Bounded Parametric Model Checking for Petri Nets

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a joint work with Michał Knapik and Agata Półrola

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Outline

1. Introduction to Parametric Model Checking
2. Benchmark: Mutual exclusion (MUTEX)
3. Syntax and Semantics of PRTCTL
4. Bounded parametric model checking for ENS
5. Parametric reachability for DTPN
6. Experimental Results
7. Final Remarks

Model Checking

Standard

$$M \overset{?}{\models} \varphi$$

a Kripke model a modal formula

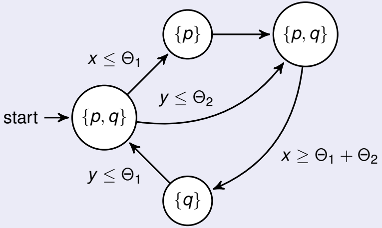
For Petri Nets

M is a model corresponding either to the marking graph of an EPN or to the concrete state graph of a TPN.

Parametric Model Checking

Parameters can appear in:

- a (timed) model[1,5]
- a formula[2,3]
- a model and a formula[4]



$$\forall_{\Theta \leq b} EF(\neg p \wedge EG^{\leq \Theta} c_1)$$

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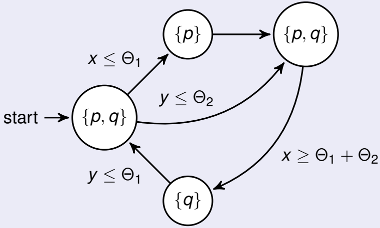

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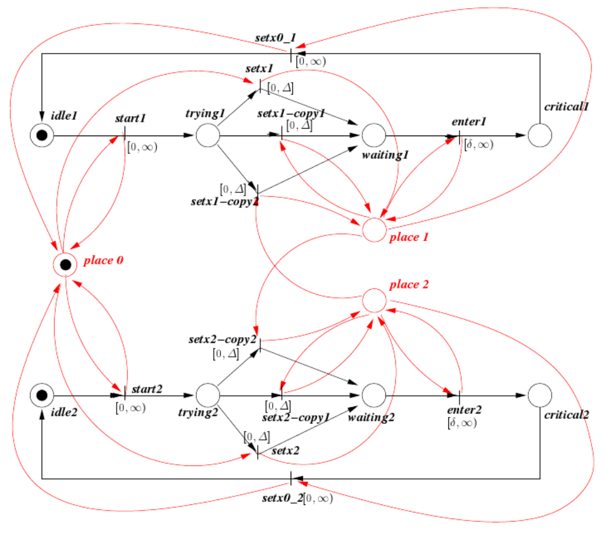
Time Petri Net: Timed Mutex



Parametric Model Checking

Complexity

If parameters are in:

- a model (e.g., TA, TPN), then reachability is undecidable,
- a formula, then for TECTL – *3EXPTIME*,
- both a model and a formula, then reachability is undecidable.

Idea

SAT-based Bounded Model Checking applied to parametric verification.

Applications

BMC for PRTCTL[1]:

- parameters in formulas for Elementary Petri Nets[2], and
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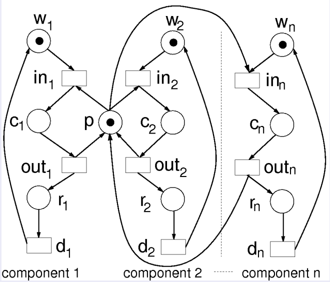
Working example: mutual exclusion

Petri Net: MUTEX

Mutual exclusion:

- n processes compete for access to the shared resource p,
- token in:
  - w_i: the i–th process is waiting,
  - c_i: the i–th process in a critical section,
  - r_i: the i–th process is in an unguarded section,
  - p: the resource is available.



Syntax and Semantics

Syntax of vRTCTL

- $\mathcal{PV}$ – propositional formulas, containing the symbol *true*,
- *Parameters* = $\{\Theta_1, \ldots, \Theta_n\}$ – *parameter variables,*
- *Linear expressions* – $\eta = \sum_{i=1}^{n} c_i\Theta_i + c_0$, *where* $c_0, \ldots, c_n \in \mathbb{N}$.

vRTCTL syntax:

- $\mathcal{PV} \subseteq$ vRTCTL,
- if $\alpha, \beta \in$ vRTCTL, then $\neg\alpha, \alpha \vee \beta, \alpha \wedge \beta \in$ vRTCTL,
- if $\alpha, \beta \in$ vRTCTL, then $EX\alpha,\ EG\alpha,\ E\alpha U\beta \in$ vRTCTL,
- if $\alpha, \beta \in$ vRTCTL, then $EG^{\leq\eta}\alpha,\ E\alpha U^{\leq\eta}\beta \in$ vRTCTL.

