



Partial Orders Fit For Work

Jörg Desel

FernUniversität in Hagen

Partial Orders Fit For Work

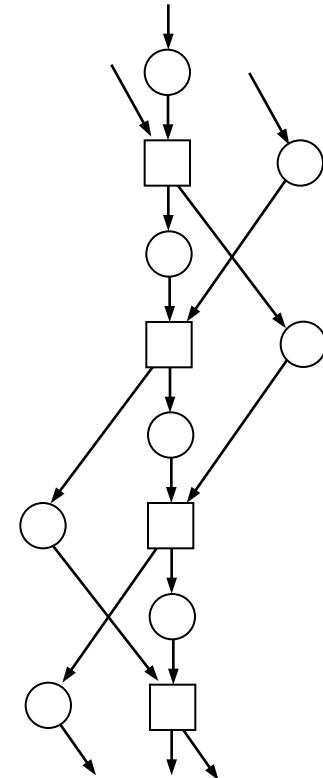
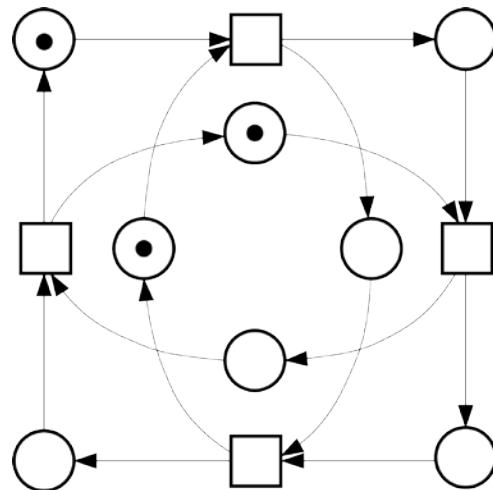
PART I:
A proof using partial orders
(occurrence nets)
work done in 1988 at GMD

PART II:
Process Model Synthesis
From Partial Orders
(ViPtool)

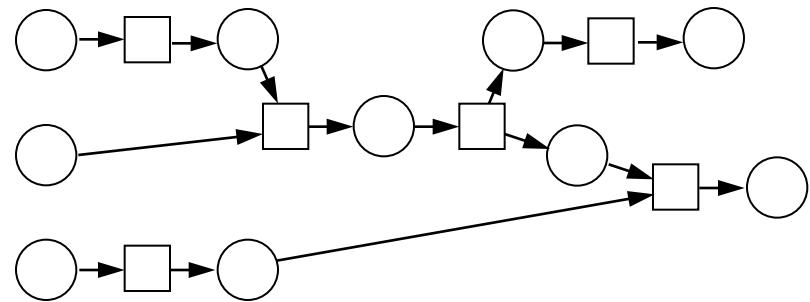
PART I:

A proof using partial orders (occurrence nets)

work done in 1988 at GMD

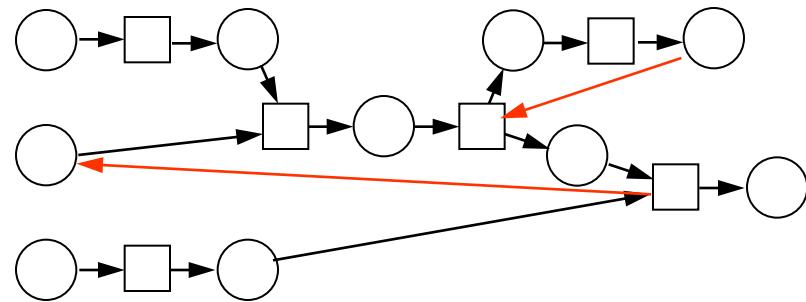


An occurrence net (B,E,K)



An **occurrence net** (B, E, K)

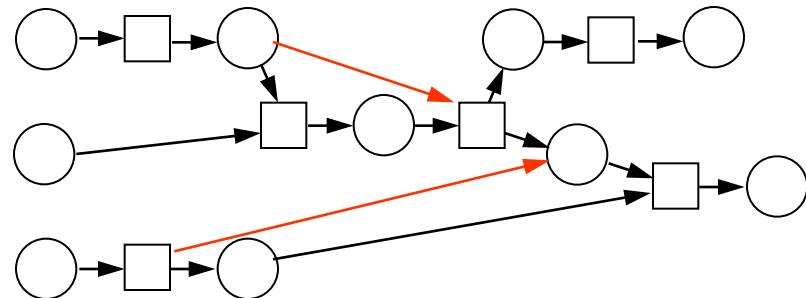
acyclic



An **occurrence net** (B,E,K)

