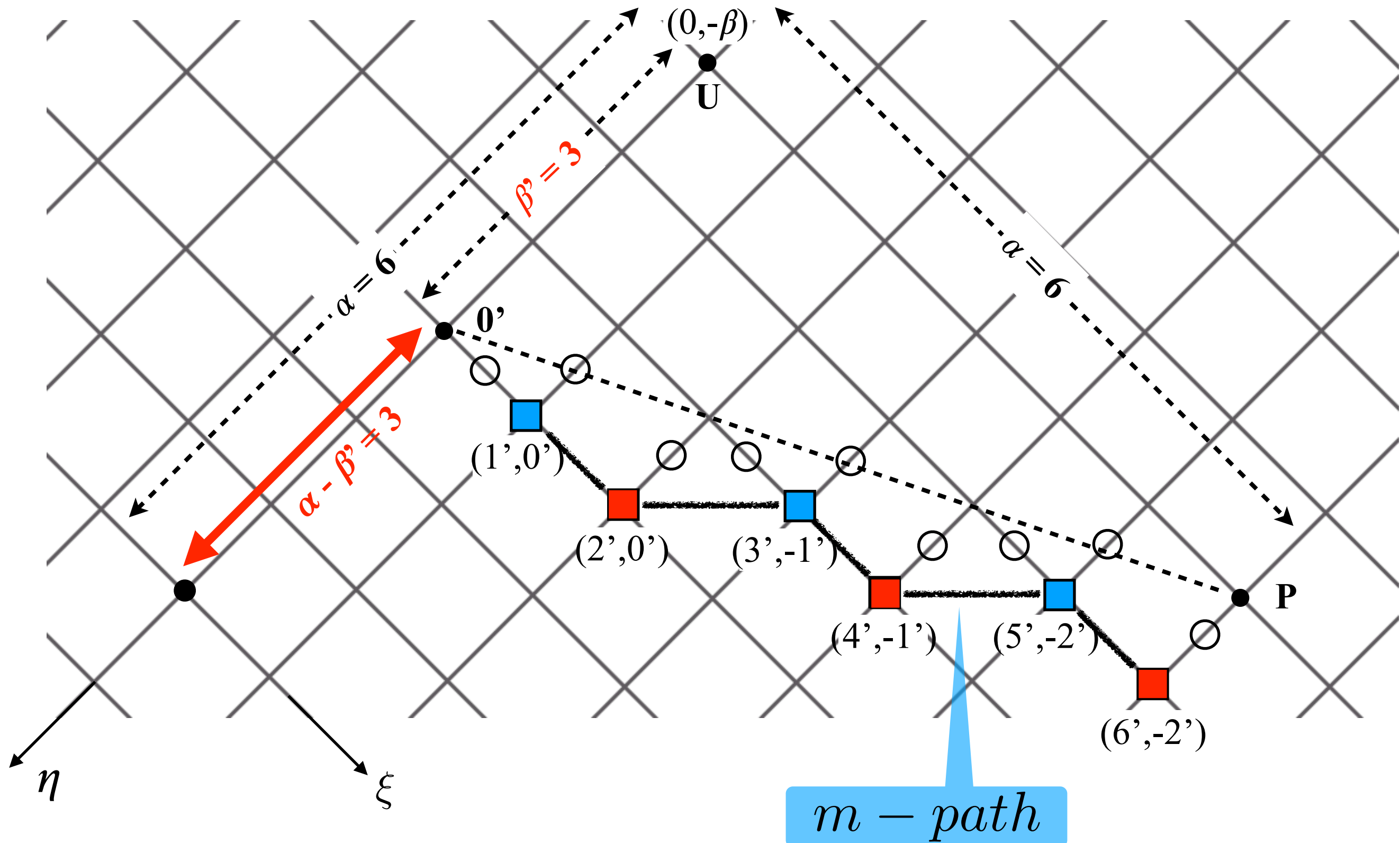
μ is number of tokens activating initially semi-active transitions.

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μ is number of tokens activating initially semi-active transitions.

$$\mu = |\alpha - \beta|$$



$$\textit{slowness} = \frac{|\alpha - \beta|}{\alpha + \beta} = \frac{\mu}{\mu_0} = \frac{\textit{number of semi-active tokens}}{\textit{number of all tokens}}$$

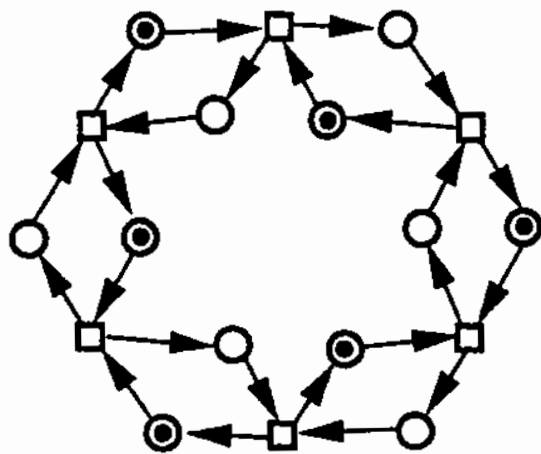


Fig. 18: (3,3,1,1) – Slowness = 0

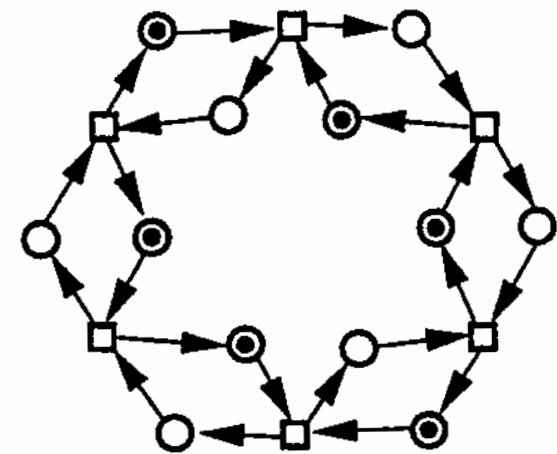


Fig. 19: (4,2,1,1) – Slowness = 1/3

$$\text{slowness} = \frac{|\alpha - \beta|}{\alpha + \beta} = \frac{\mu}{\mu_0} = \frac{\text{number of semi-active tokens}}{\text{number of all tokens}}$$

active
marked

$$\tau_a = 3 \quad \alpha = 3$$

$$\tau_0 = 3 \quad \beta = 3$$

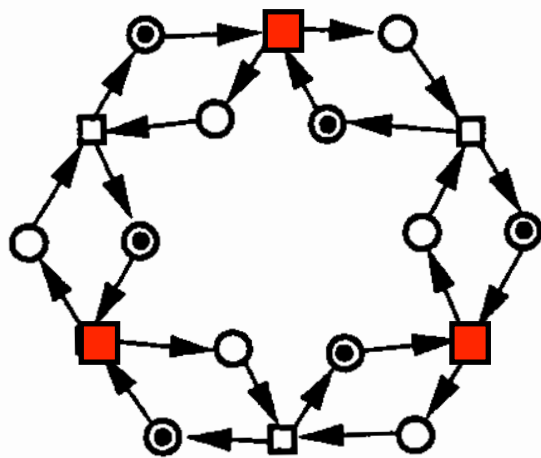


Fig. 18: (3,3,1,1) – Slowness = 0

$$\tau_a = 2 \quad \beta = 2$$

$$\tau_0 = 4 \quad \alpha = 4$$

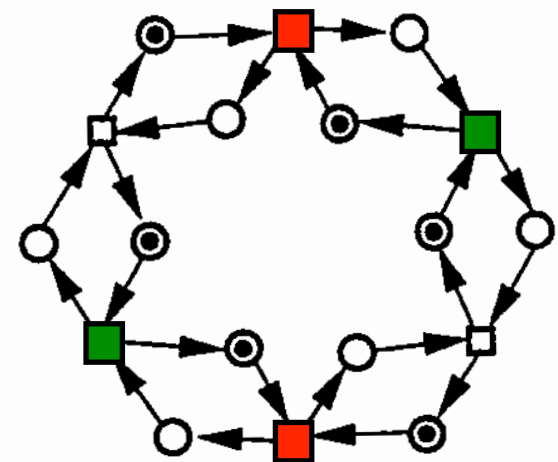


Fig. 19: (4,2,1,1) – Slowness = 1/3

$$\text{slowness} = \frac{|\alpha - \beta|}{\alpha + \beta} = \frac{\mu}{\mu_0} = \frac{\text{number of semi-active tokens}}{\text{number of all tokens}}$$

$$\tau_a = \min\{\alpha, \beta\}$$

$$\tau_0 = \max\{\alpha, \beta\}$$

active

$$\tau_a = 3 \quad \alpha = 3$$

marked

$$\tau_0 = 3 \quad \beta = 3$$

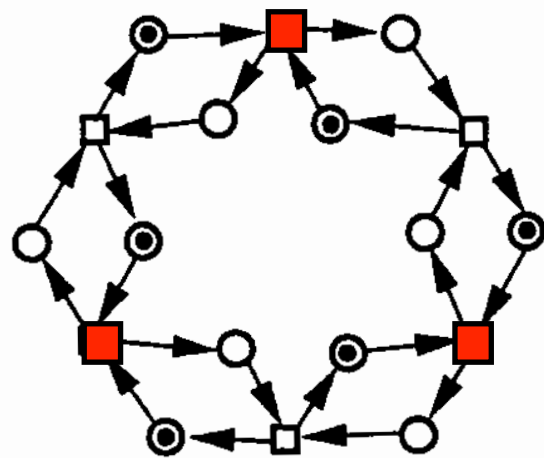


Fig. 18: (3,3,1,1) – Slowness = 0

$$\tau_a = 2 \quad \beta = 2$$

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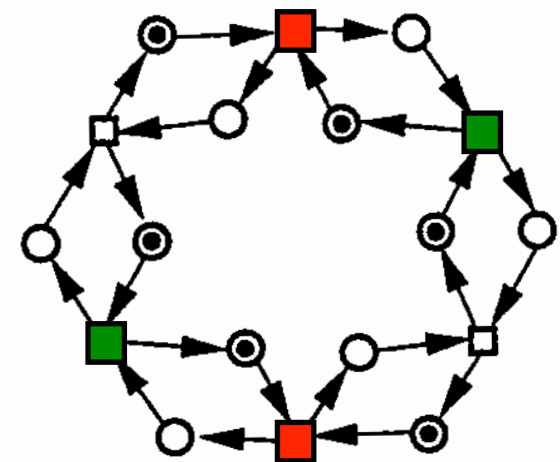


Fig. 19: (4,2,1,1) – Slowness = 1/3

$$\text{slowness} = \frac{|\alpha - \beta|}{\alpha + \beta} = \frac{\mu}{\mu_0} = \frac{\text{number of semi-active tokens}}{\text{number of all tokens}}$$

$$\tau_a = \min\{\alpha, \beta\}$$

$$\tau_0 = \max\{\alpha, \beta\}$$

$$\gamma = ?$$

$$\delta = ?$$

active

$$\tau_a = 3 \quad \alpha = 3$$

marked

$$\tau_0 = 3 \quad \beta = 3$$

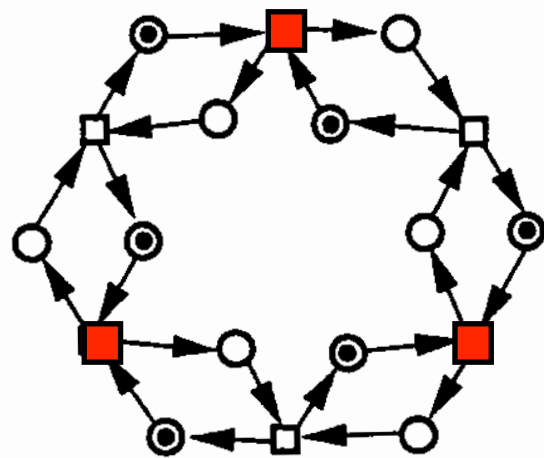


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$$\tau_a = 2 \quad \beta = 2$$

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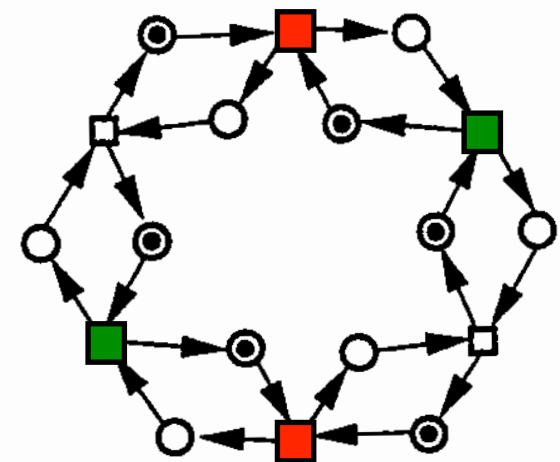
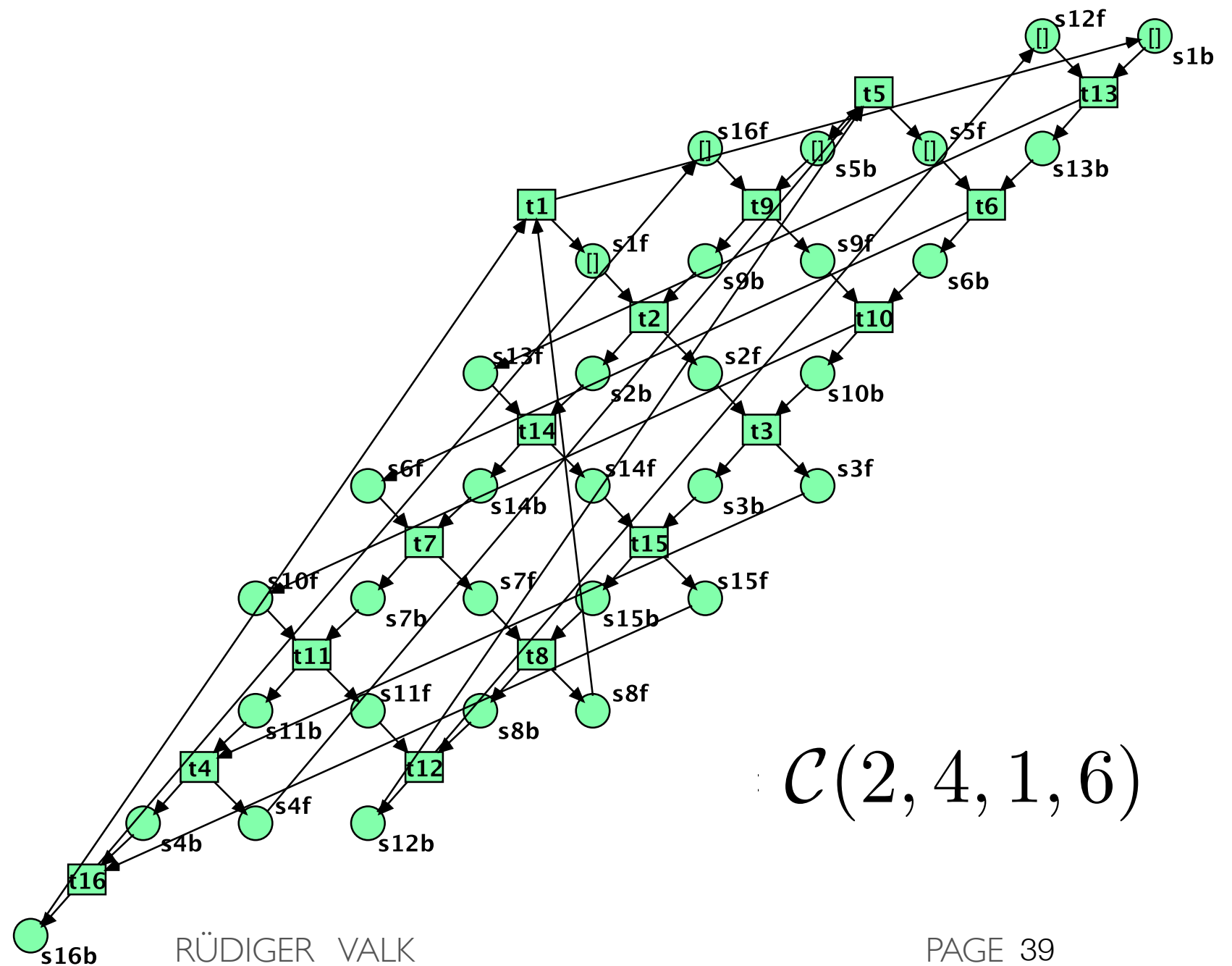


Fig. 19: (4,2,1,1) – Slowness = 1/3

Contents:

1. Introduction and Examples
2. From Petri-Space to Cycloids (informal)
3. Cycloids $\mathcal{C}(\alpha, \beta, \gamma, \delta)$ (formal)
4. The Initial Parameters α and β
- 5. The Final Parameters γ and δ**
6. Computing Cycloid Parameters α , β , γ and δ
7. Outlook