Example

$$\varphi(\Theta) = EF(\neg p \wedge EG^{\leq\Theta} c_1)$$

($EF\alpha = EtrueU\alpha$ – a derived modality)

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Syntax and semantics

Model for vRTCTL and PRTCTL

A Kripke structure $M = (S, \rightarrow, \mathcal{L})$ is a model, where

- S – a finite set of states,
- $\rightarrow \subseteq S \times S$ – a transition relation s.t. $\forall_{s \in S} \exists_{s' \in S}\ s \rightarrow s'$,
- $\mathcal{L} : S \longrightarrow 2^{\mathcal{PV}}$ – a labelling function s.t. $\forall_{s \in S}\ true \in \mathcal{L}(s)$.

Parameter valuations

vRTCTL formulae are interpreted under parameter valuations:

- $\upsilon : Parameters \rightarrow \mathbb{N}$,
- υ is extended to the linear expressions η.

Example

For $\varphi(\Theta) = EF(\neg p \wedge EG^{\leq \Theta} c_1)$ and υ s.t. $\upsilon(\Theta) = 2$

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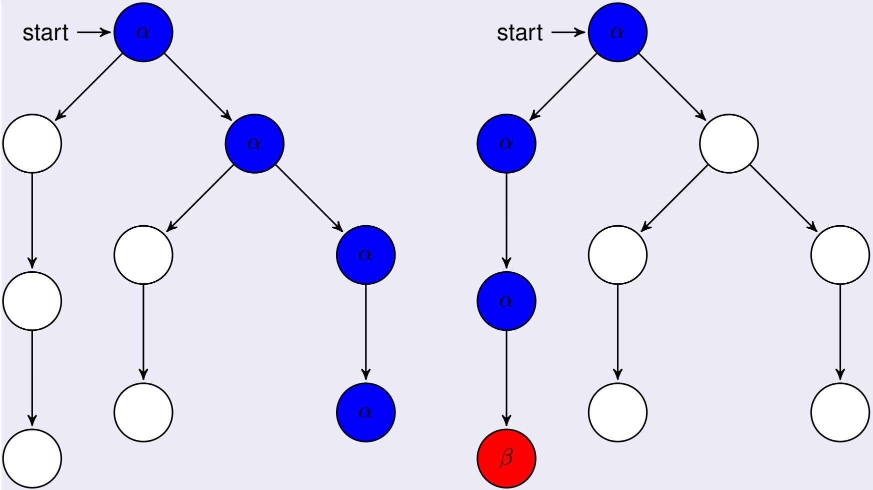

$M, start \models EG^{\leq 3}\alpha$ $M, start \models E\alpha U^{\leq 3}\beta$

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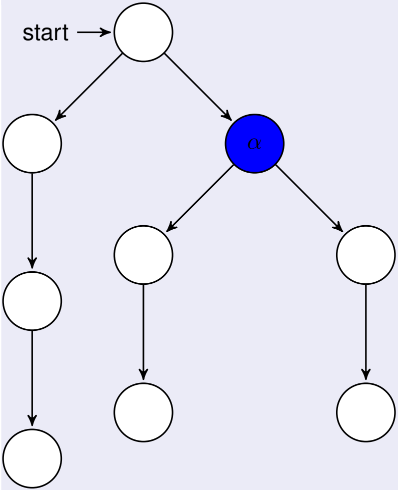

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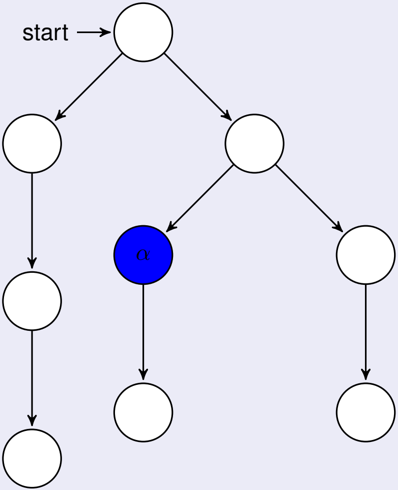

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Syntax

Syntax of PRTCTL

- vRTCTL $\subseteq$ PRTCTL,
- if $\alpha(\Theta) \in$ vRTCTL $\cup$ PRTCTL, then $\forall_\Theta \alpha(\Theta), \exists_\Theta \alpha(\Theta), \forall_{\Theta \leq a} \alpha(\Theta), \exists_{\Theta \leq a} \alpha(\Theta) \in$ PRTCTL for $a \in \mathbb{N}$.