acyclic

places unbranched



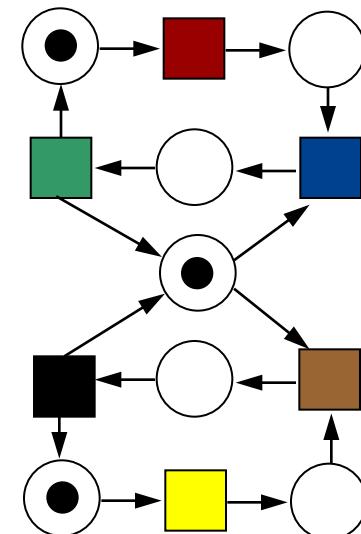
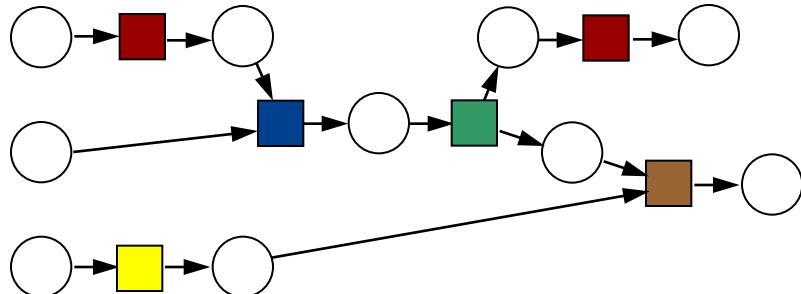
An **occurrence net** (B,E,K)

of a **Petri net** (S,T,F)

acyclic

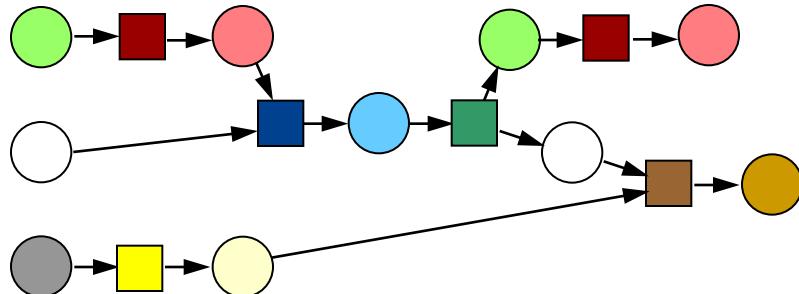
places unbranched

maps to the Petri net

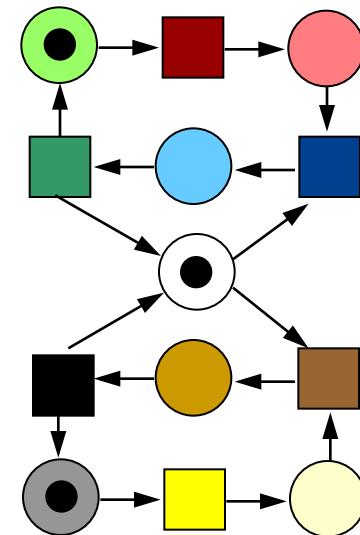


An **occurrence net** (B,E,K)

acyclic
places unbranched
maps to the Petri net



of a **Petri net** (S,T,F)



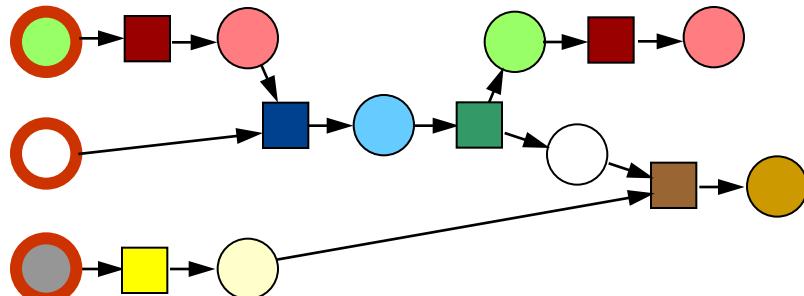
An occurrence net (B,E,K)

acyclic

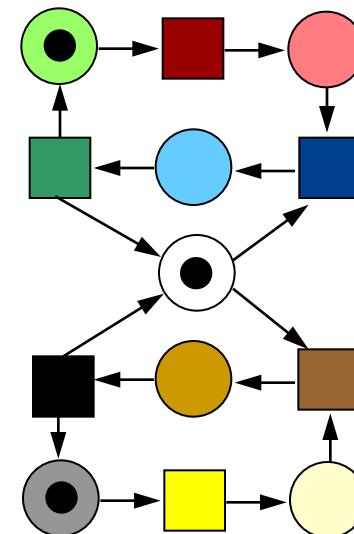
places unbranched

maps to the Petri net

minimal places map to tokens



of a Petri net (S,T,F)



An **occurrence net** (B,E,K)