$$\gamma = ? \quad \delta = ?$$

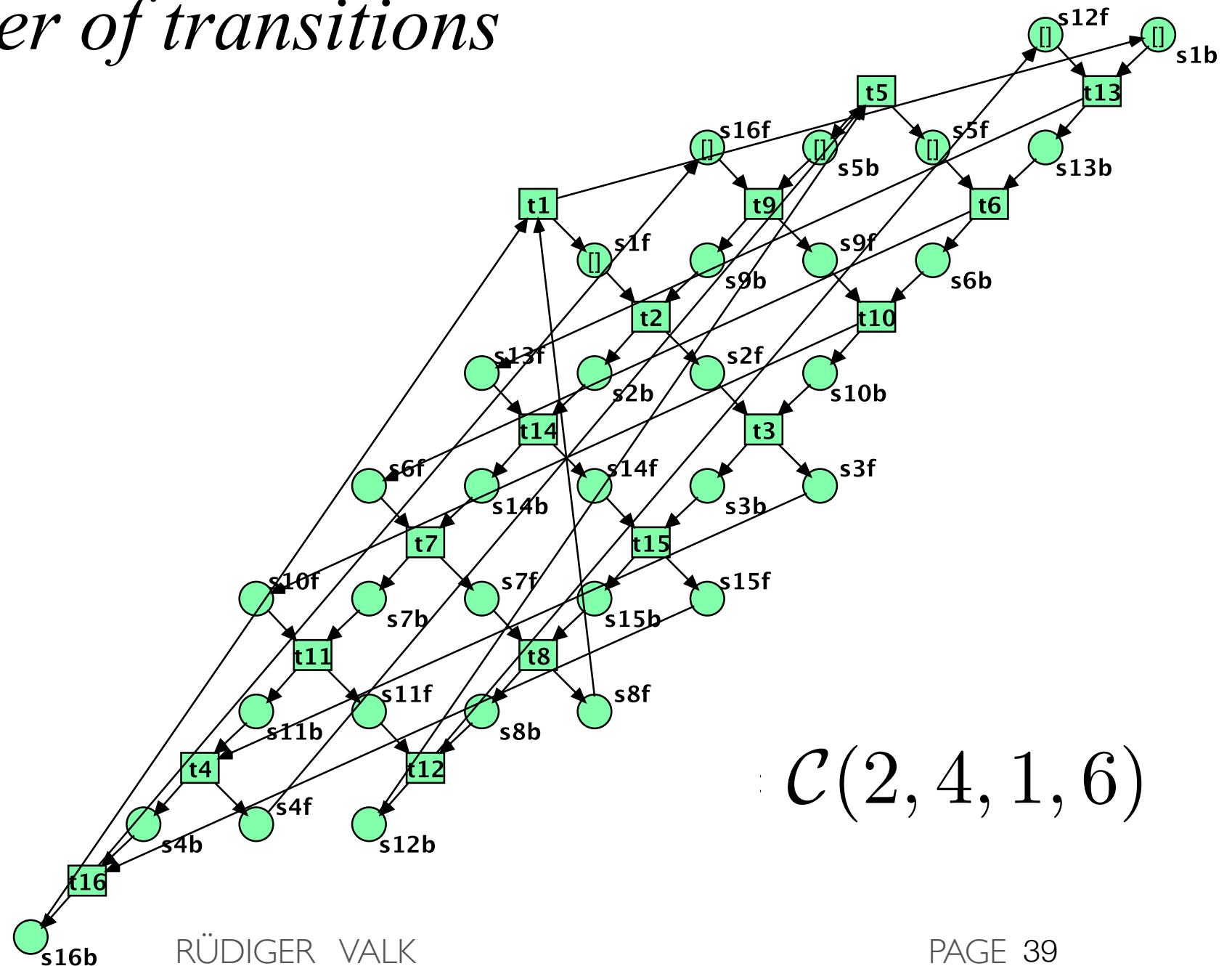


$$\mathcal{C}(2, 4, 1, 6)$$

$$\gamma = ? \quad \delta = ?$$

$$A = \alpha \cdot \delta + \beta \cdot \gamma = 2 \cdot 6 + 4 \cdot 1 = 16$$

Area: Number of transitions



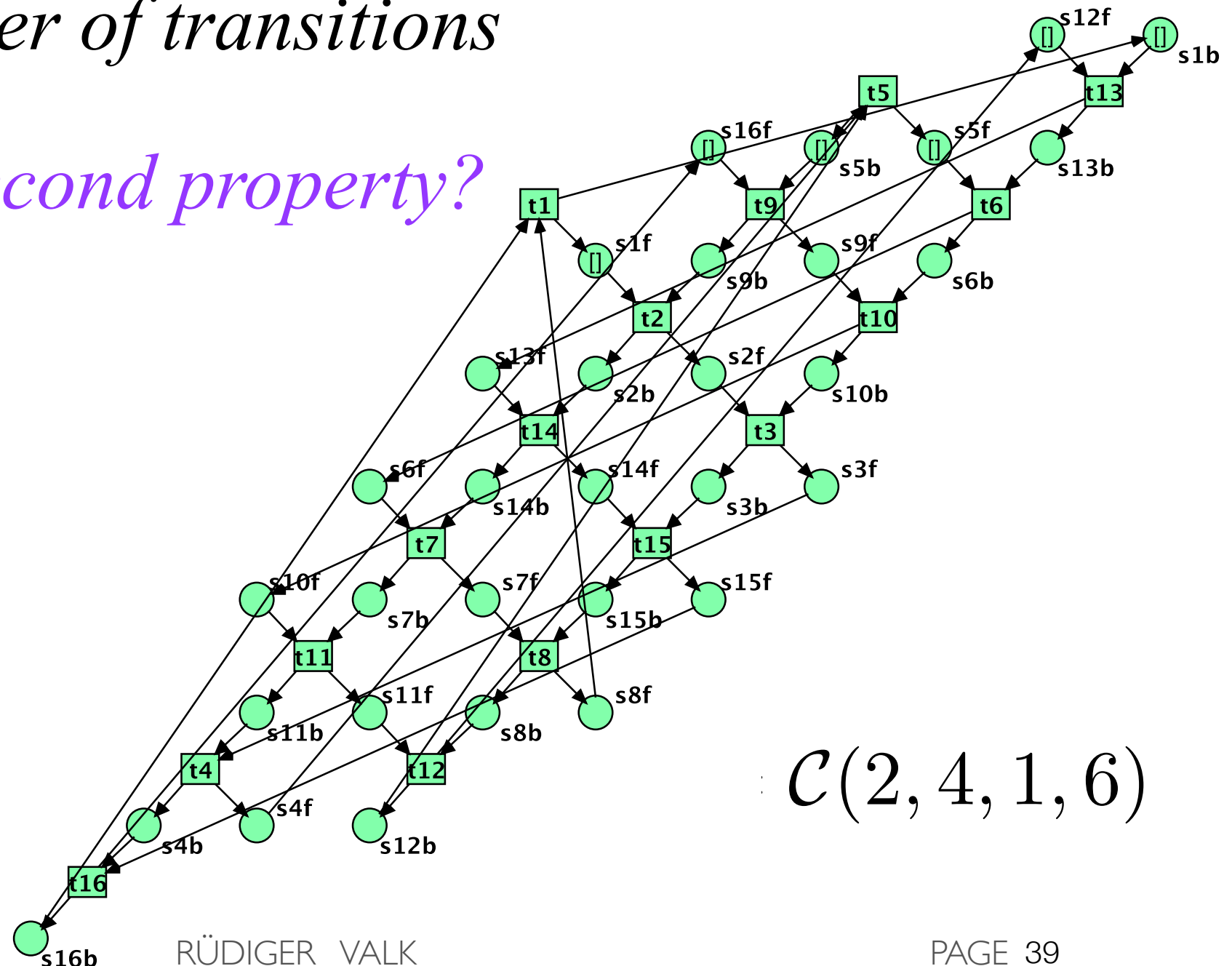
$$\mathcal{C}(2, 4, 1, 6)$$

$$\gamma = ? \quad \delta = ?$$

$$A = \alpha \cdot \delta + \beta \cdot \gamma = 2 \cdot 6 + 4 \cdot 1 = 16$$

Area: Number of transitions

What to take as a second property?



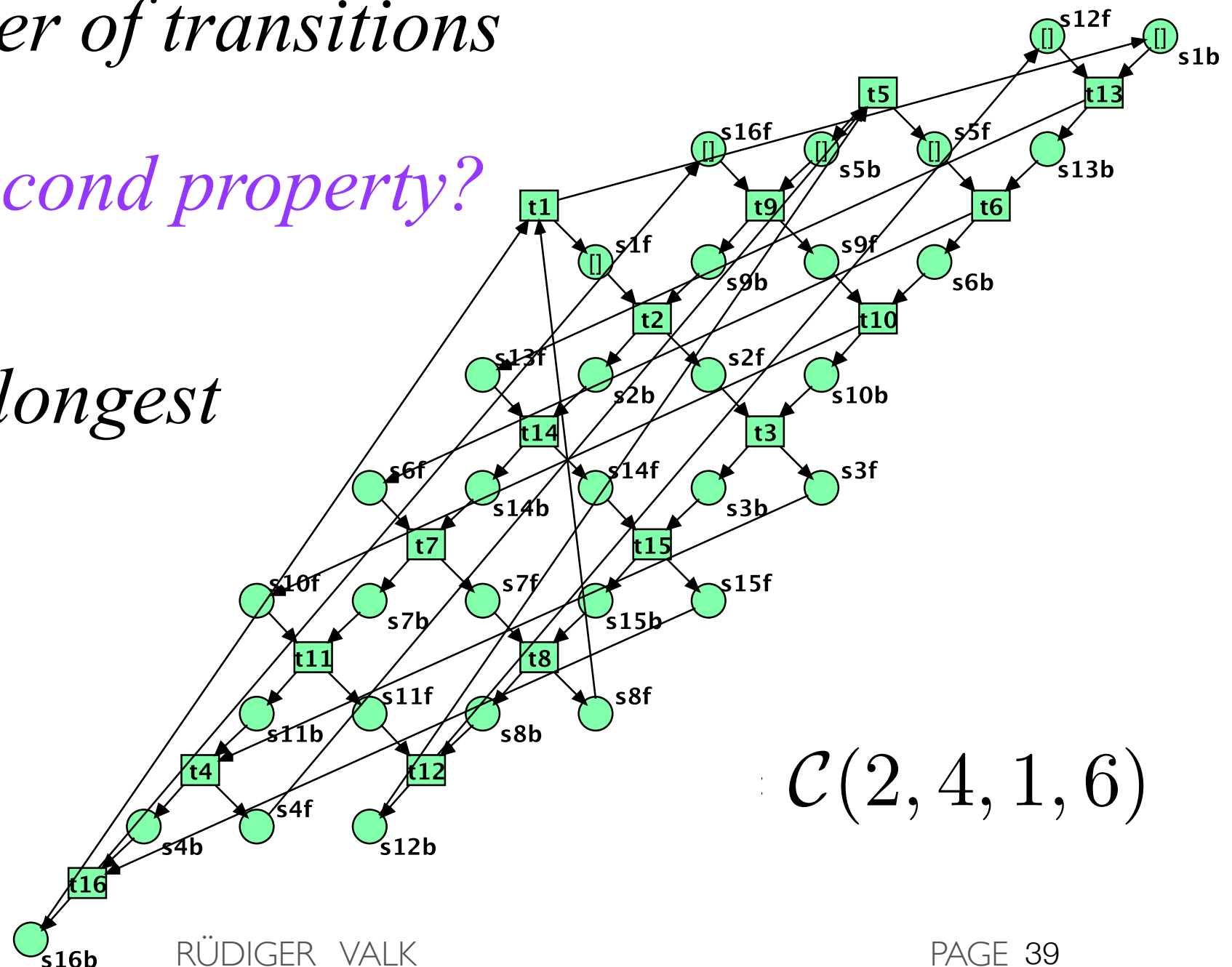
$$\gamma = ? \quad \delta = ?$$

$$A = \alpha \cdot \delta + \beta \cdot \gamma = 2 \cdot 6 + 4 \cdot 1 = 16$$

Area: Number of transitions

What to take as a second property?

*length of shortest/longest
cycle?*



$$\mathcal{C}(2, 4, 1, 6)$$

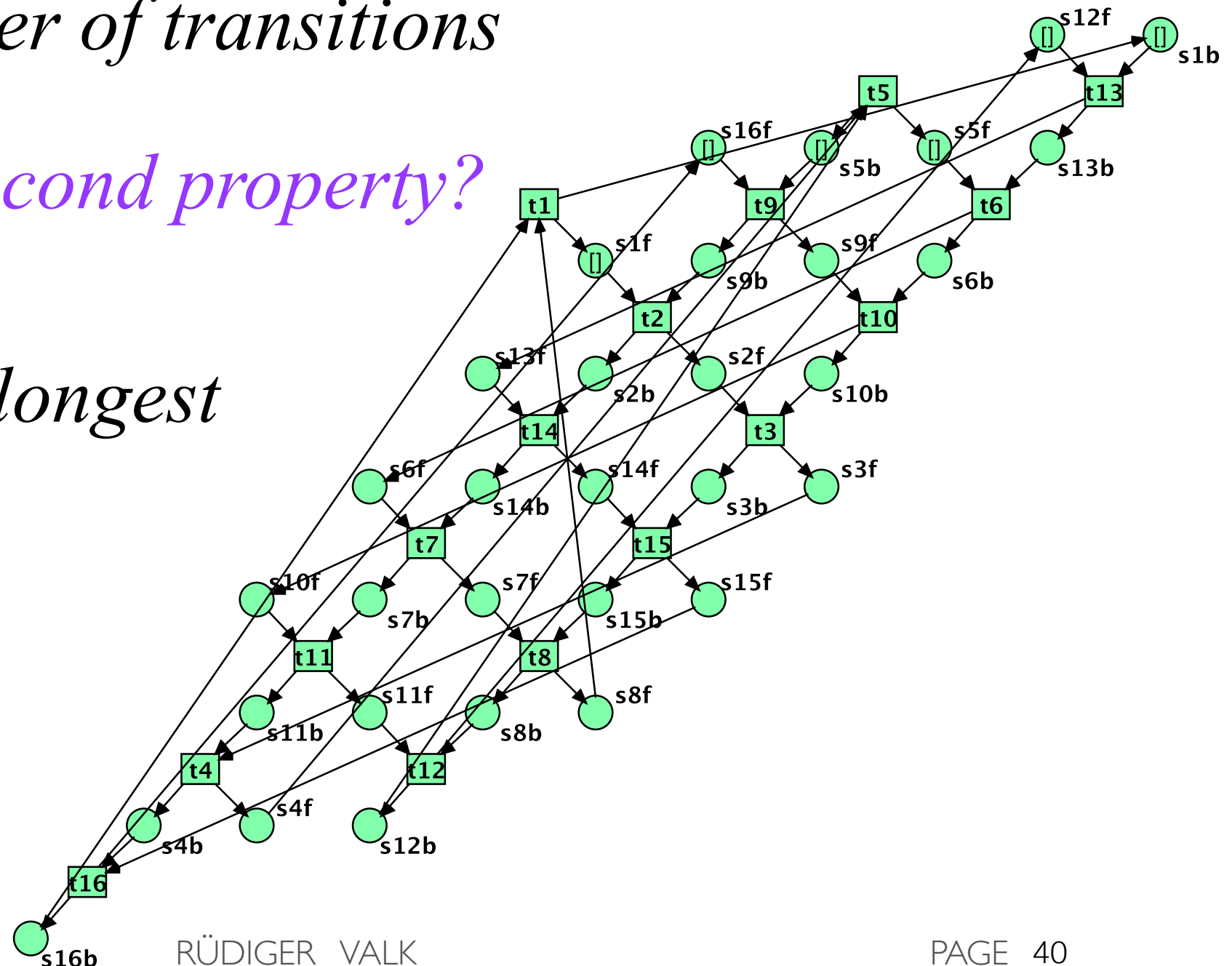
$$\gamma =? \quad \delta =?$$

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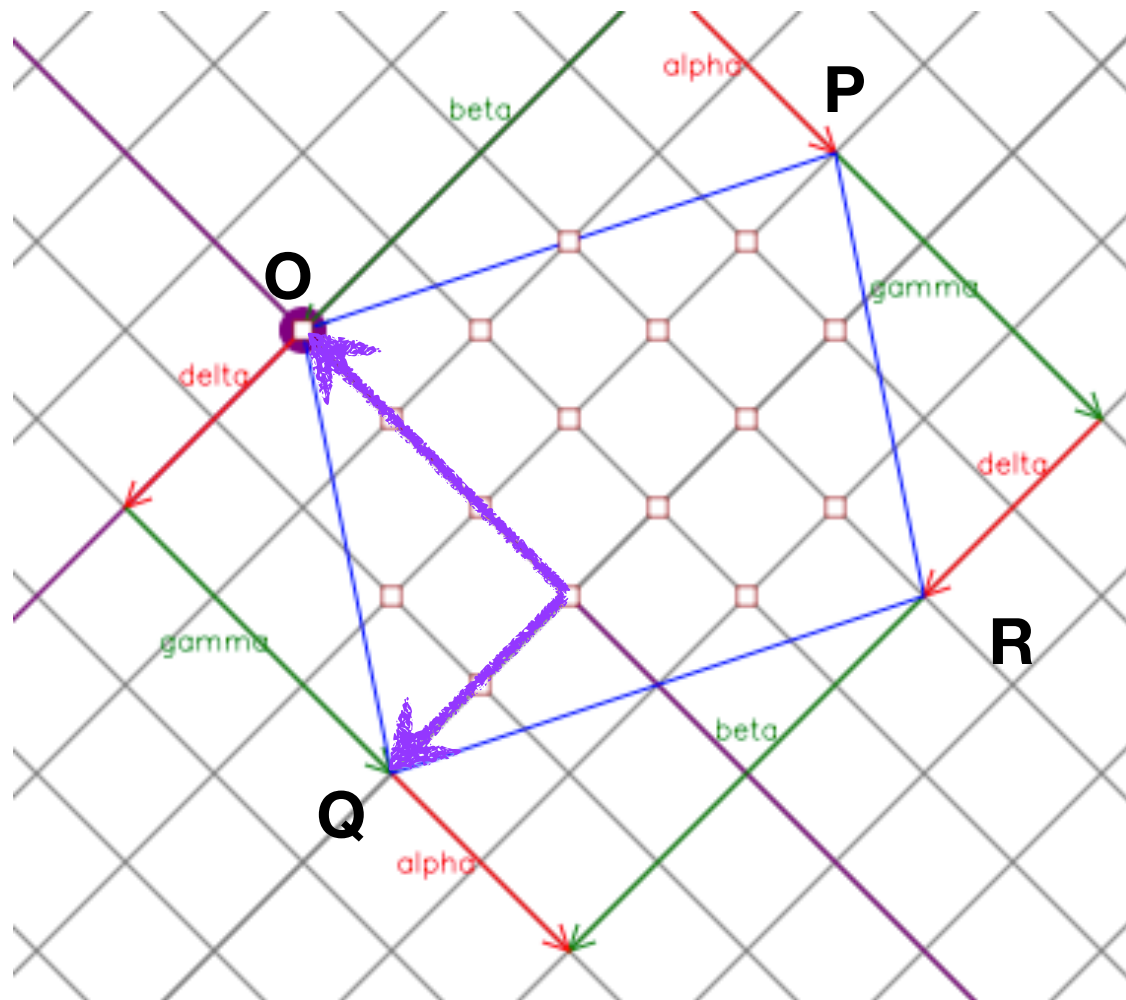
Area: Number of transitions

What to take as a second property?

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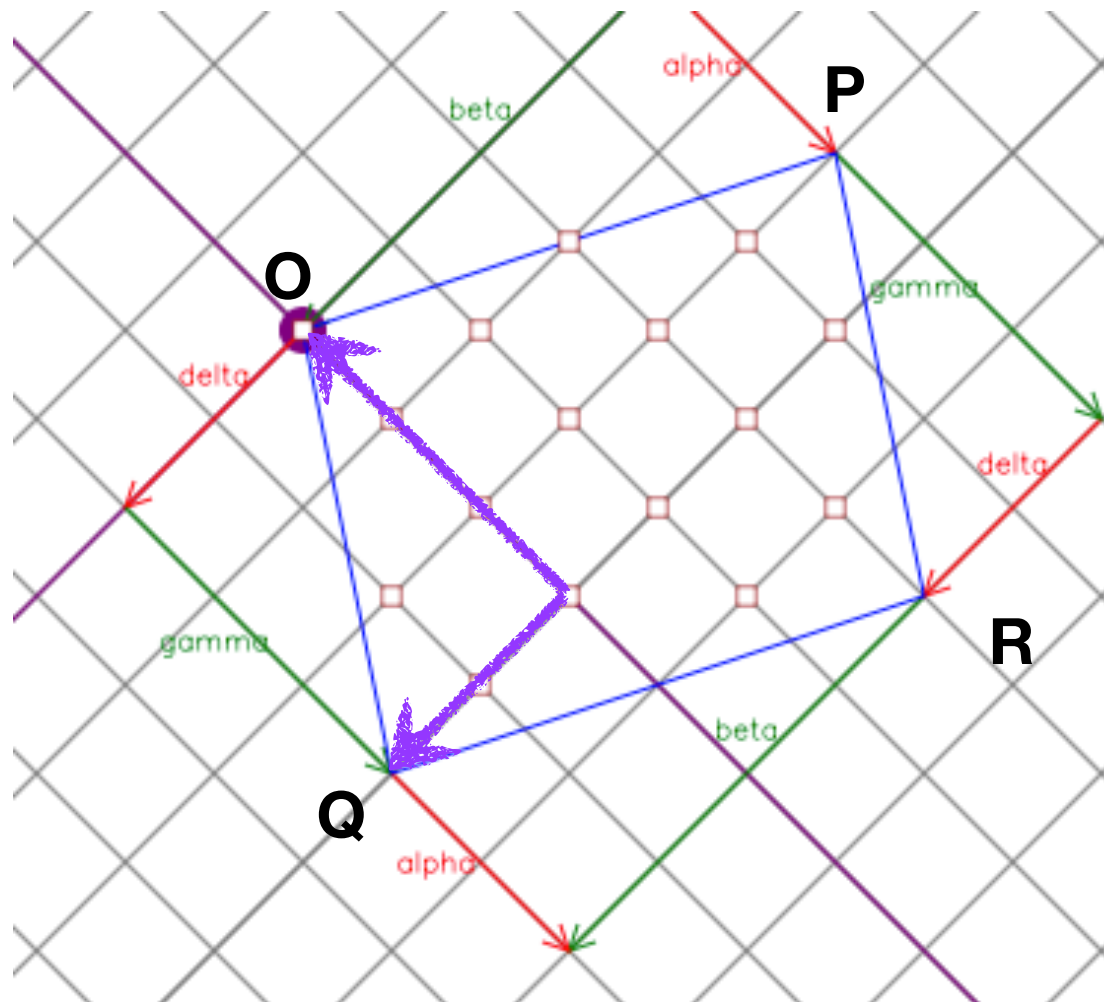
$$\mathcal{C}(2, 4, 3, 2)$$



