

# Petri Net (versus) State Spaces

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# My experience with state spaces

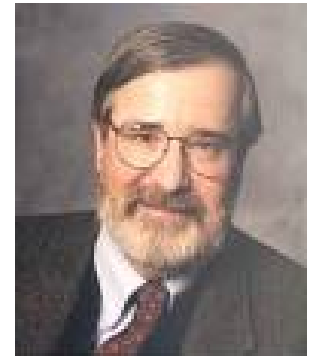
- INA Integrated Net Analyzer
- LoLA A Low Level Analyzer
- The service-technology.org tool family

## Case studies and applications:

- Finding hazards in a GALS wrapper
- Integration into Pathway Logic Assistent
- Soundness check for 700+ industrial business process models in (avg) 2 msec
- Verification of web service choreographies
- Verification of parameterized Boolean programs
- Solving AI planning challenges
- Integration into BP related tools like ProM, Oryx
- Integration into model checking platforms (MC Kit, PEP, CPN-AMI,...)
- .....To be continued



Why state spaces?



Why Petri nets?



Verification based on state space



Why state spaces?

-Consider asynchronously communicating components rather than global state changes

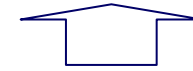
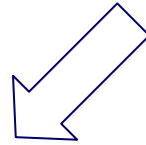
-Consider causality of events rather than their ordering in time!

# Petri net principles

Monotonicity of firing

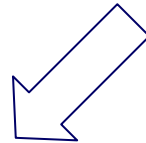


Presence or absence of resources  
rather than reading / writing variables



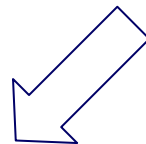
Linearity of firing rule

-Consider asynchronously communicating  
components rather than global state changes



Locality

Partially ordered  
event structures



-Consider causality of events rather than their  
ordering in time!

# Petri net specific verification

Monotonicity of firing

Coverability graphs  
Siphons / traps

Linearity of firing rule

invariants

Locality

Net reduction

Partially ordered  
event structures

Branching prefixes

# State space generation

1. Checking enabledness
2. Firing a transition
3. Backtracking
4. Managing the visited states

# State space generation

## 1. Checking enabledness

After firing, only check:

previously enabled transitions which have lost tokens

previously disabled transitions which have gained tokens

... managed through explicitly stored lists

... typical: reduction from linear to constant time

## 2. Firing a transition

## 3. Backtracking

## 4. Managing the visited states





# State space generation

1. Checking enabledness

2. Firing a transition

Marking changed via list of pre-, list of post-places

- effort does not depend on size of net
- Typically: constant effort



3. Backtracking

4. Managing the visited states

# State space generation

1. Checking enabledness

2. Firing a transition

3. Backtracking

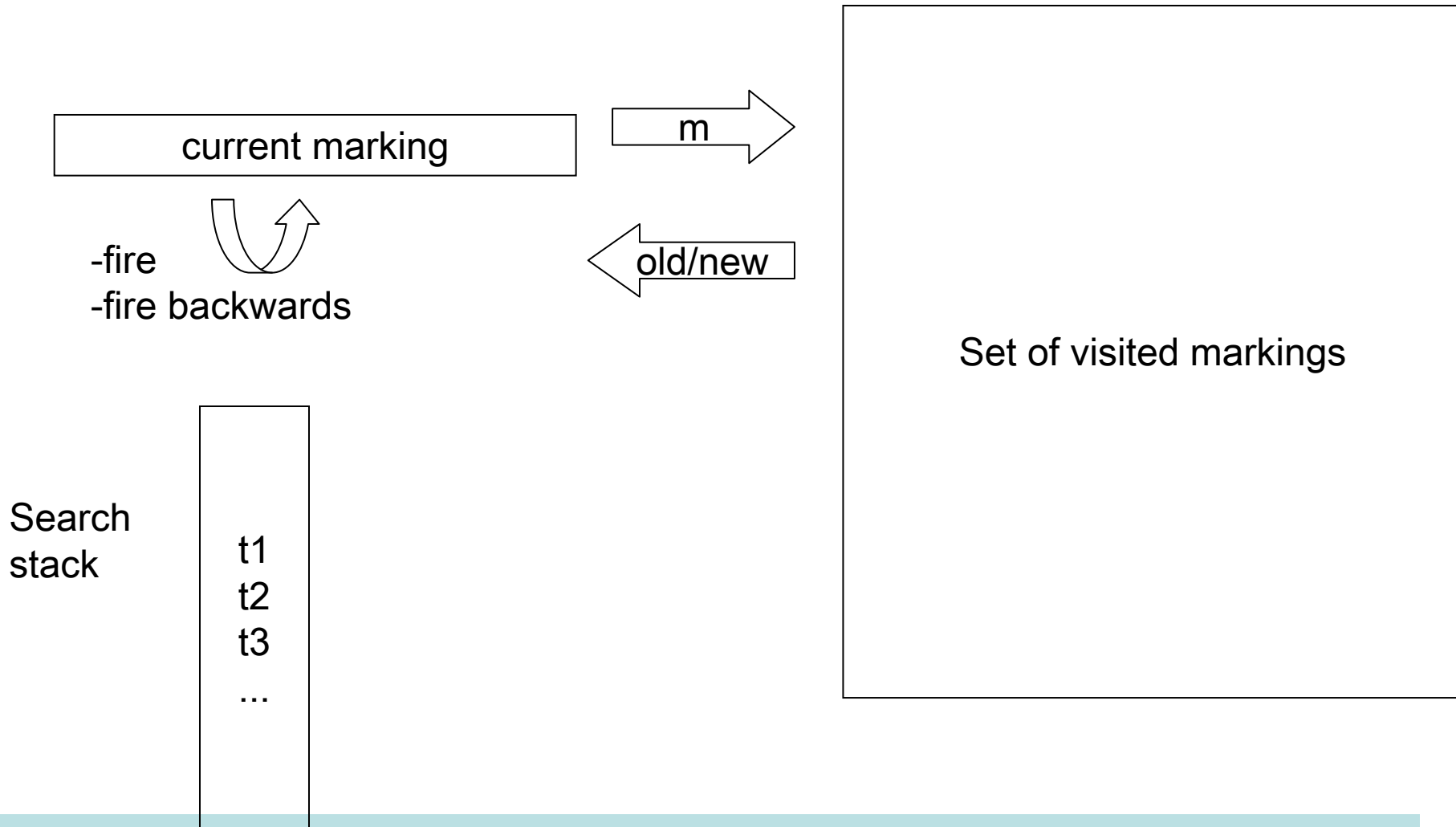
In depth-first search: fire transition backwards

In breadth-first search: implemented as incremental depth-first search

4. Managing the visited states

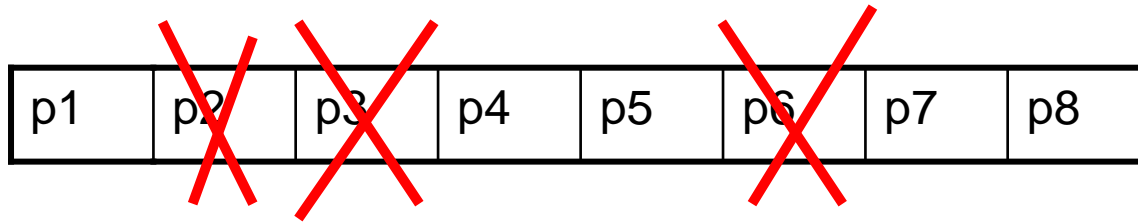


# Consequence: „write-only“ storage of markings



# 4. Managing the visited states

only performed actions: search, insert



$$a_1 p_1 + a_2 p_2 + a_3 p_3 = \text{const.}$$

$$b_2 p_2 + b_4 p_4 + b_6 p_6 = \text{const.}$$

$$c_3 p_3 + c_7 p_7 + c_8 p_8 = \text{const.}$$



Place invariants

30-60% less memory  
preprocessing <1sec  
run time gain: 30-60%

# Reduction techniques

1. Linear Algebra
2. The Sweep-Line Method
3. Symmetries
4. Stubborn Sets

# 1. Linear algebra

- The invariant calculus
  - originally invented for *replacing* state spaces
  - in LoLA: used for *optimizing* state spaces

Already seen: place invariants

Transition invariant: firing vector of a potential cycle

# Transition invariants

Linearity

for termination sufficient: store one state per cycle of occurrence graph

implementation in LoLA:

transition invariants

- set of transitions that occur in every cycle
- store states where those transitions enabled

saves space, if applied in connection with stubborn sets, costs time

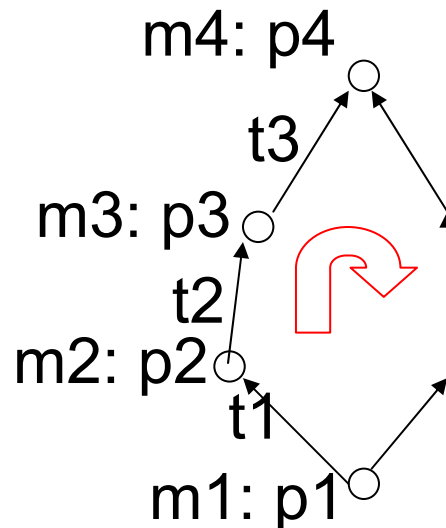
## 2. The sweep-line method

- Relies on progress measure

LoLA computes measure automatically:



$p_2 = p_1 + \Delta t_1$   
 $p_3 = p_2 + \Delta t_2$   
...



transition invariant



## 3. The symmetry method

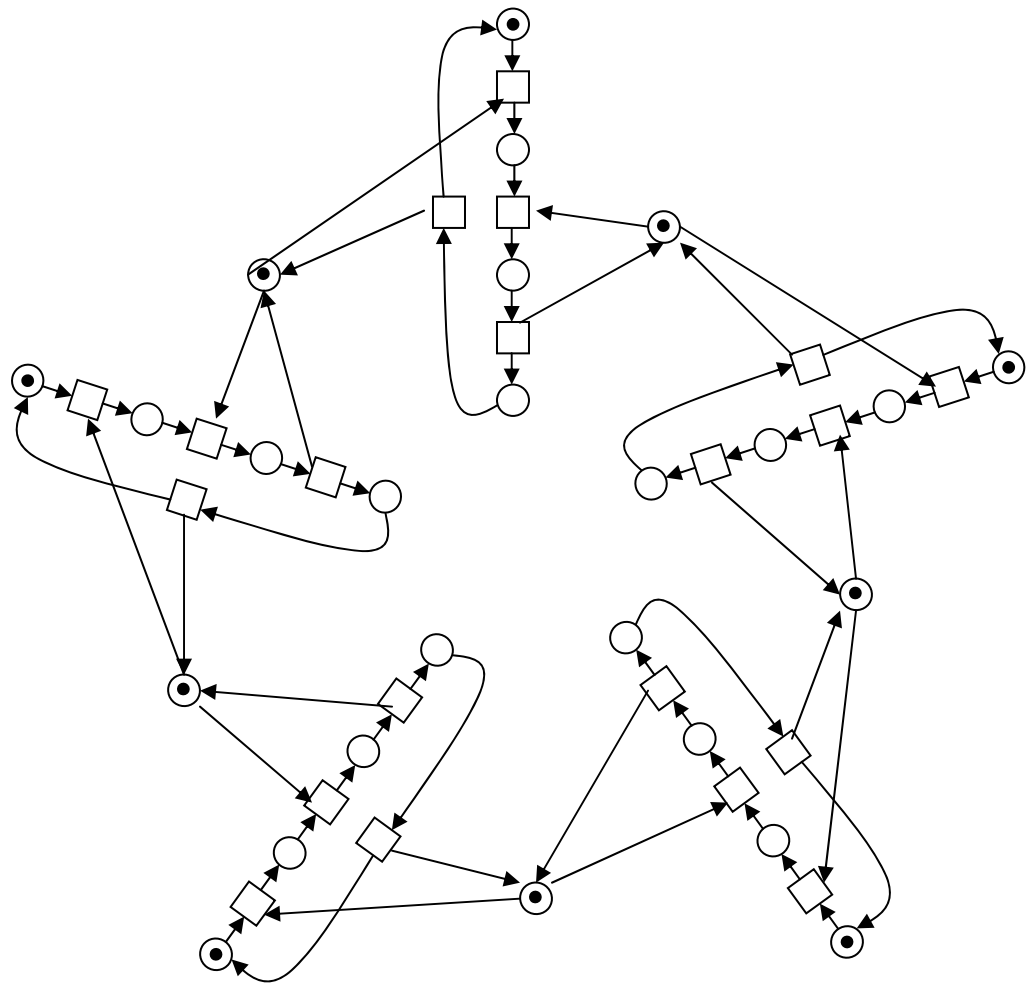


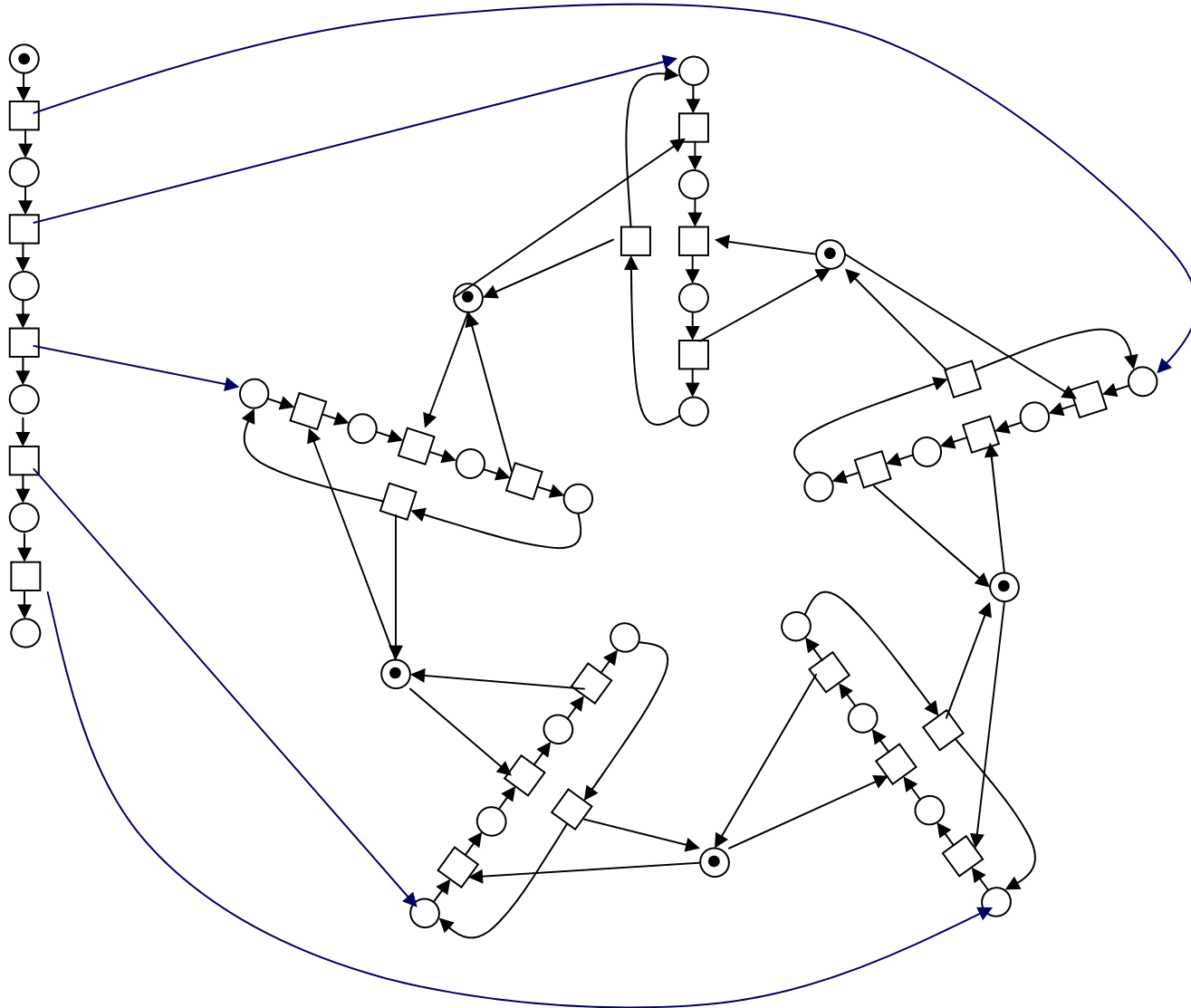
LoLA: A symmetry = a graph automorphism of the PT-Net

All graph automorphisms = a group (up to exponentially many members)  
- stored in LoLA: polynomial generating set

A marking class: all markings that can be transformed into each other by a symmetry  
- executed in LoLA: polynomial time approximation

# Example





## 4. Stubborn set method

- Dedicated method for each supported property

traditional LTL-preserving method:

- one enabled transition
- the basic stubbornness principal
- only *invisible* transitions
- at least once, on every cycle, all enabled transitions

LoLA:

- can avoid some of the criteria, depending on property



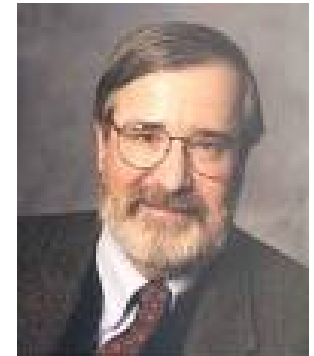
# Conclusion



Why state spaces?



That's why



Why Petri nets?

Further reading:

- Tools: [www.service-technology.org](http://www.service-technology.org)
- Group / Papers: [www.informatik.uni-rostock.de/tpp/](http://www.informatik.uni-rostock.de/tpp/)