Notation: $\alpha(\Theta_1, \ldots, \Theta_n)$ denotes that $\Theta_1, \ldots, \Theta_n$ are free in α.

Example

$$\varphi_1^3 = \forall_{\Theta \leq 3} EF(\neg p \wedge EG^{\leq \Theta} c_1)$$

We consider the closed formulae (sentences) of PRTCTL only.

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Semantics

Semantics of PRTCTL (the closed formulae)

- $M, s \models \forall_\Theta \alpha(\Theta)$ iff $\bigwedge_{0 \leq i_\Theta} M, s \models \alpha(\Theta \leftarrow i_\Theta)$,
- $M, s \models \forall_{\Theta \leq a} \alpha(\Theta)$ iff $\bigwedge_{0 \leq i_\Theta \leq a} M, s \models \alpha(\Theta \leftarrow i_\Theta)$,
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Example

$$M, s \models \varphi_1^3 \text{ iff } \bigwedge_{i_{\Theta} \le 3} M, s \models EF(\neg p \wedge EG^{\le i_{\Theta}} c_1)$$

Example of a PRTCTL formula

$$\forall_{\Theta}[AG(request \Rightarrow AF^{\leq\Theta} receive) \Rightarrow AG(request \Rightarrow AF^{\leq 2\times\Theta} grant)]$$

expresses much more than the corresponding CTL formula

$$[AG(request \Rightarrow AFreceive) \Rightarrow AG(request \Rightarrow AFgrant)]$$

Complexity of model checking

For CTL, vRTCTL, and PRTCTL

- CTL and vRTCTL can be model checked in time $O(|M| \cdot |\varphi|)$.
- PRTCTL can be model checked in time $O(|M|^{k+1} \cdot |\varphi|)$, where k is the number of parameters in φ.

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Syntax and Semantics

Existential fragments

The logics vRTECTL and PRTECTL are defined as the restrictions of, respectively, vRTCTL and the set of sentences of PRTCTL such that the negation can be applied to propositions only.

Example: $\varphi_1^4 = \forall_{\Theta \leq 4} EF(\neg p \wedge EG^{\leq \Theta} c_1)$

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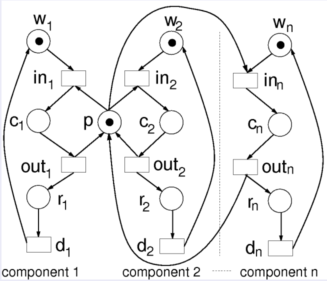
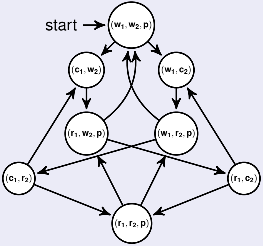
Syntax and semantics – back to MUTEX

Petri Net for MUTEX



The marking graph



Let $\varphi_1^b = \forall_{\Theta \leq b} EF(\neg p \wedge EG^{\leq \Theta} c_1)$.

Intuitive meaning of $M, start \models \varphi_1^b$:

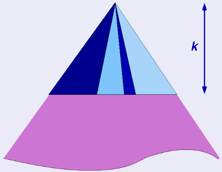
"There exists a future state, such that the resource is taken and the first process stays in the critical section for any time value bounded by b"

Bounded semantics

k–models

Idea – to unwind the computation tree of a model M up to depth k.

- M – a model, $k \in \mathbb{N}$,
- $Path_k$ – the set of all sequences $(s_0, \ldots, s_k)$, where $s_i \to s_{i+1}$.
- $M_k = (Path_k, \mathcal{L})$ is called the *k-model*.
- If an existential formula φ holds in M_k, then φ holds in M.
- The problem $M_k \models \varphi$ is translated to checking satisfiability of the propositional formula $[M_k] \wedge [\varphi]$ using a SAT-solver.