Petri: “number of segments
in local basic circuit“

$$d(O, Q) = \gamma + \delta = 3 + 2 = 5$$

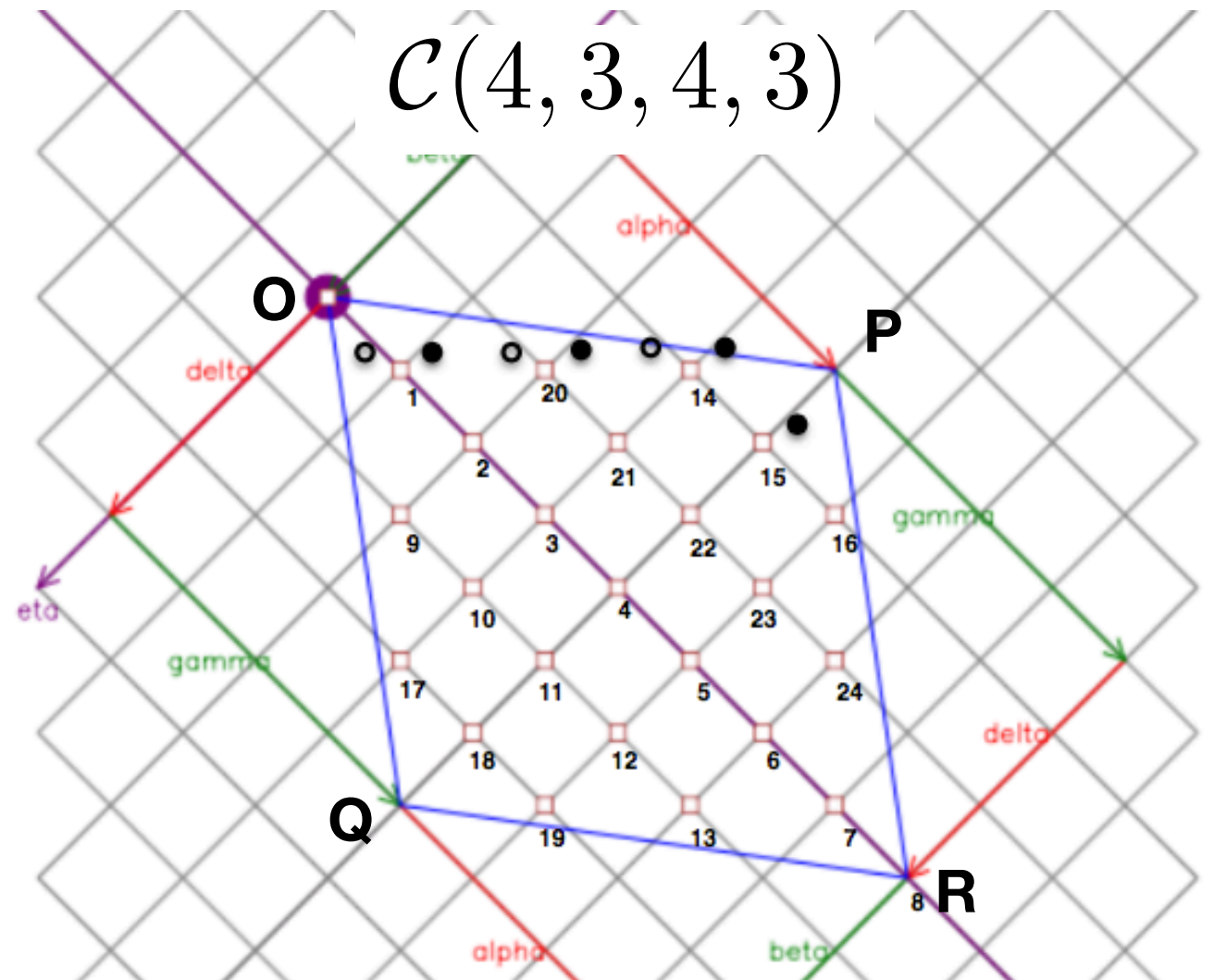
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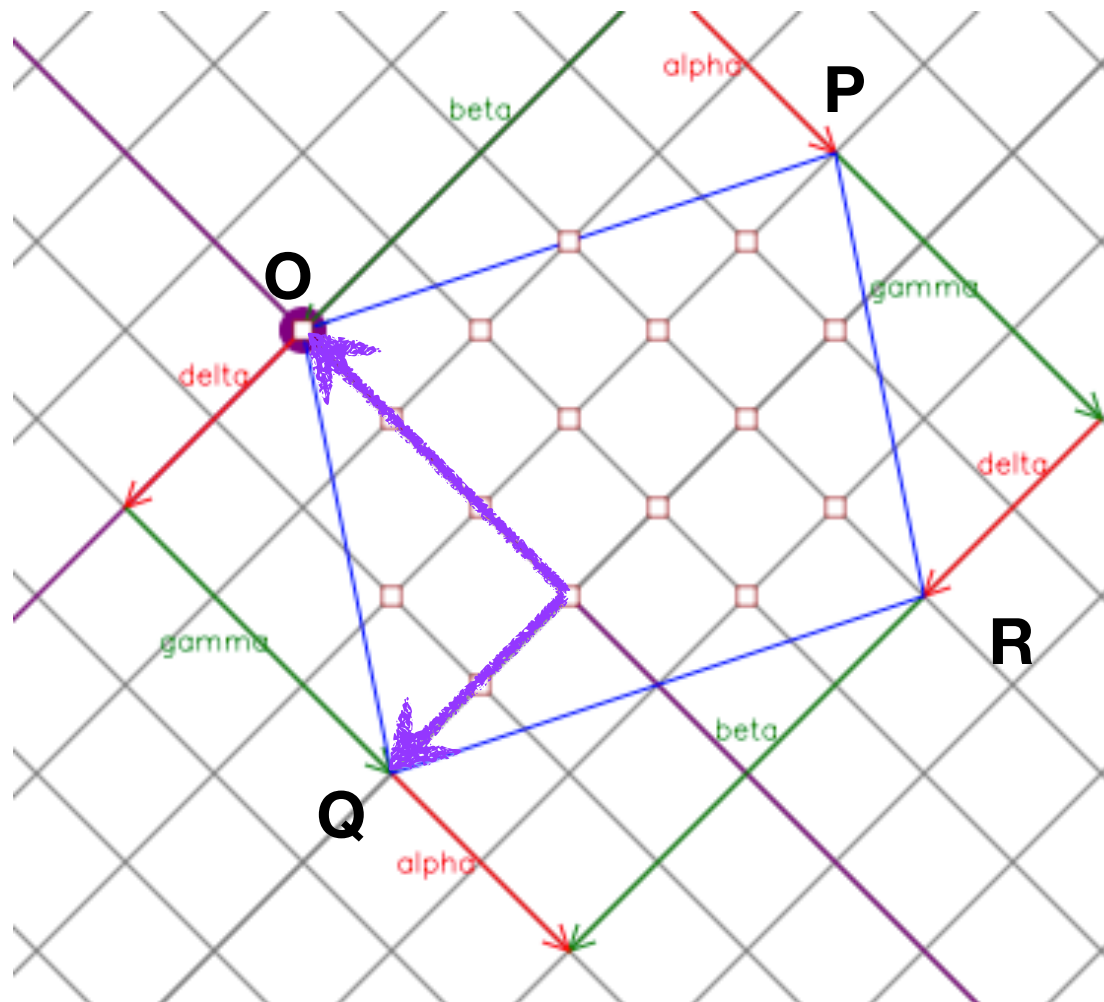
$$d(O, Q) = \gamma + \delta = 3 + 2 = 5$$

$$\mathcal{C}(4, 3, 4, 3)$$



$$d(O, Q) = \gamma + \delta = 4 + 3 = 7$$

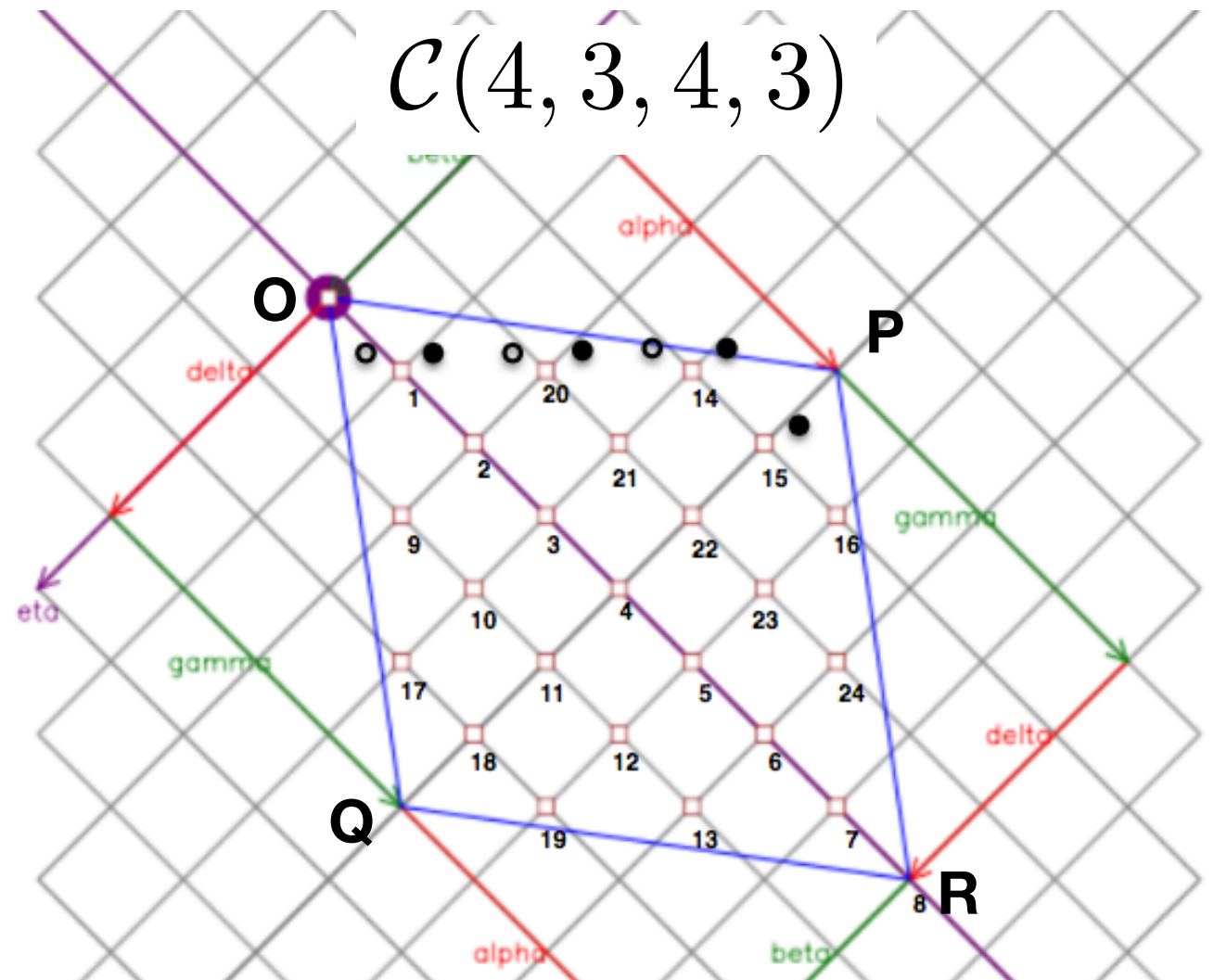
$$\mathcal{C}(2, 4, 3, 2)$$



Petri: “number of segments
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$$d(O, Q) = \gamma + \delta = 3 + 2 = 5$$

$$\mathcal{C}(4, 3, 4, 3)$$

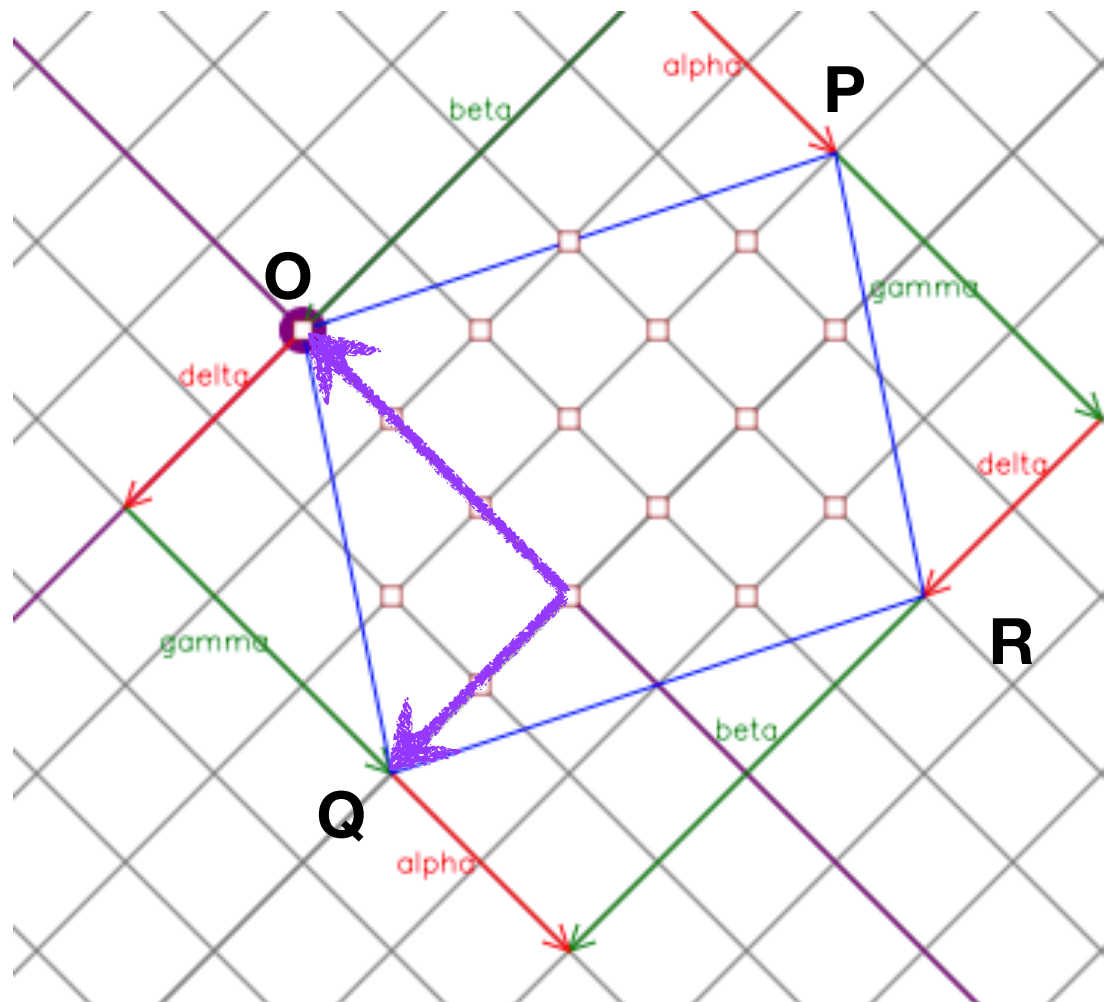


$$d(O, Q) = \gamma + \delta = 4 + 3 = 7$$

$$d(P, Q) = |\alpha - \gamma| + \beta + \delta =$$

$$|4 - 4| + 3 + 3 = 6$$

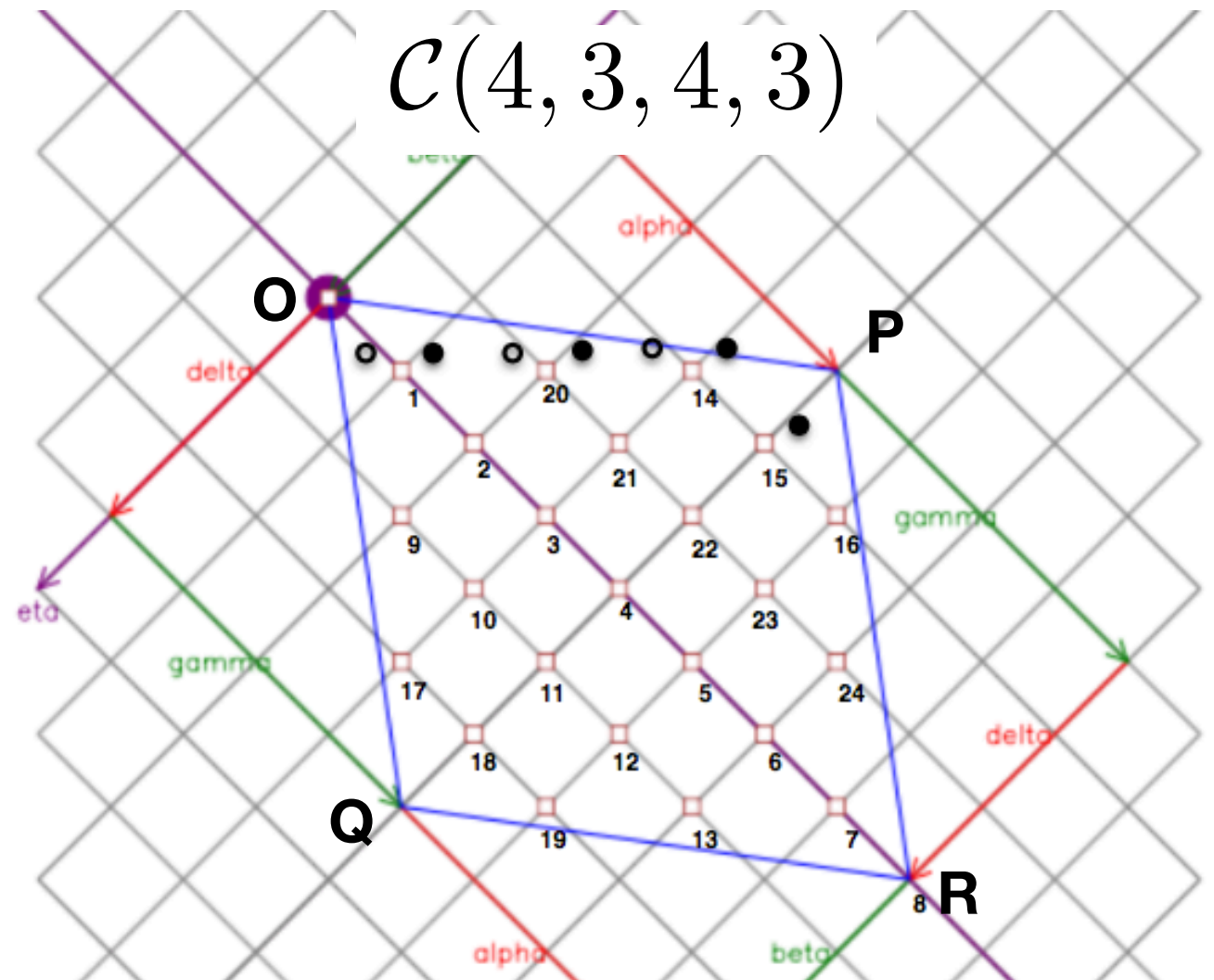
$$\mathcal{C}(2, 4, 3, 2)$$



Petri: “number of segments
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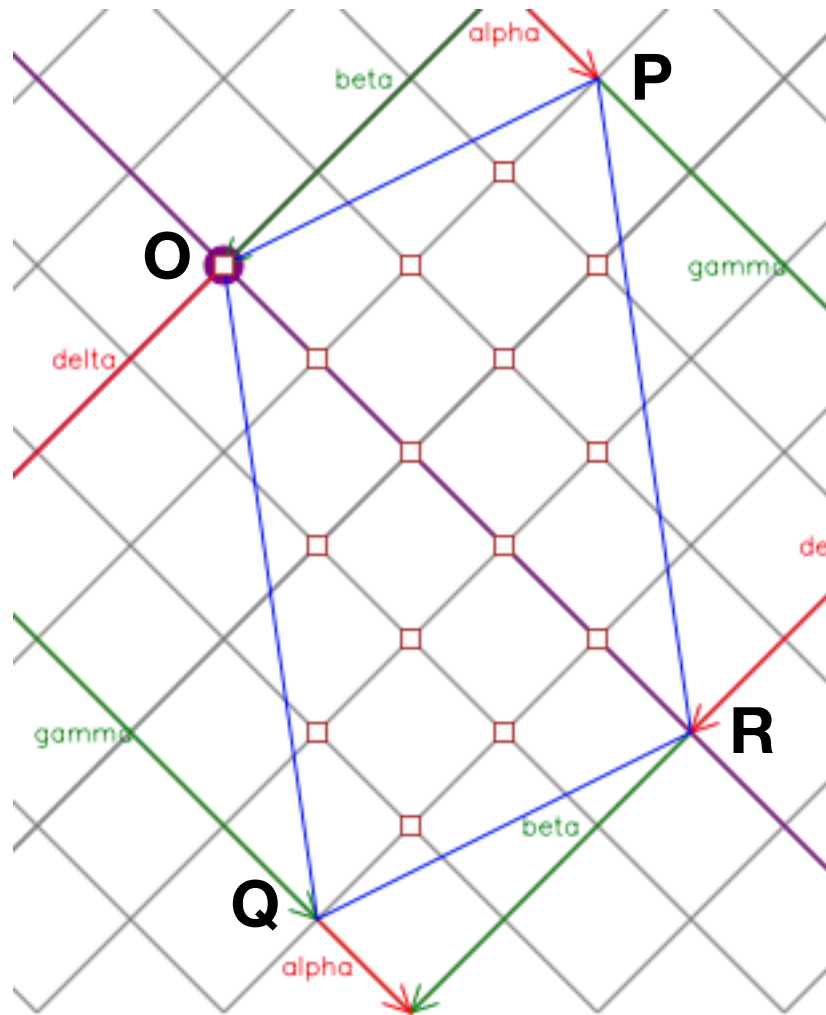
$$\mathcal{C}(4, 3, 4, 3)$$



$$d(O, Q) = \gamma + \delta = 4 + 3 = 7$$

$$d(P, Q) = |\alpha - \gamma| + \beta + \delta = |4 - 4| + 3 + 3 = 6$$

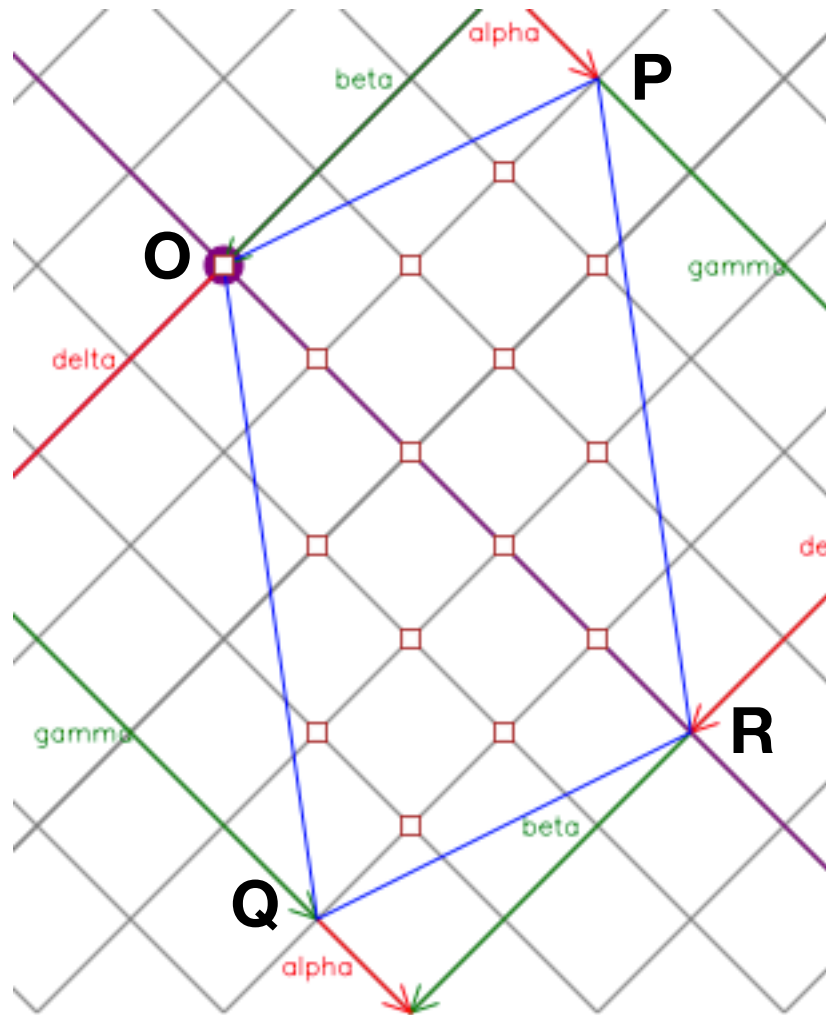
$$d(O, R) = |\beta - \delta| + \alpha + \gamma = |3 - 4| + 4 + 3 = 7$$



$\mathcal{C}(1, 3, 4, 3)$

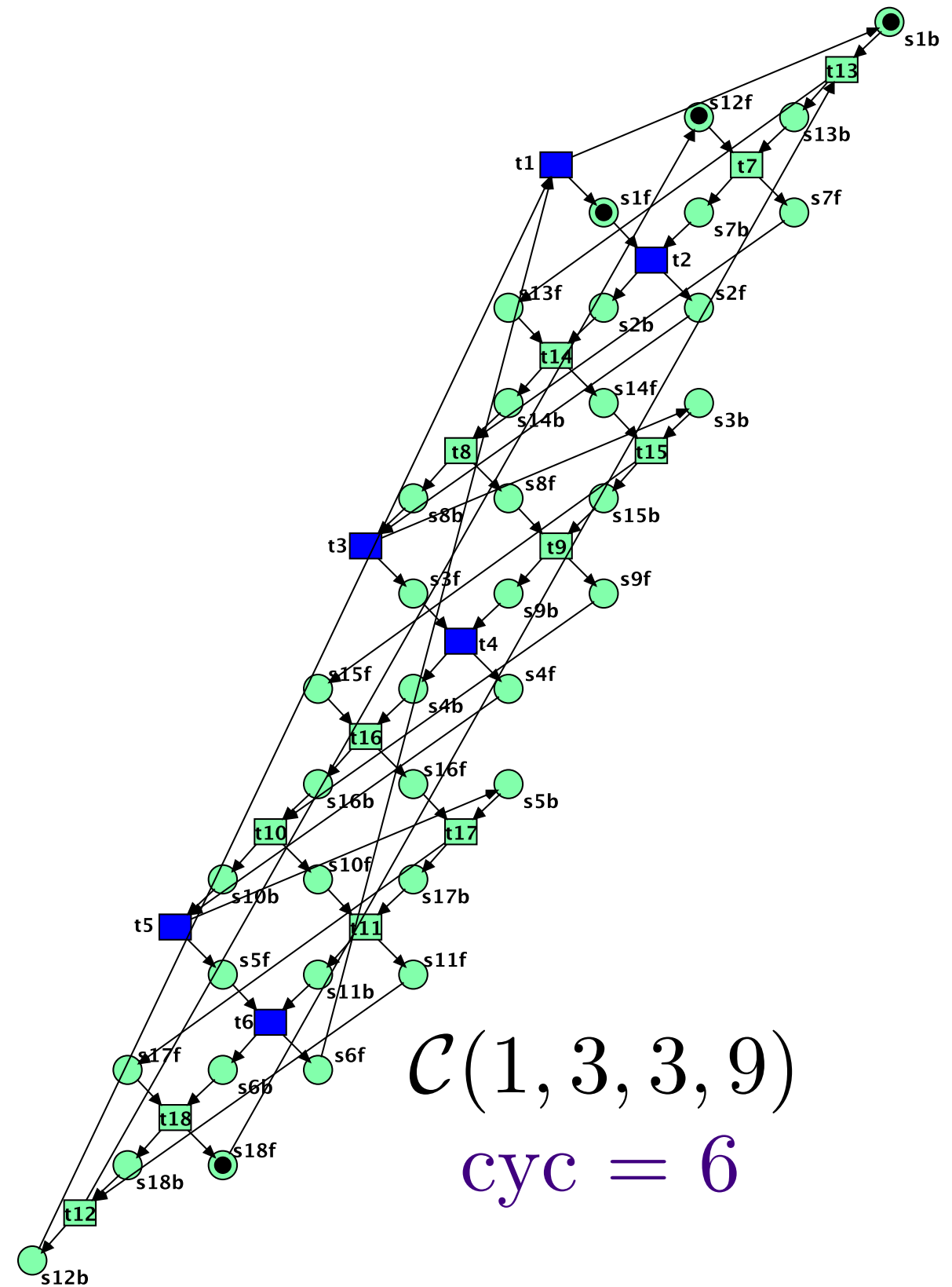
$$d(O, R) = |\beta - \delta| + \alpha + \gamma =$$

$$|3 - 3| + 1 + 4 = 5$$



$$\mathcal{C}(1, 3, 4, 3)$$

$$d(O, R) = |\beta - \delta| + \alpha + \gamma = |3 - 3| + 1 + 4 = 5$$



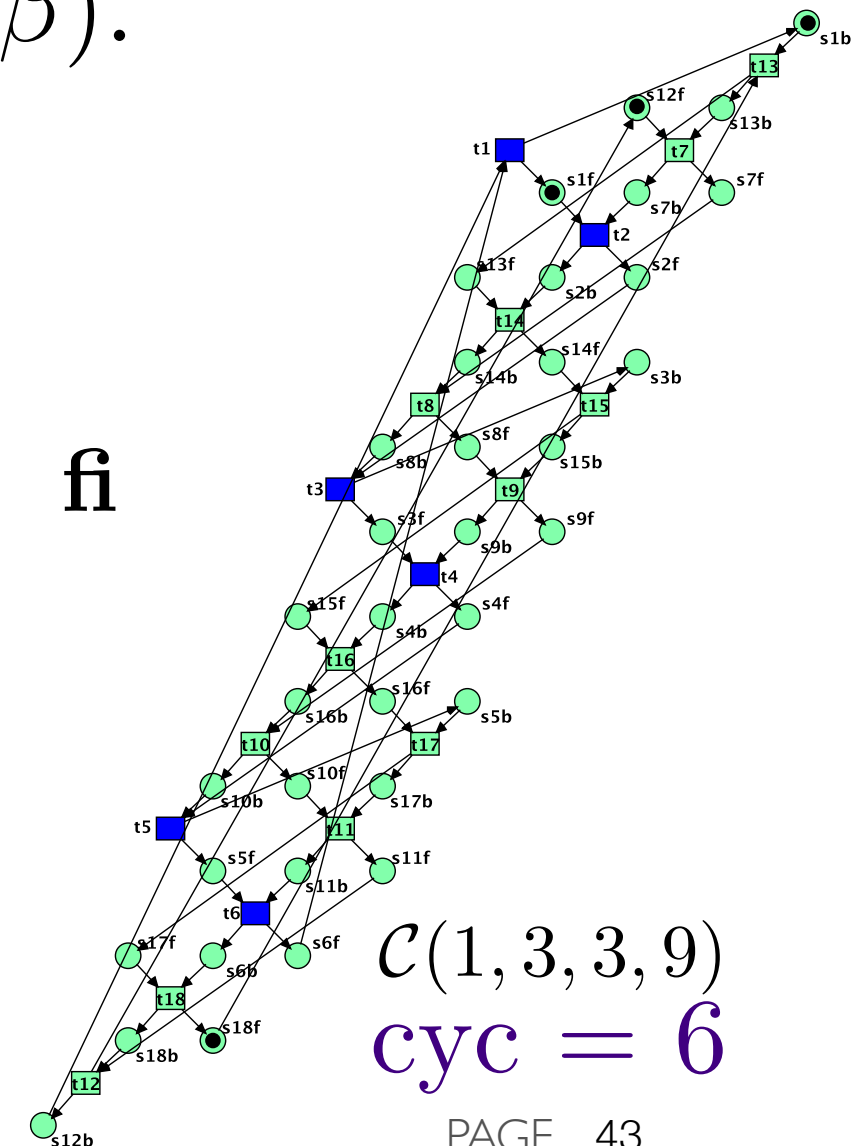
$$\mathcal{C}(1, 3, 3, 9)$$

$$\text{cyc} = 6$$

The length of a minimal cycle of a cycloid $\mathcal{C}(\alpha, \beta, \gamma, \delta)$ is

$$cyc(\alpha, \beta, \gamma, \delta) = \gamma + \delta + i \cdot (\alpha - \beta).$$

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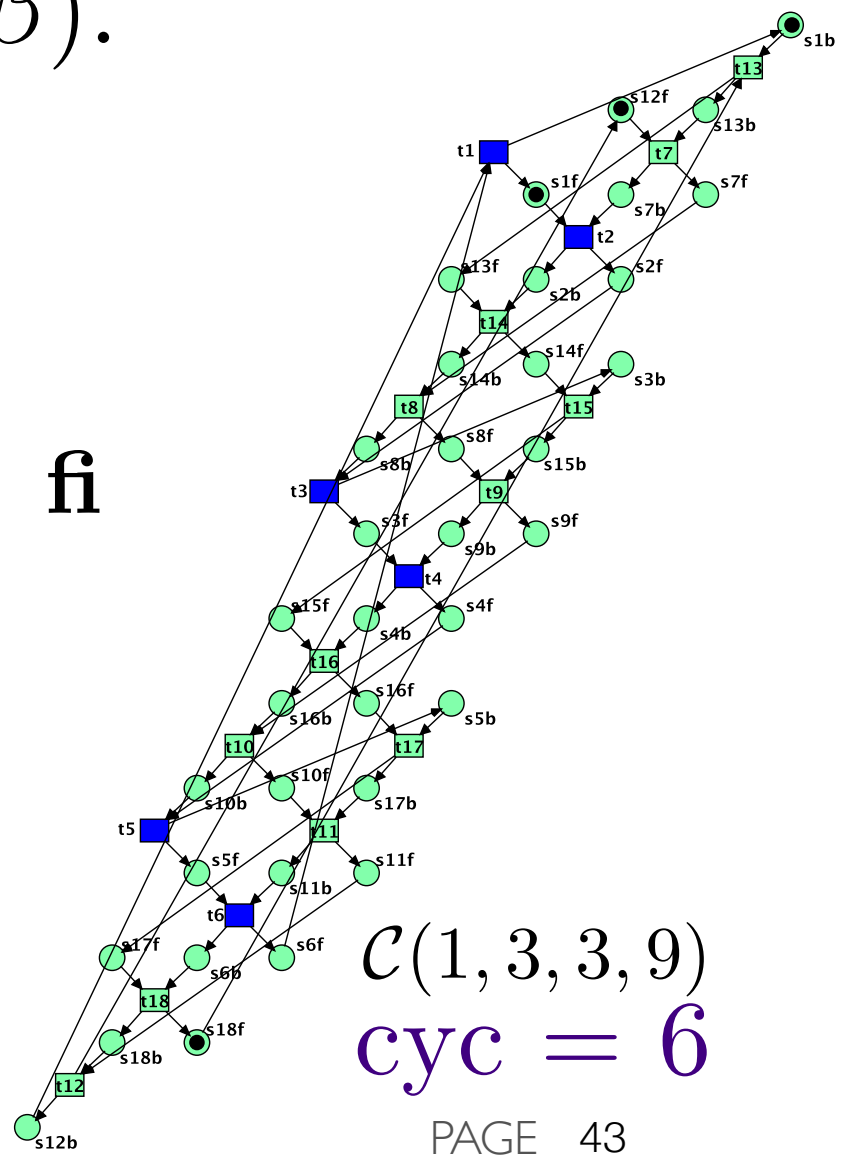
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$\mathcal{C}(1, 3, 3, 9)$
cyc = 6

τ : number of semi-active transitions.

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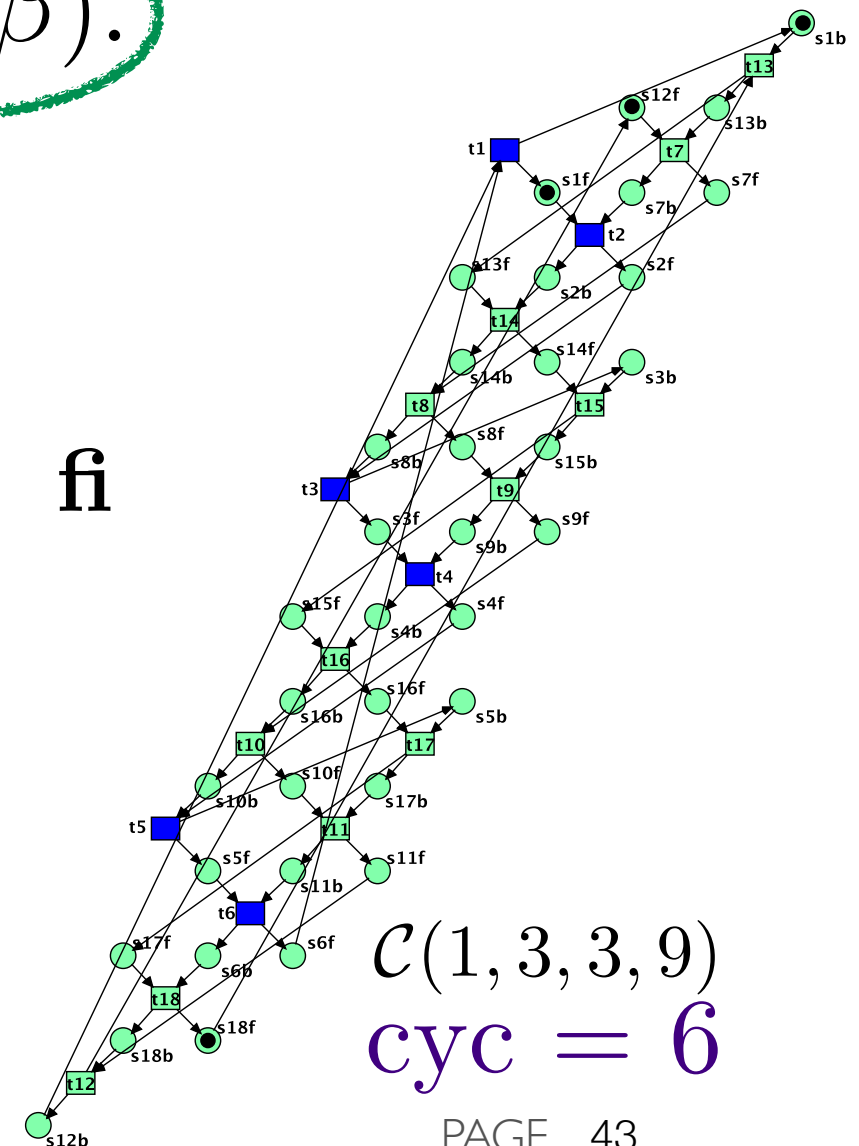
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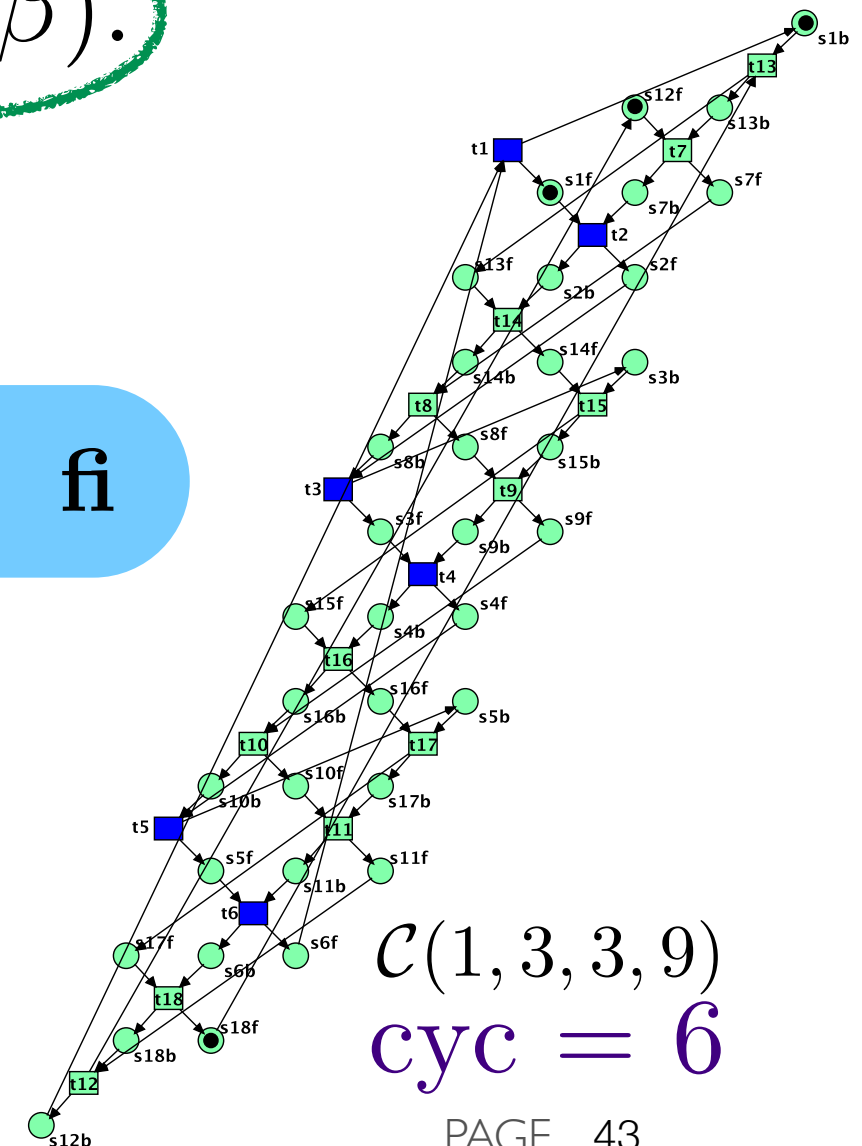
index of the cycloid

i = number of windings

$$12 + 3 \cdot (1 - 3)$$

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Contents:

1. Introduction and Examples
2. From Petri-Space to Cycloids (informal)
3. Cycloids $\mathcal{C}(\alpha, \beta, \gamma, \delta)$ (formal)
4. The Initial Parameters α and β
5. The Final Parameters γ and δ
- 6. Computing Cycloid Parameters α , β , γ and δ**
7. Outlook

6. From System Parameters to Cycloid Parameters

τ_0, τ_a, A, cyc

$\alpha, \beta, \gamma, \delta$

6. From System Parameters to Cycloid Parameters

$$\tau_0, \tau_a, A, cyc$$

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Solution:

$$\alpha = \tau_0$$

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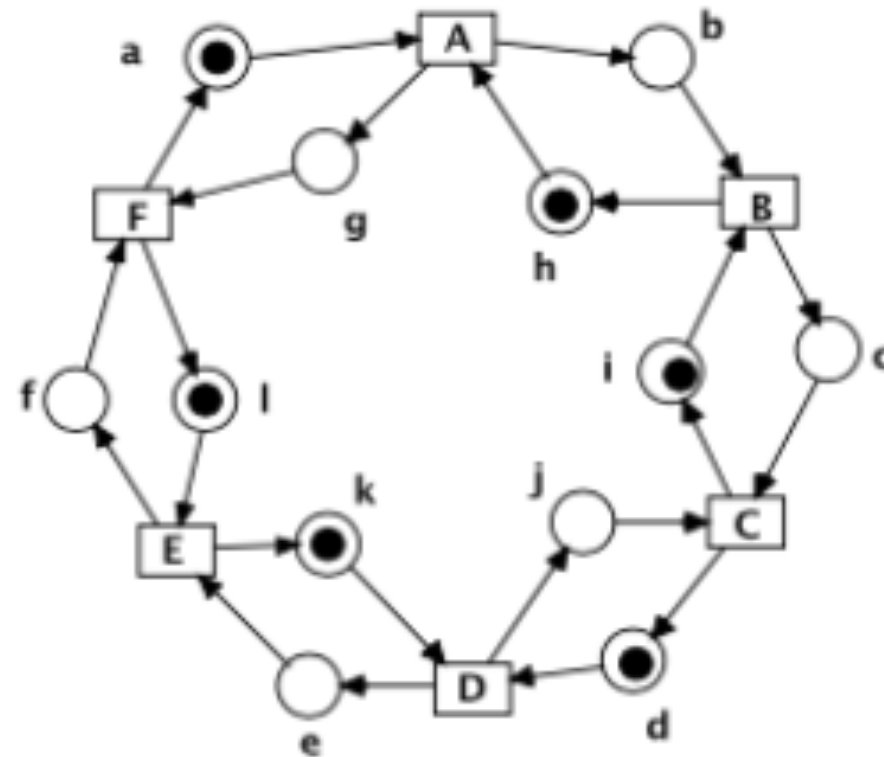
all solutions lead to isomorphic cycloids

6. From System Parameters to Cycloid Parameters

Applications to Cases

$$\mathcal{C}(4, 2, 1, 1)$$

$A = 6$ transitions
and $cyc = 2$

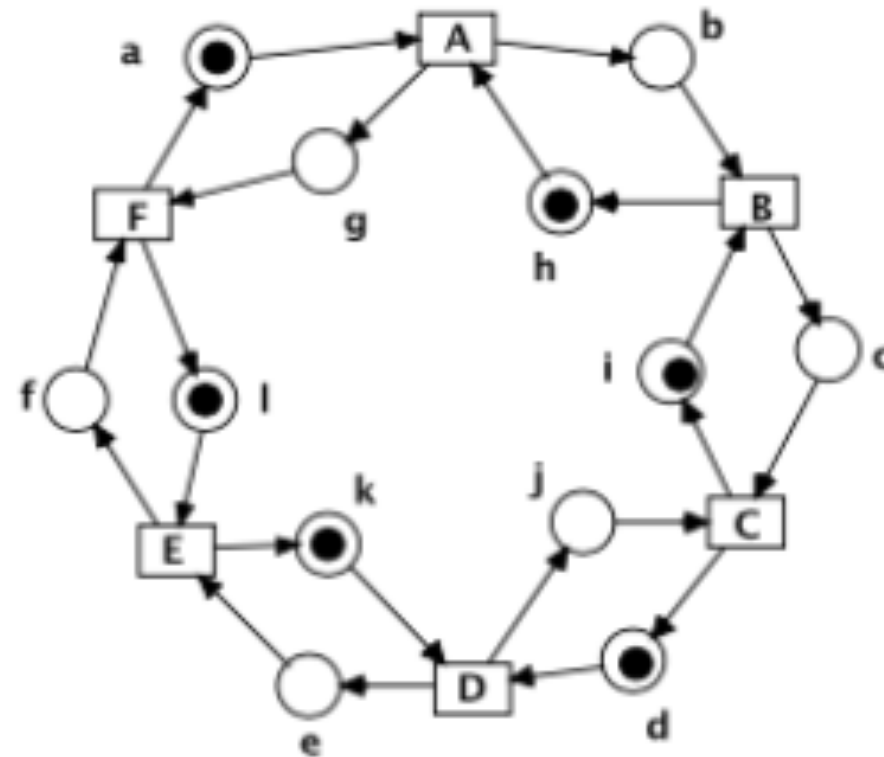


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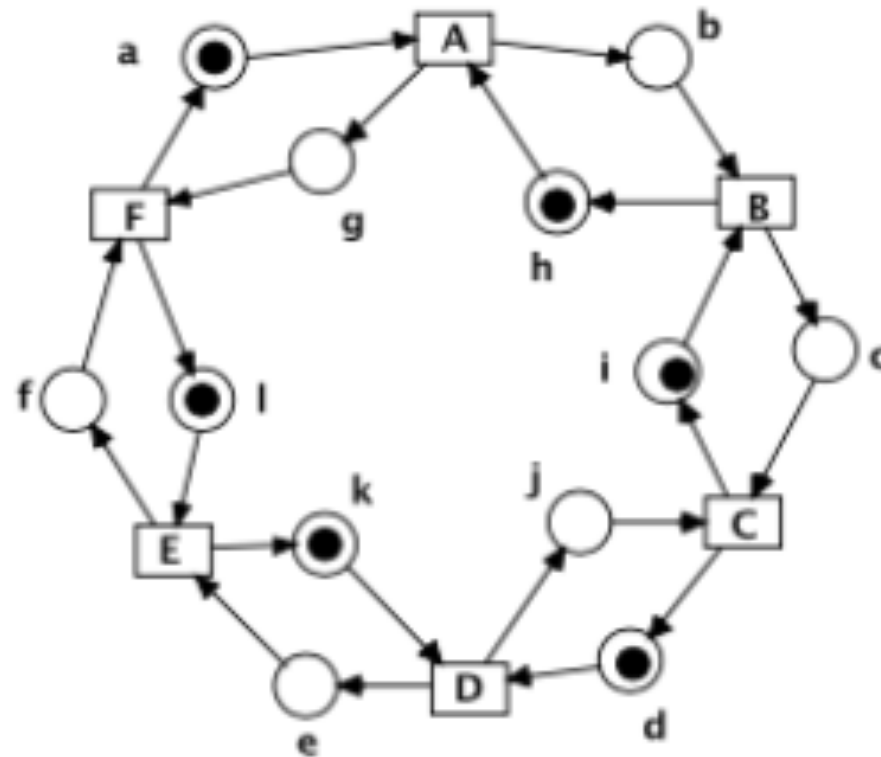
$$\gamma \bmod \alpha = \frac{\alpha \cdot cyc - A}{\alpha - \beta} = \frac{4 \cdot 2 - 6}{4 - 2} = 1.$$

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The set of positive solutions for γ is

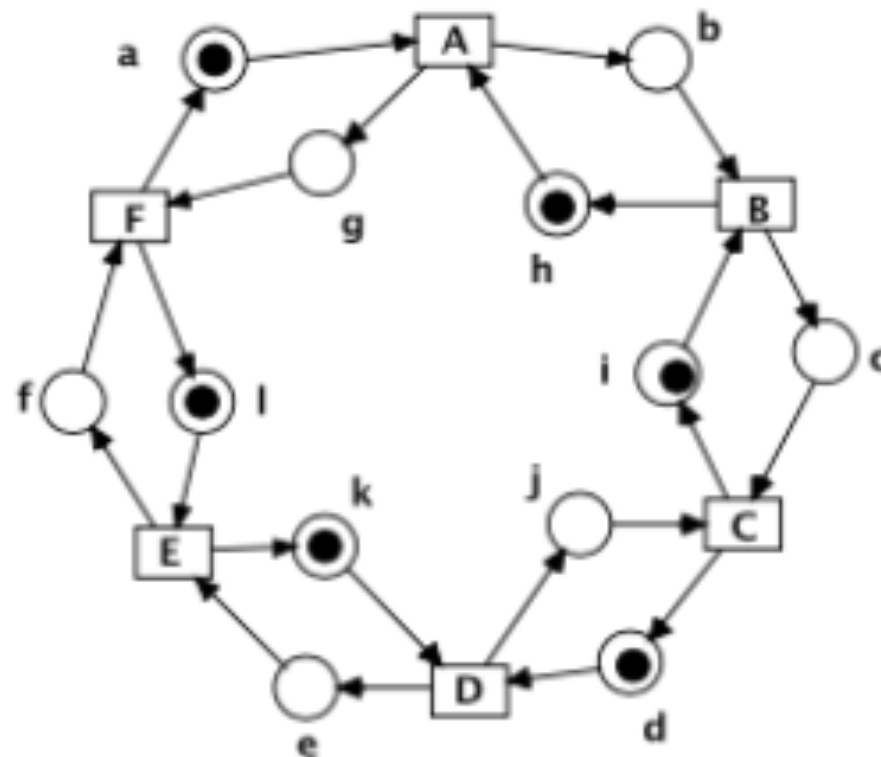
$$\{4n + 1 | n \in \mathbb{N}\} = \{1, 5, 9, \dots\}$$

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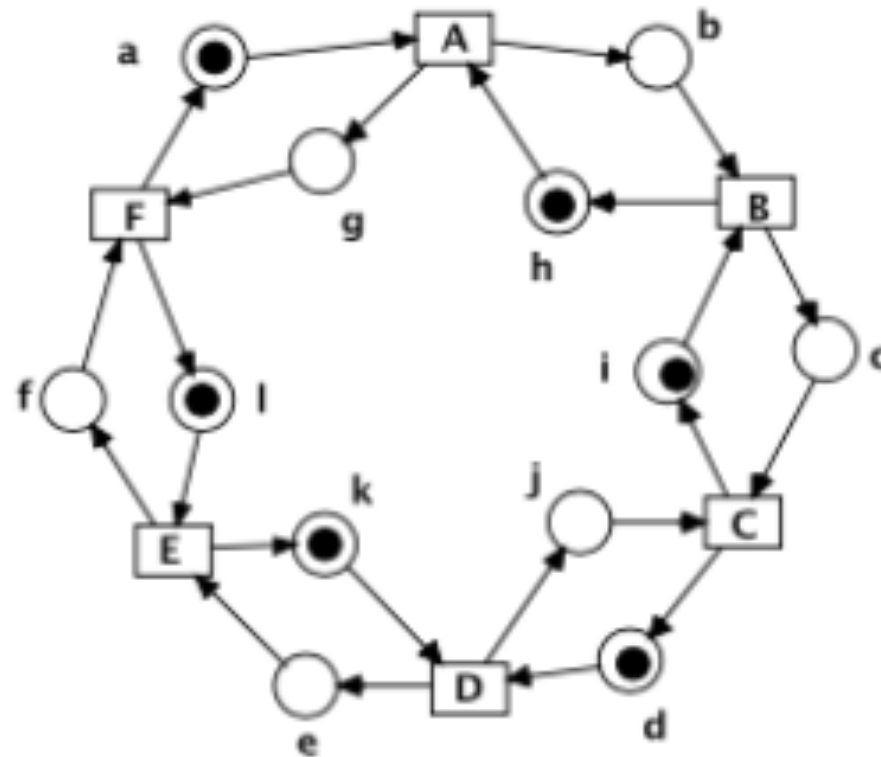
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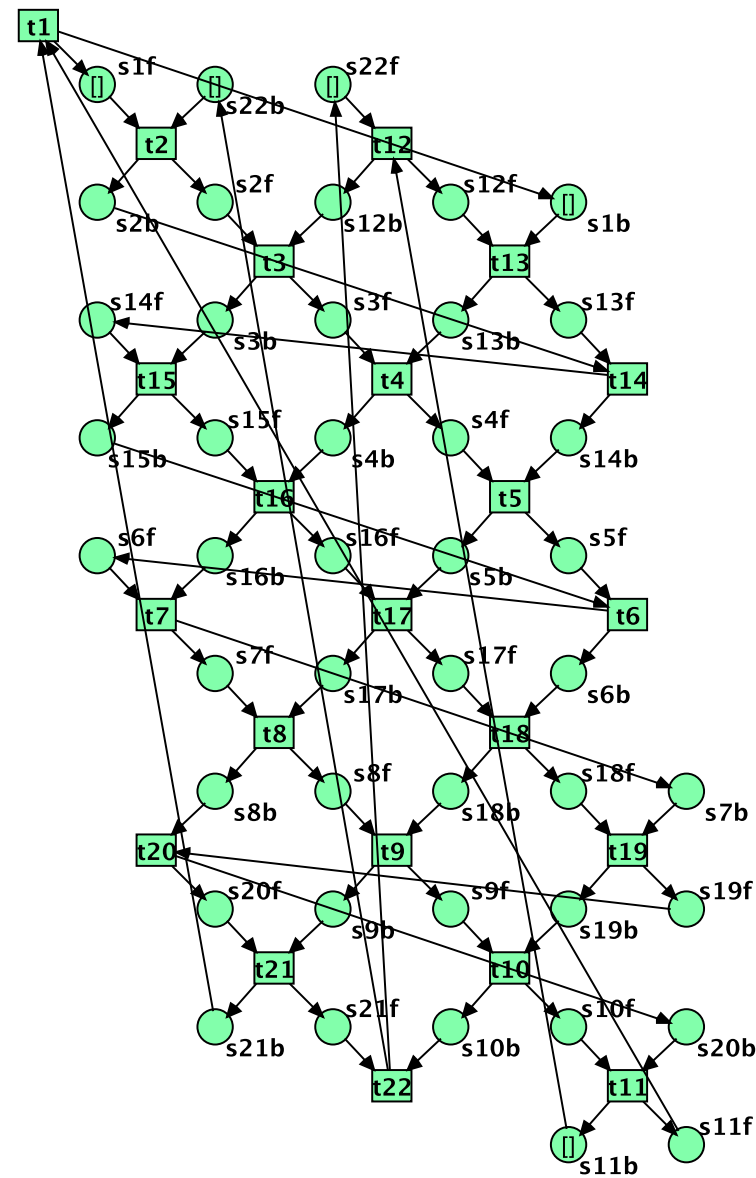
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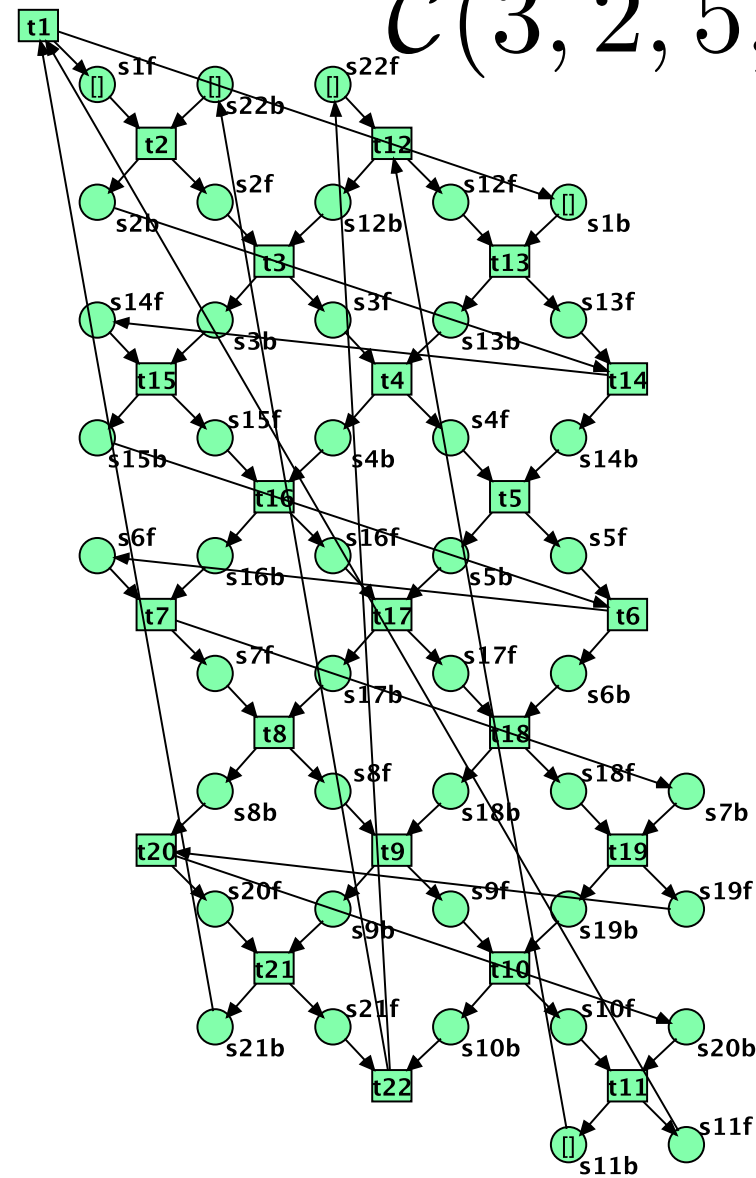
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3 solutions:

$$\mathcal{C}(3, 2, 8, 2)$$

$$\mathcal{C}(3, 2, 5, 4)$$

$$\mathcal{C}(3, 2, 2, 6)$$



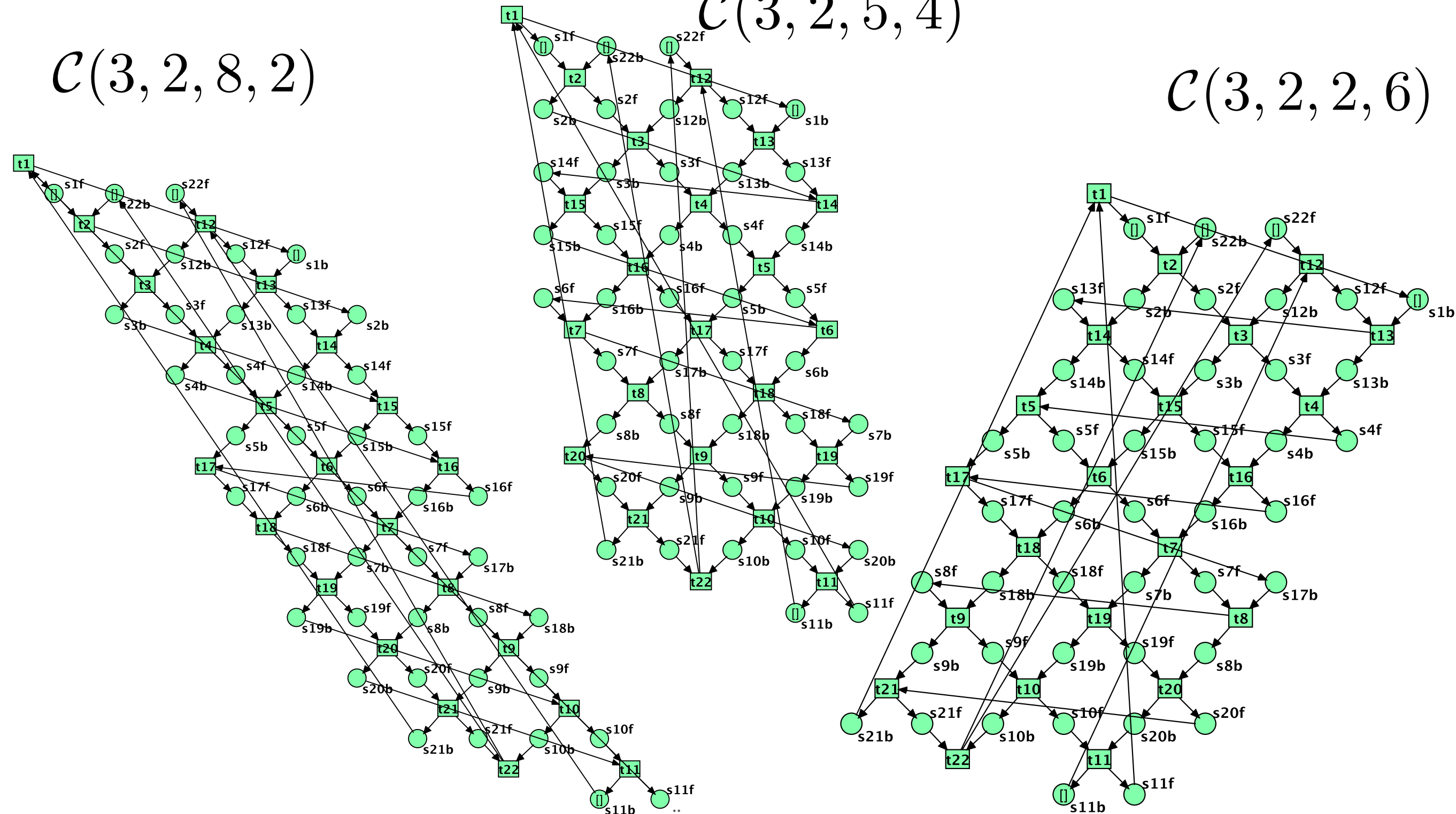
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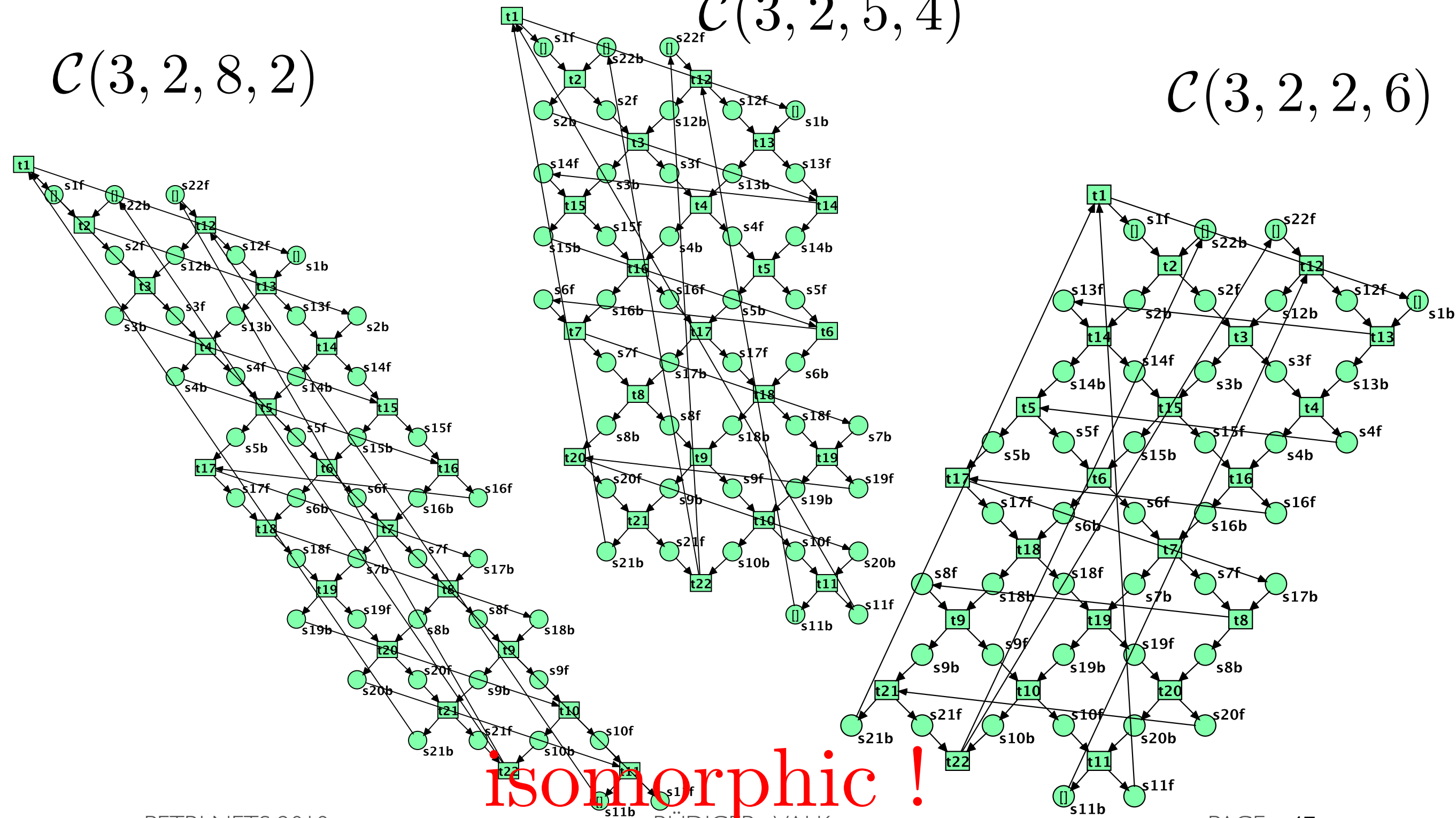
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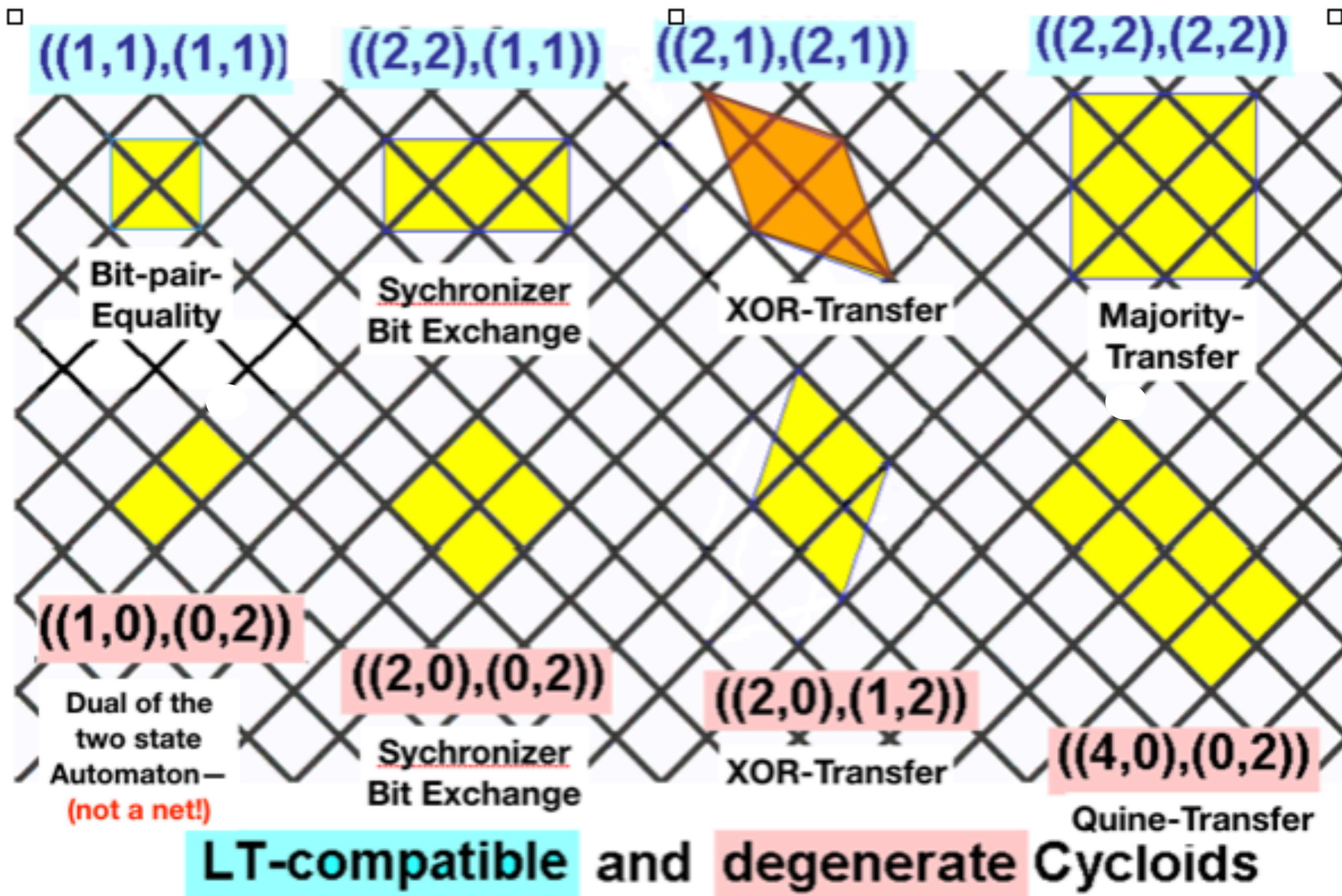
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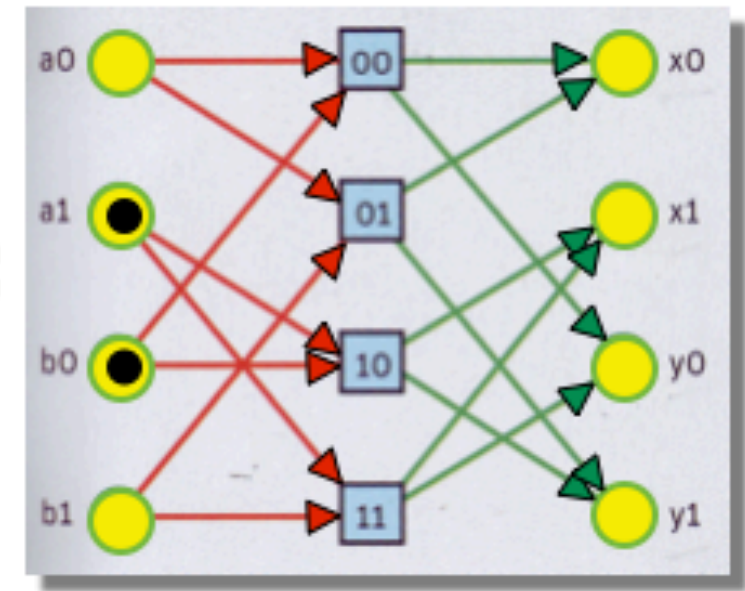
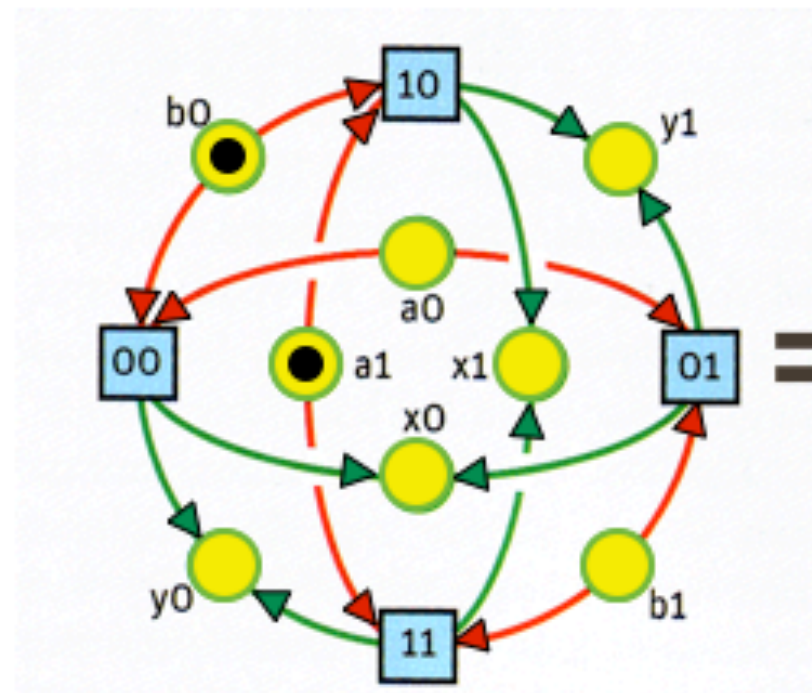
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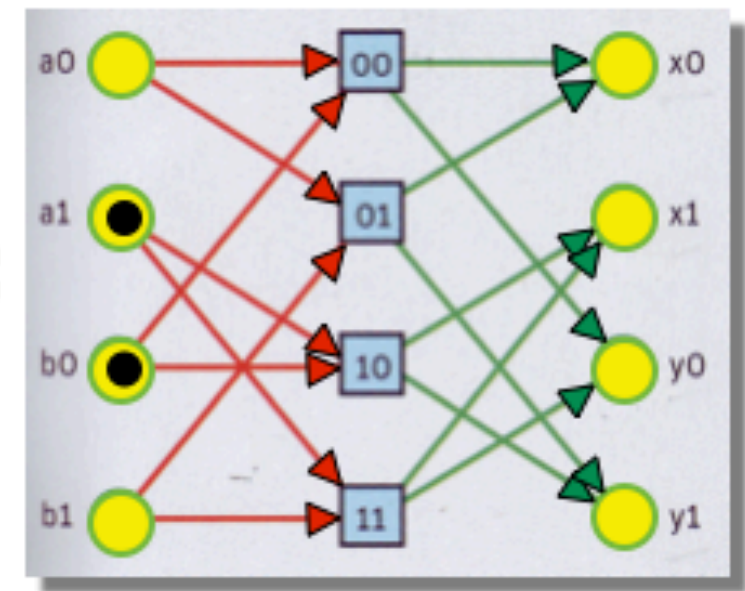
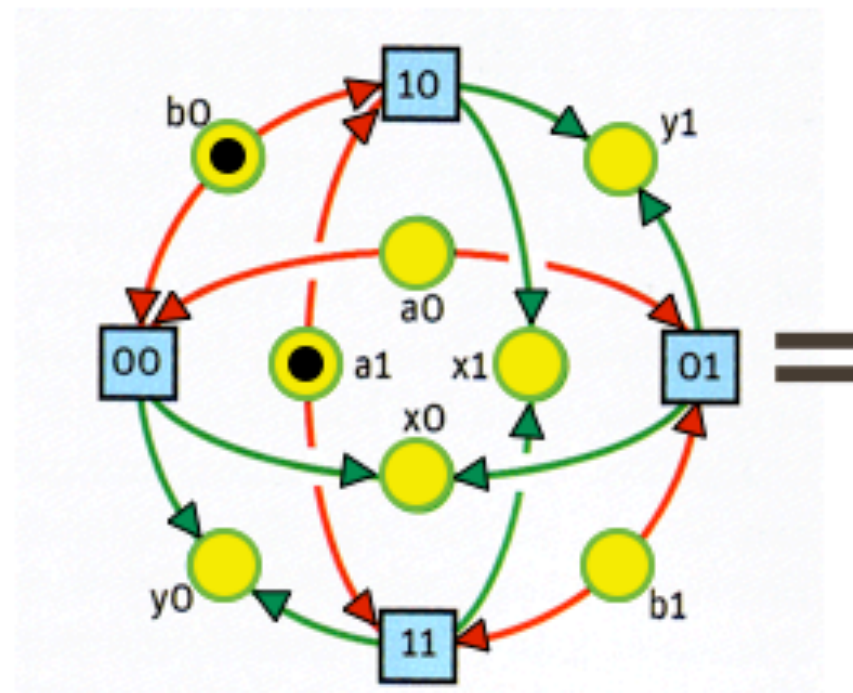
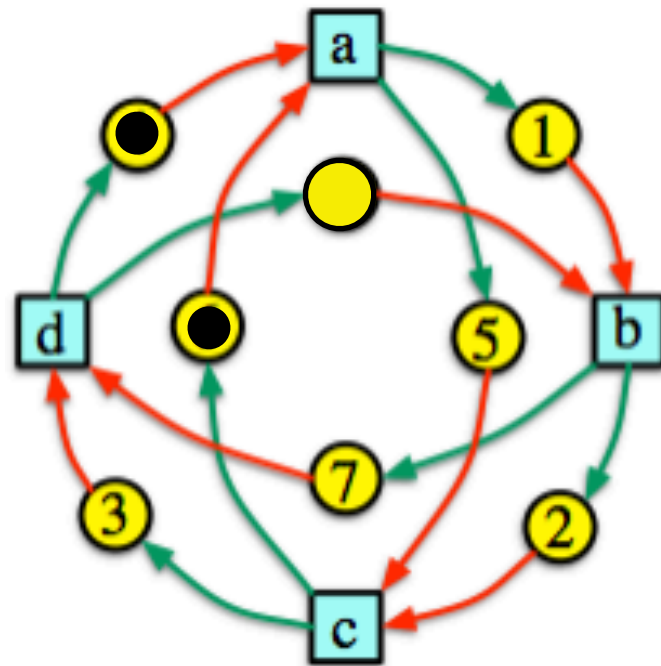


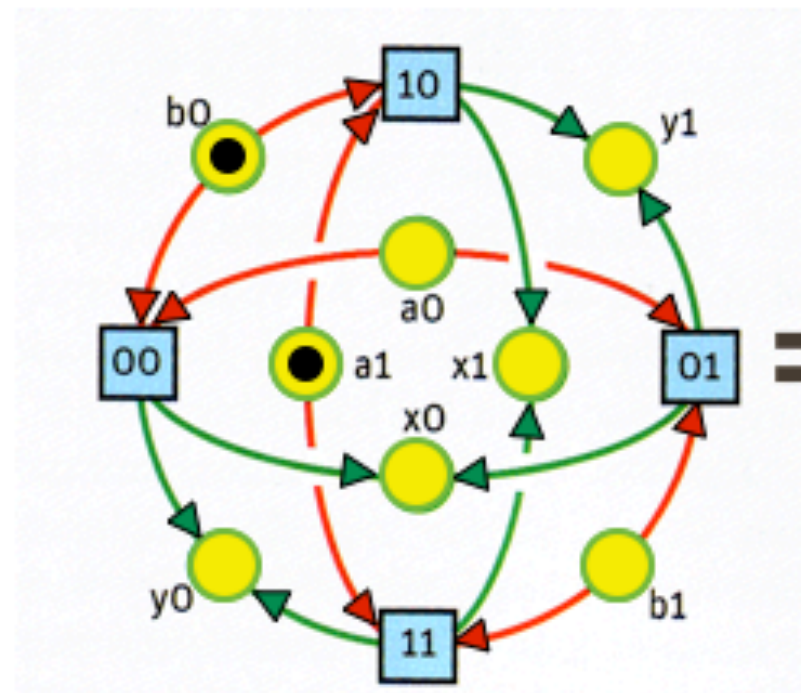
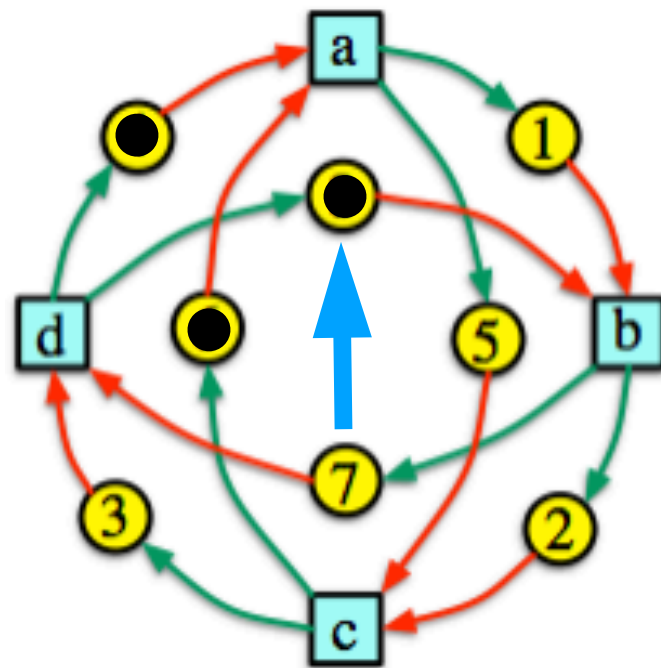