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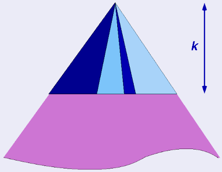

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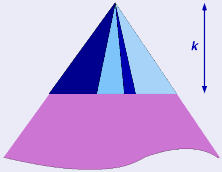

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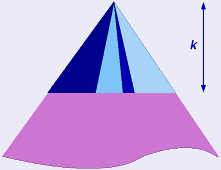

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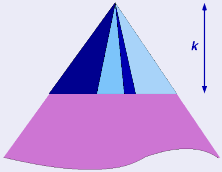

Translation to boolean formula

Encoding submodels

$$[M]_k^A := \bigwedge_{j \in A} \bigwedge_{i=0}^{k-1} T(w_{i,j}, w_{i+1,j})$$

Where A – a set of path indices determined by function[5] f_k.

$$V \models [M]_k^A \text{ iff } V \text{ encodes } k\text{–model}$$

Encoding formulae

φ – a PRTCTL formula

$\Downarrow$

$[\varphi]_k$ – a propositional formula

Testing formula

$$[M]_k^{F_k(\alpha)} \wedge I_s(w_{0,0}) \wedge [\varphi]_k$$

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Distributed Time Petri Nets

Time Petri Nets

A Time Petri Net (TPN) - a tuple $N = (P, T, F, m_0, Eft, Lft)$, where:

- P, T, F, m_0 - like before,
- $Eft : T \to \mathbb{N}$, $Lft : T \to \mathbb{N} \cup \{\infty\}$ - earliest and latest firing times of transitions ($Eft(t) \leq Lft(t)$ for each $t \in T$)

Distributed Time Petri Nets

A Distributed Time Petri Net (DTPN) - a set of sequential$^{(*)}$ TPNs, of pairwise disjoint sets of places, and communicating via joint transitions.

$^{(*)}$ *a net is sequential if none of its reachable markings concurrently enables two transitions*

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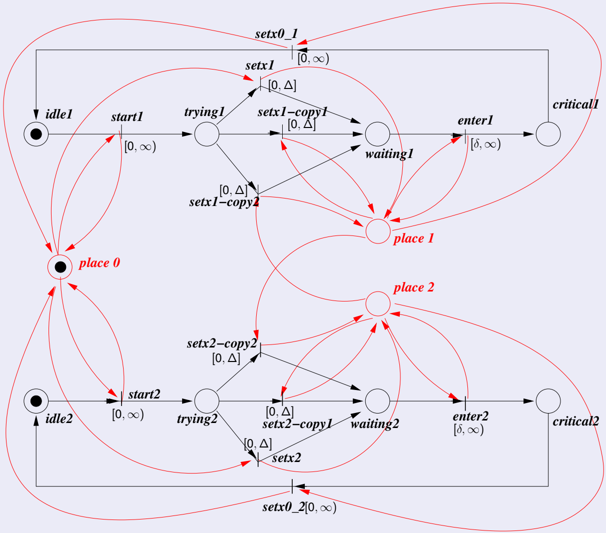
Example: Fischer's mutual exclusion protocol



Parametric verification for DTPNs

Parametric reachability - a general problem

Given a property *p*, we want to find:

- the minimal time $c \in N$ at which a state satisfying *p* can be reached

  (corresponds to finding the minimal *c* s.t. $EF^{\leq c}p$ or $EF^{<c}p$ holds),

Details of the verification method:

W.Penczek, A.Półrola, A.Zbrzezny: SAT-Based (Parametric) Reachability for a Class of Distributed Time Petri Nets, T. Petri Nets and Other Models of Concurrency 4: 72-97 (2010).

Parametric reachability

A general solution

Searching for a minimal $c \in \mathbb{N}$ s.t. $\mathrm{EF}^{\leq c} p$:

1. test whether p is reachable
2. if so, extract the time x at which it has been reached (we know that $c \leq \lceil x \rceil$)
3. check whether there is a path of a shorter time at which p is reachable
4. if such a path exists return to 2, otherwise return $\lceil x \rceil$

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Parametric Reachability

Solving the problem using BMC

Searching for a minimal $c \in \mathbb{N}$ s.t. $\mathrm{EF}^{\leq c} p$:

- we run the standard reachability test to find the first time value x at which p can be reached

  we obtain a shortest path (of a length k_0), but not necessarily of the shortest time