$x = a$
 $y = a \text{ XOR } b$ **Exclusive OR**

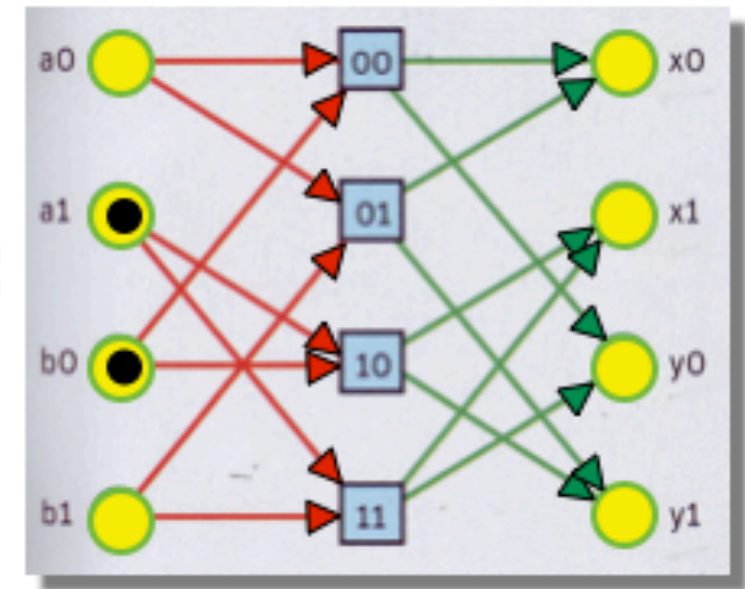


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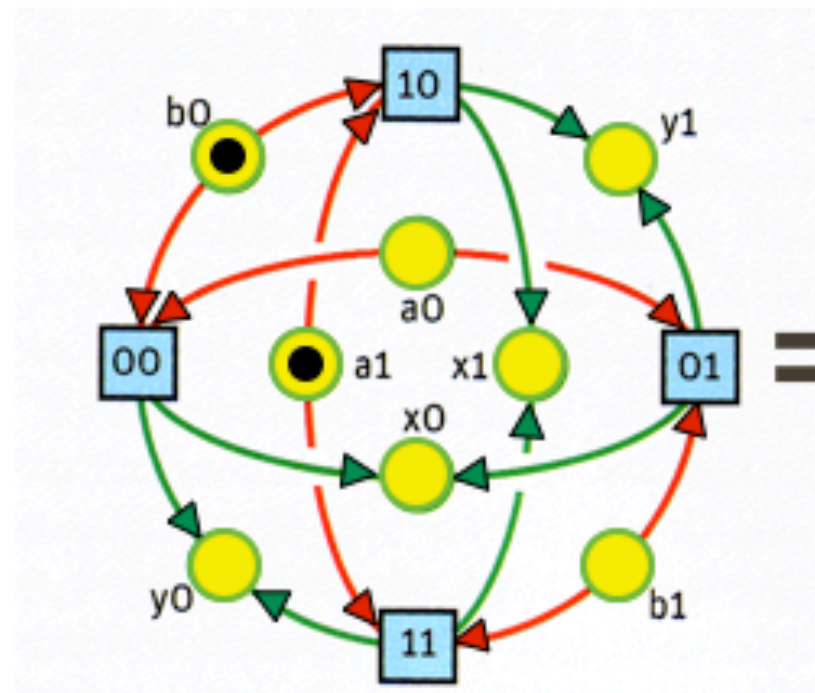
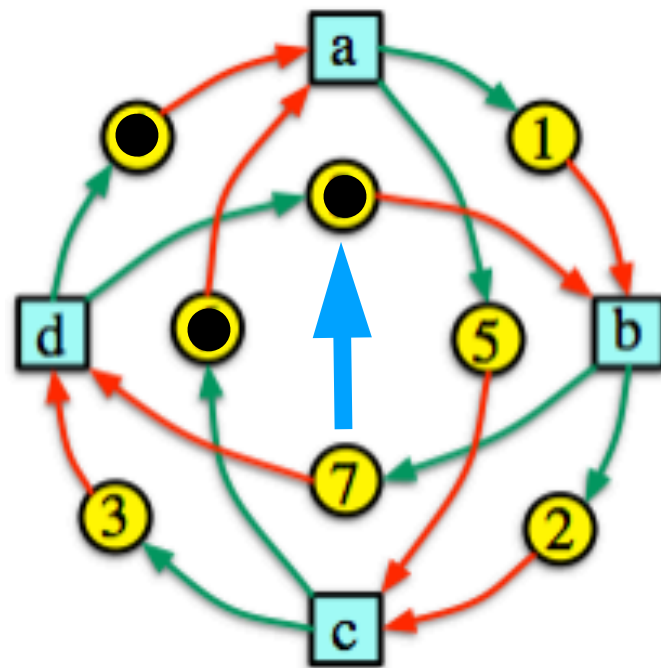




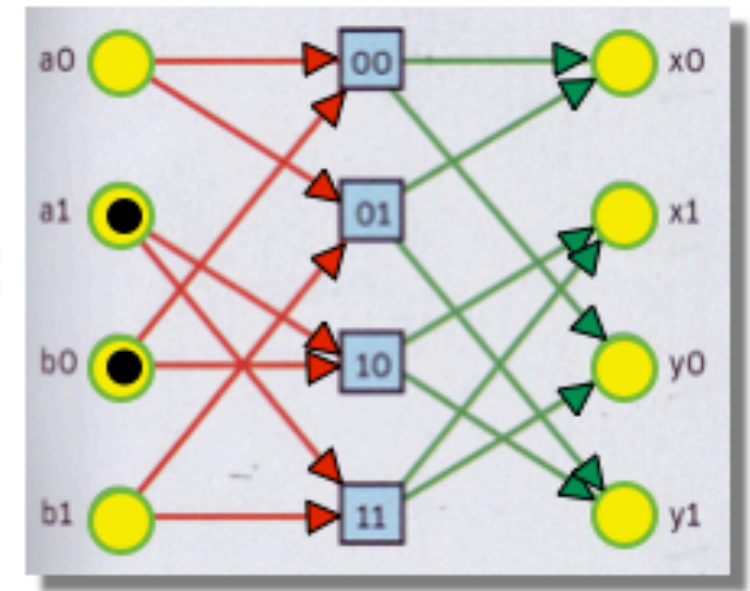
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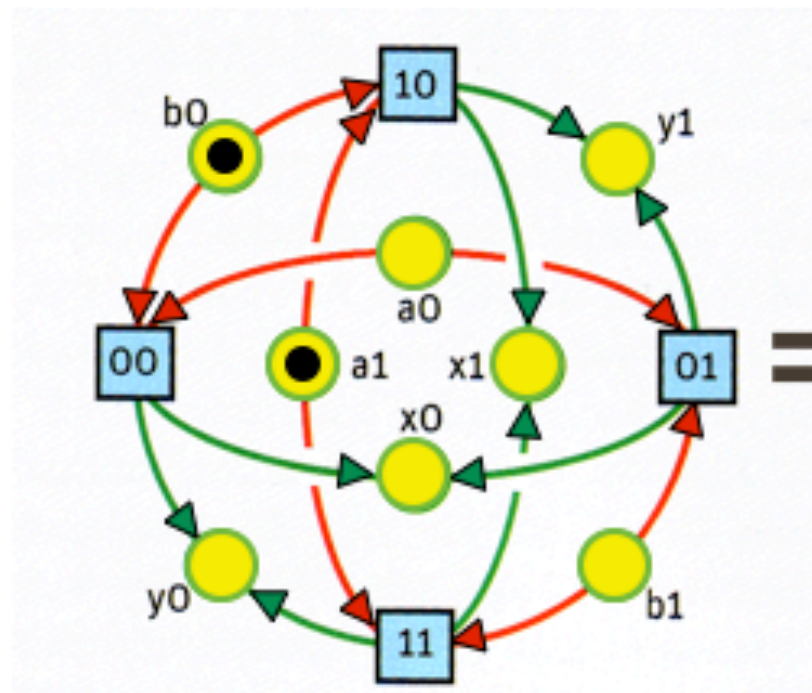
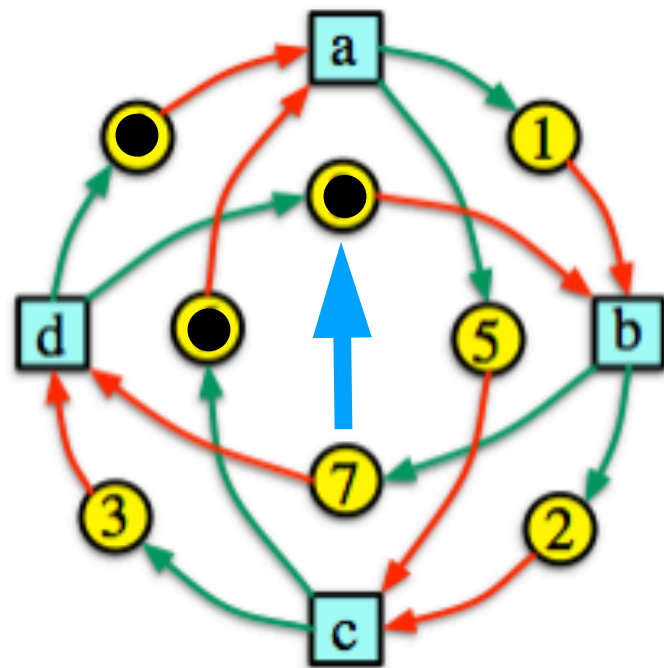


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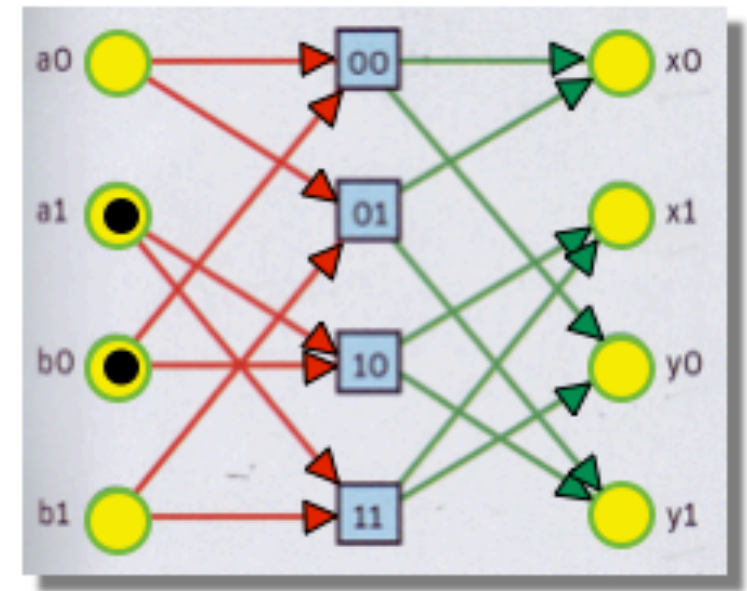


$\alpha = 2, \beta = 1$ two marked, one active transition

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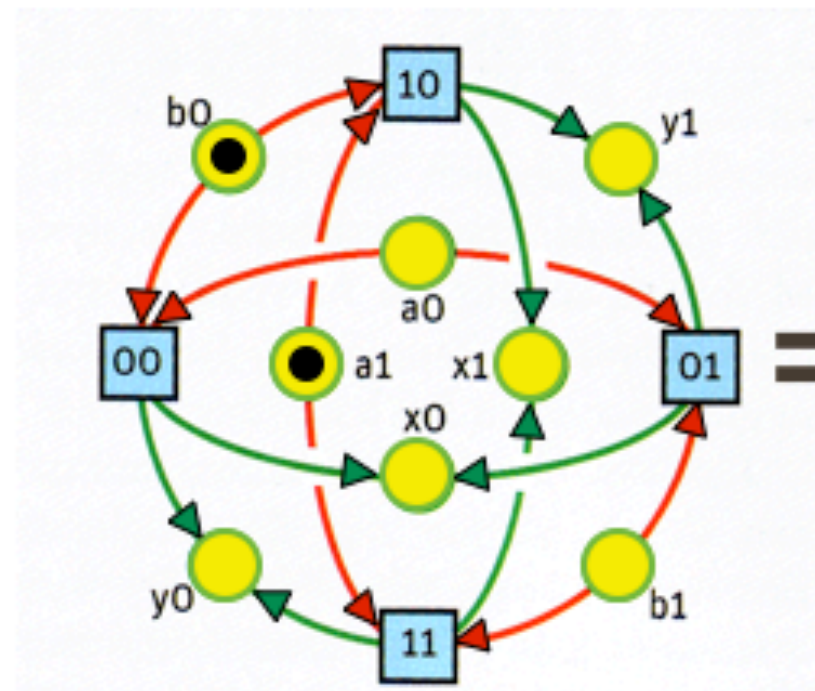
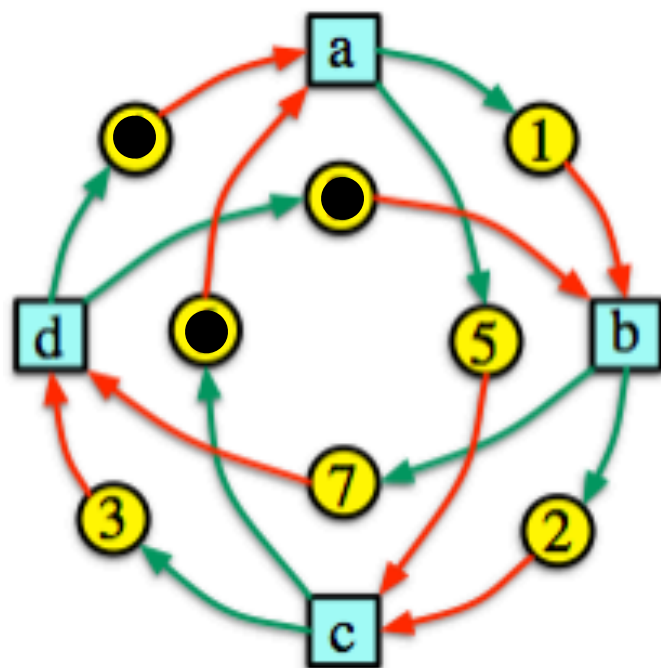
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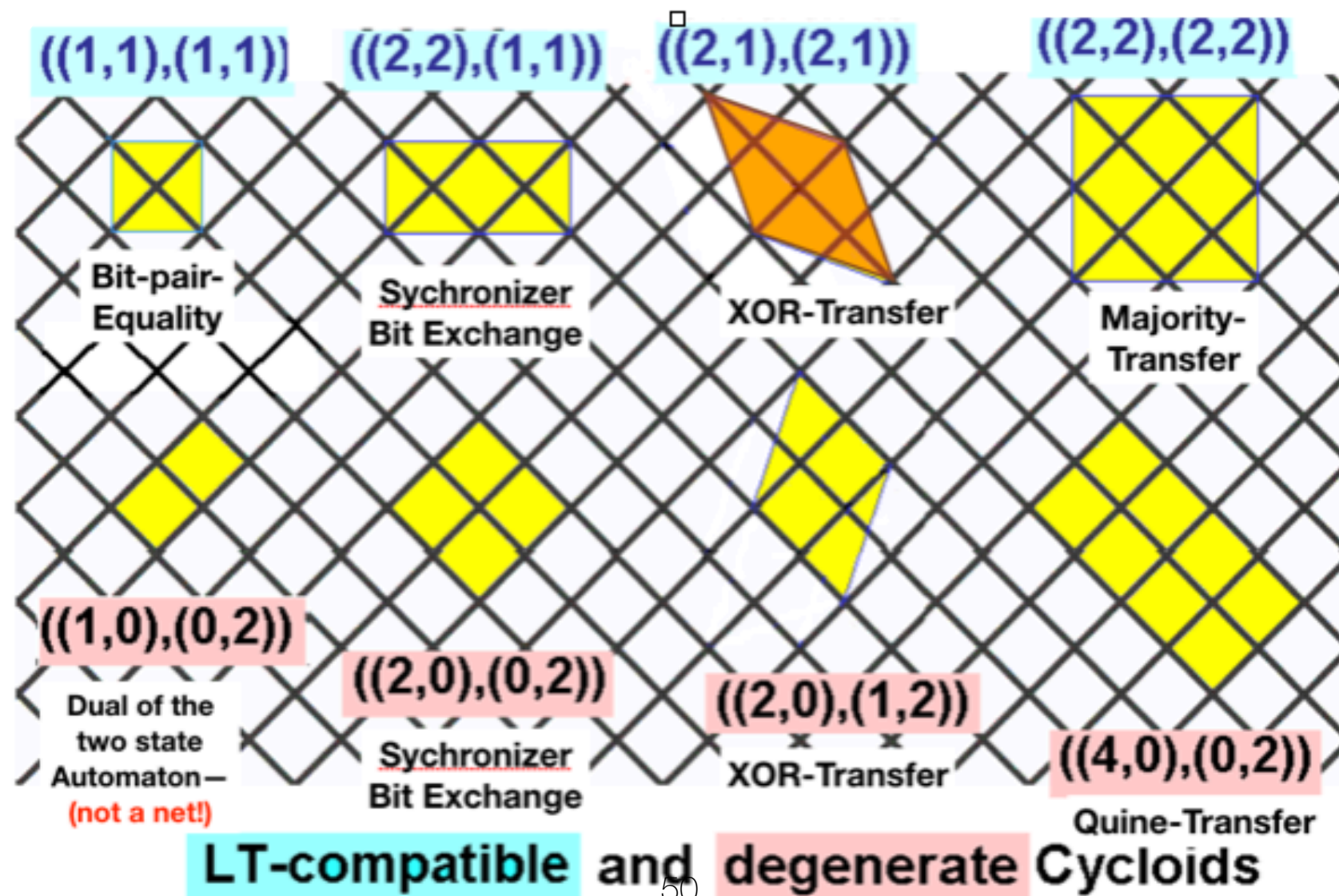
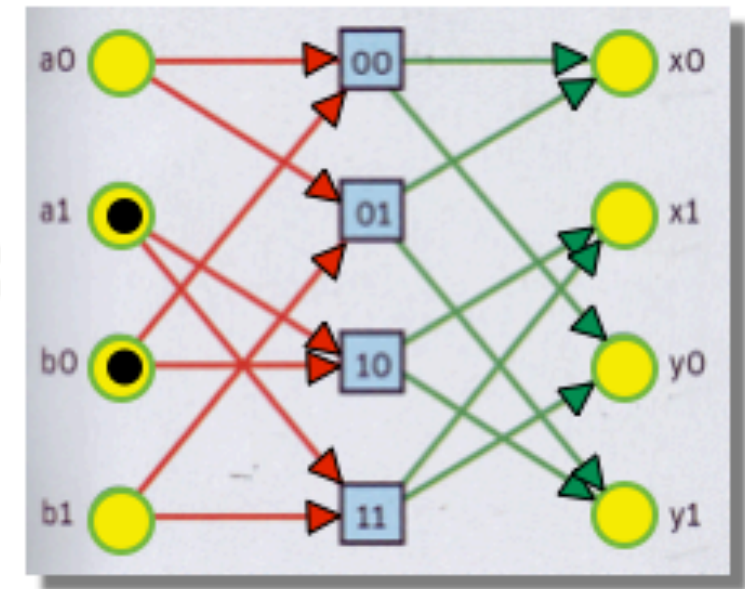
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$$(\gamma, \delta) \in \{\cdots (0, 2), (2, 1), (4, 0), \cdots\}$$