- in order to test whether p can be reached at the time shorter than n, we augment the net with an additional component and test reachability of $p \wedge p_{in}$

  we can start with $k = k_0$

- in order to know that a state is unreachable, we need either to run proving unreachability, or to find an upper bound on the path

  for certain types of nets such an upper bound can be deduced

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  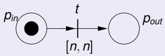

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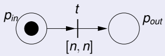

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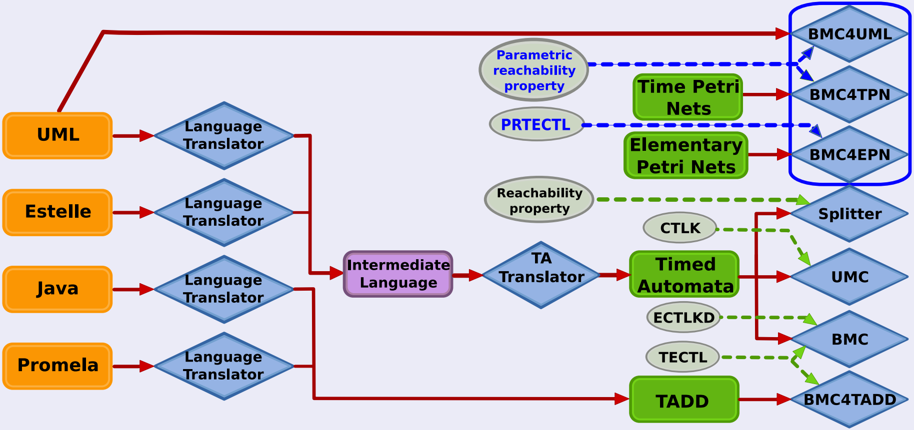
VerICS: architecture



Experimental Results

EPNs: mutex of NoP processes; $\varphi_1^b = \forall_{\Theta \leq b} EF(\neg p \wedge EG^{\leq \Theta} c_1)$

formula	NoP	k	PBMC				MiniSAT	SAT?
			vars	clauses	sec	MB	sec	
φ_1^1	3	2	1063	2920	0.01	1.3	0.003	NO
φ_1^1	3	3	1505	4164	0.01	1.5	0.008	YES
φ_1^2	3	4	2930	8144	0.01	1.5	0.01	NO
φ_1^2	3	5	3593	10010	0.01	1.6	0.03	YES
φ_1^2	30	4	37825	108371	0.3	7.4	0.2	NO
φ_1^2	30	5	46688	133955	0.4	8.9	0.52	YES
φ_1^3	4	6	8001	22378	0.06	2.5	0.04	NO
φ_1^3	4	7	9244	25886	0.05	2.8	0.05	YES

DTPNs: Fischer's protocol of 25 processes; $\Delta = 2$, $\delta = 1$; searching for minimal c s.t. $EF^{\leq c}p$, where p - violation of mutual exclusion

		tpnBMC				RSat		
k	n	variables	clauses	sec	MB	sec	MB	sat
0	-	840	2194	0.0	3.2	0.0	1.4	NO
2	-	16263	47707	0.5	5.2	0.1	4.9	NO
4	-	33835	99739	1.0	7.3	0.6	9.1	NO
6	-	51406	151699	1.6	9.6	1.8	13.8	NO
8	-	72752	214853	2.4	12.3	20.6	27.7	NO
10	-	92629	273491	3.0	14.8	321.4	200.8	NO
12	-	113292	334357	3.7	17.5	14.3	39.0	YES
12	7	120042	354571	4.1	18.3	45.7	59.3	YES
12	6	120054	354613	4.0	18.3	312.7	206.8	YES
12	5	120102	354763	4.0	18.3	64.0	77.7	YES
12	4	120054	354601	4.1	18.3	8.8	35.0	YES
12	3	115475	340834	3.9	17.7	24.2	45.0	YES
12	2	115481	340852	3.9	17.8	138.7	100.8	YES
12	1	115529	341008	3.9	17.7	2355.4	433.4	NO
				40.1	18.3	3308.3	433.4	

Final Remarks

- Parametric BMC for EPN and DTPN,
- New modules of VerICS are aimed at SAT-based parametric verification of Elementary Petri Nets, Distributed Time Petri Nets, and UML,
- Avaialable at http://pegaz.ipipan.waw.pl/verics/

The End

Thank You