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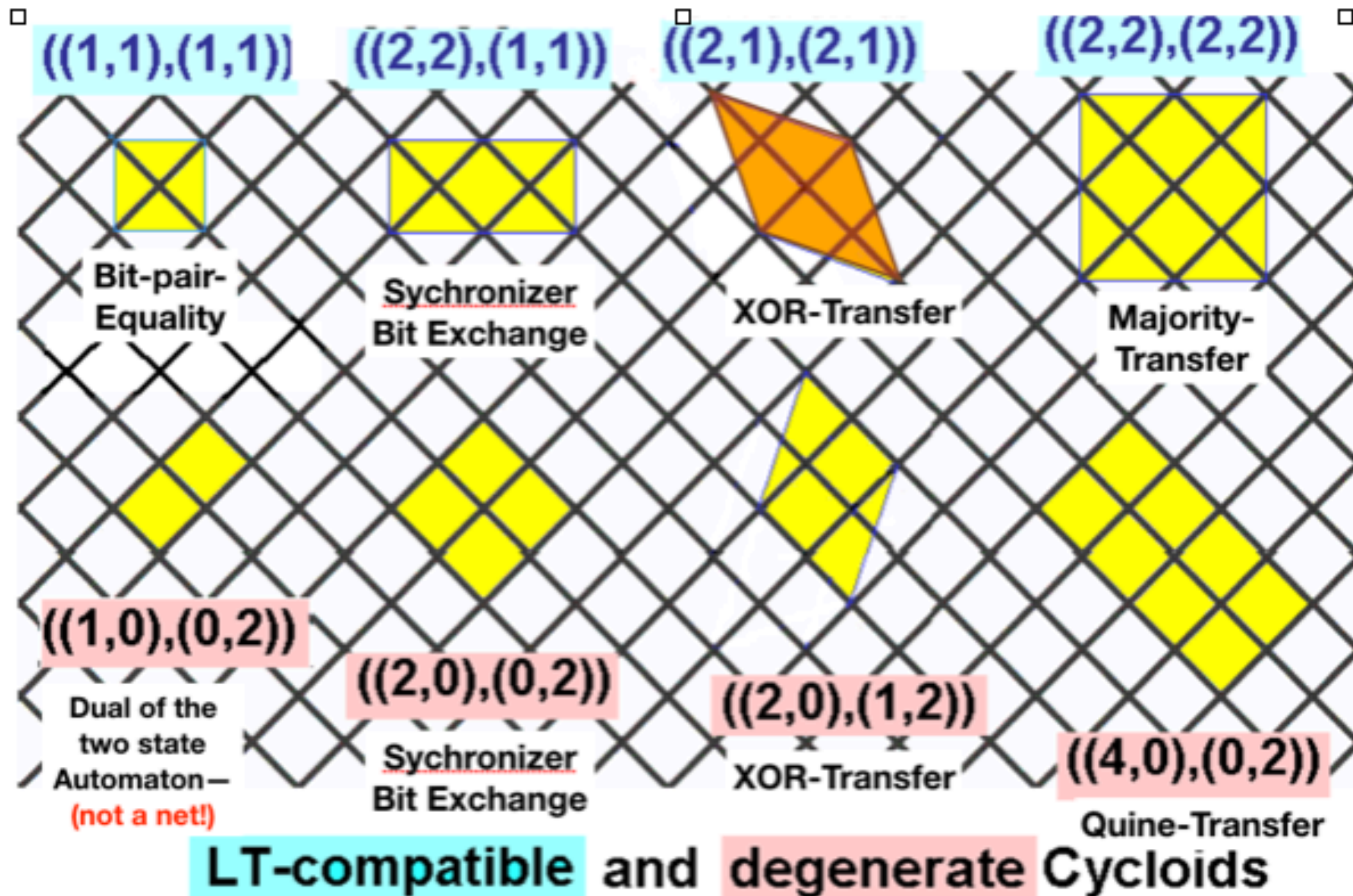
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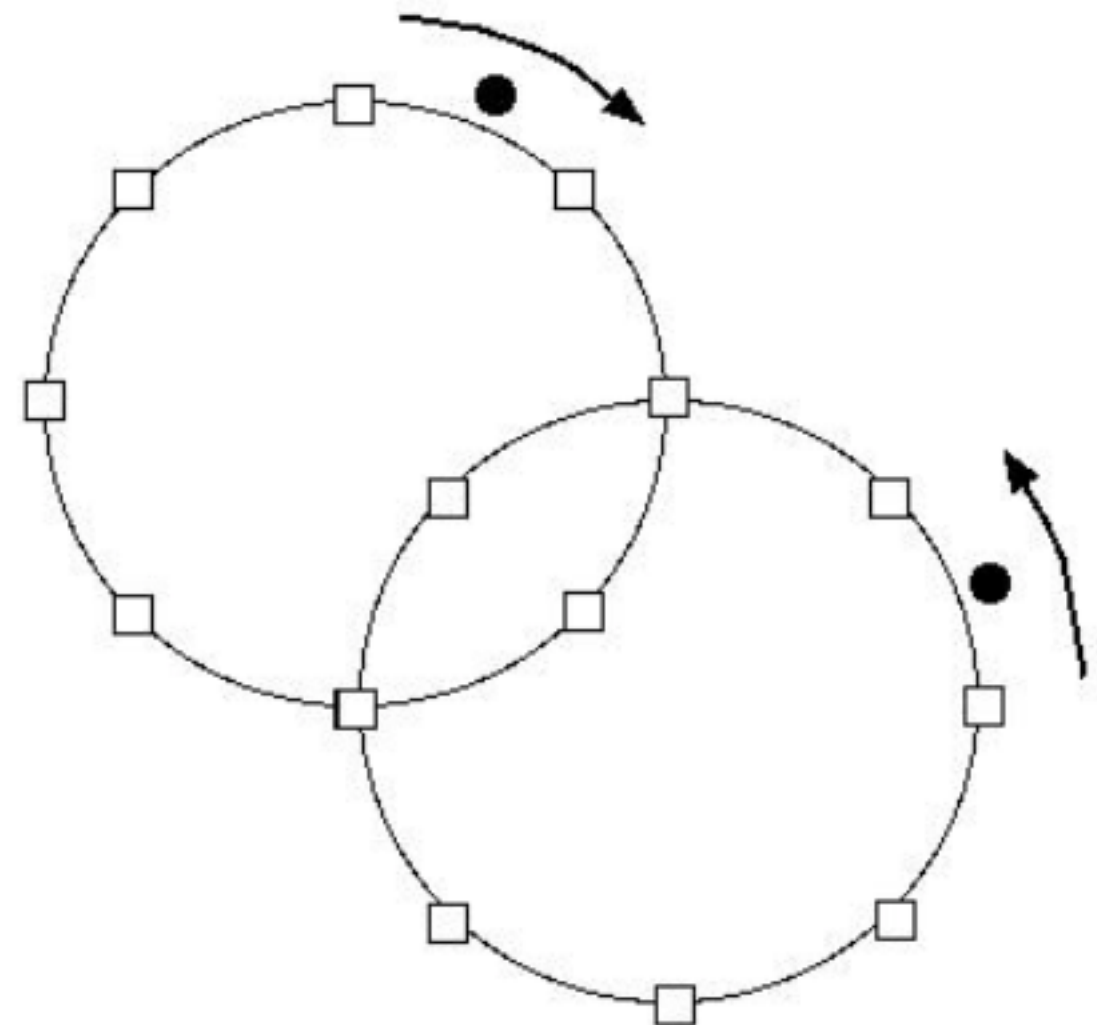
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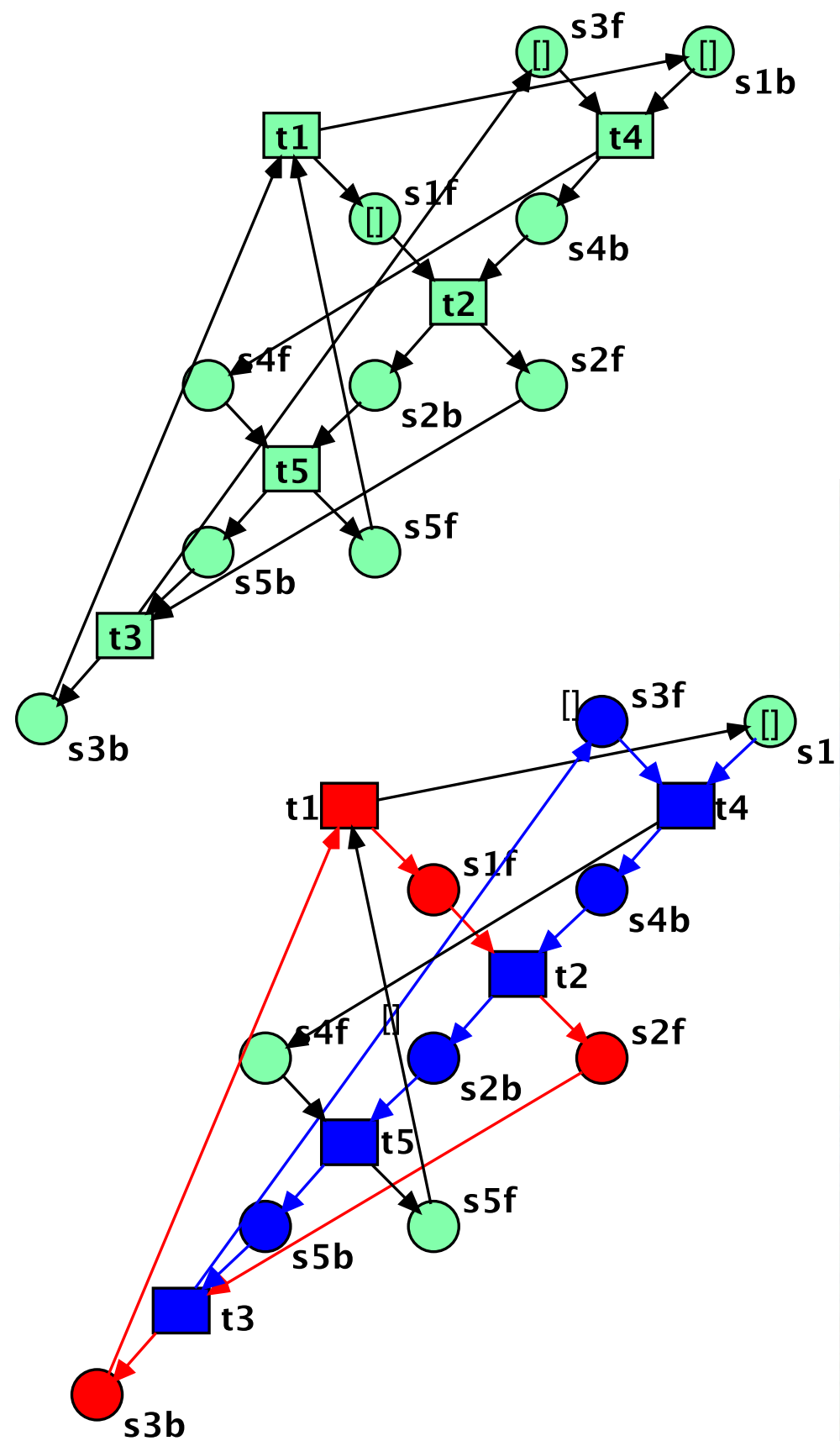
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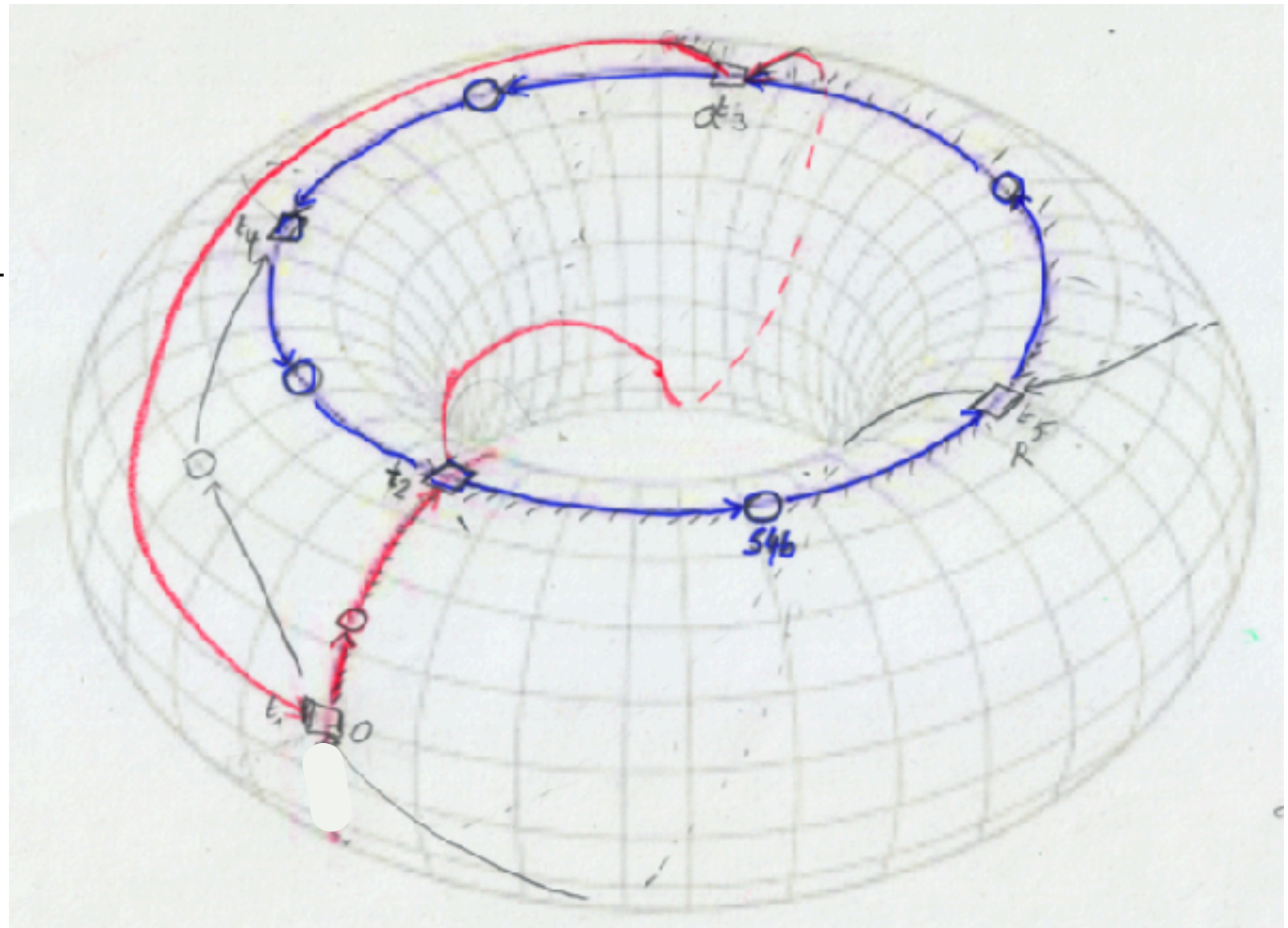
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Electron orbitals are nothing else than 4-dimensional cycloids

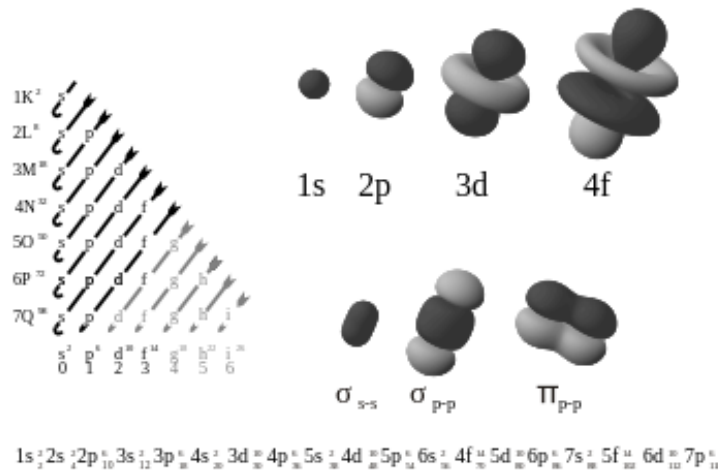
The End



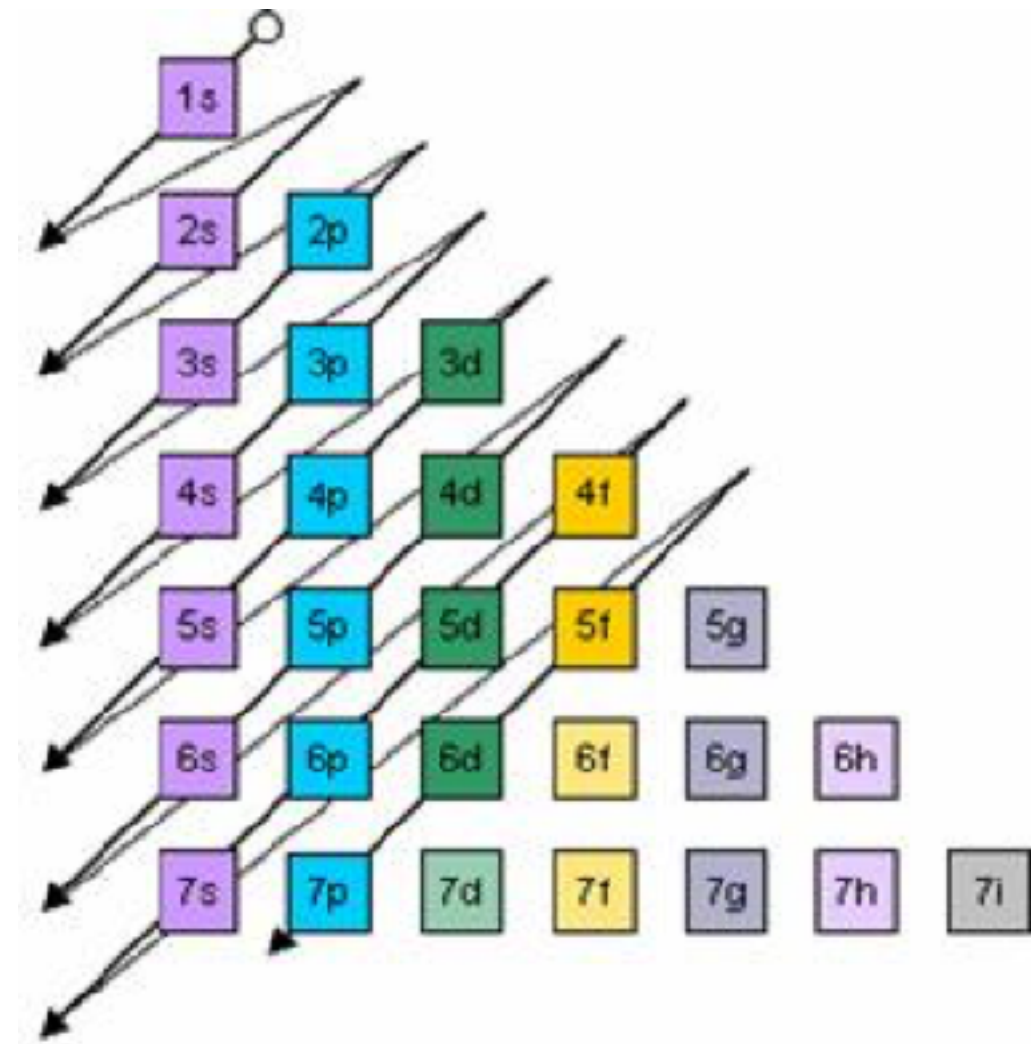
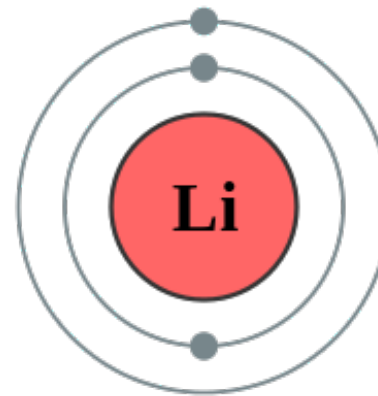
$$C(1, 2, 1, 3)$$



Electron configuration



In [atomic physics](#) and [quantum chemistry](#), the **electron configuration** is the distribution of [electrons](#) of an [atom](#) or [molecule](#) (or other physical structure) in [atomic](#) or [molecular orbitals](#).^[1] For example, the electron configuration of the [neon](#) atom is $1s^2 2s^2 2p^6$, using the [notation explained below](#).



Electronic configurations describe each electron as moving independently in an orbital, in an average field created by all other orbitals. Mathematically, configurations are described by [Slater determinants](#) or [configuration state functions](#).

According to the laws of [quantum mechanics](#), for systems with only one electron, an energy is associated with each electron configuration and, upon certain conditions, electrons are able to move from one configuration to another by the emission or absorption of a [quantum](#) of energy, in the form of a [photon](#).

Knowledge of the electron configuration of different atoms is useful in understanding the structure of the [periodic table](#) of elements. This is also