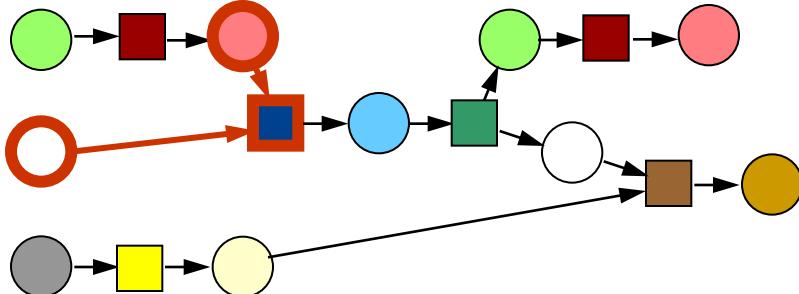
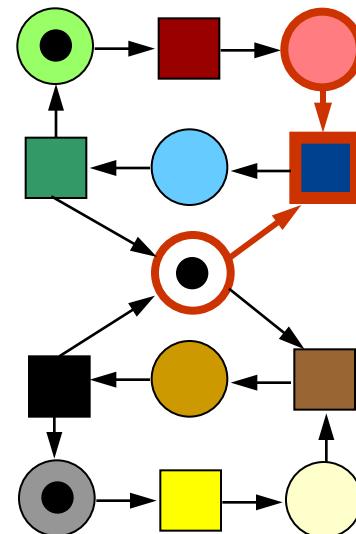
acyclic

places unbranched

maps to the Petri net

minimal places map to tokens

presets of transitions map to presets of transitions


 of a **Petri net** (S,T,F)


An **occurrence net** (B,E,K)

acyclic

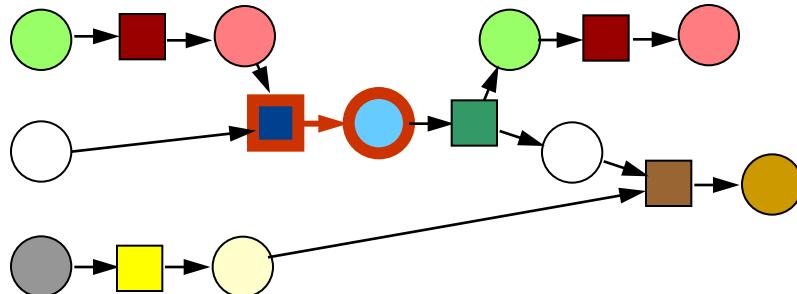
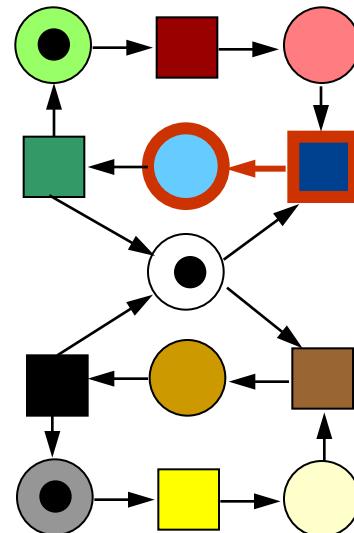
places unbranched

maps to the Petri net

minimal places map to tokens

presets of transitions map to presets of transitions

postsets of transitions map to postsets of transitions

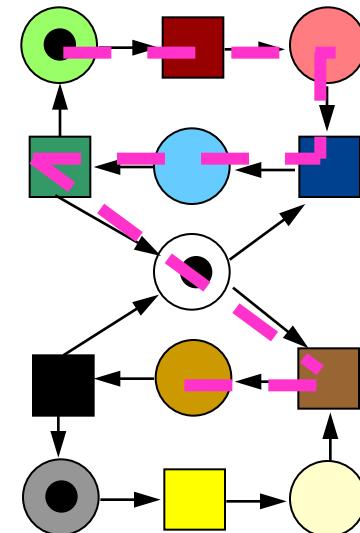
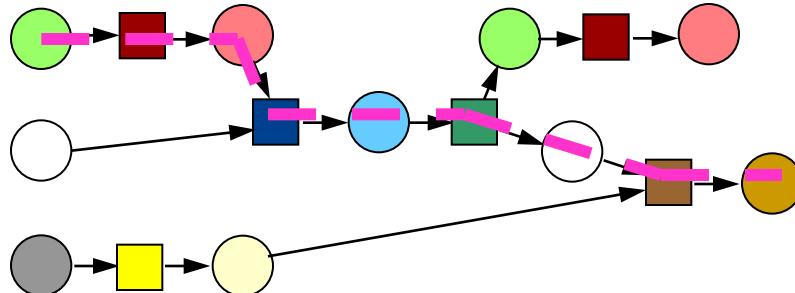

 of a **Petri net** (S,T,F)


An occurrence net (B,E,K)

of a Petri net (S, T, F)

Lemma: paths map to

paths

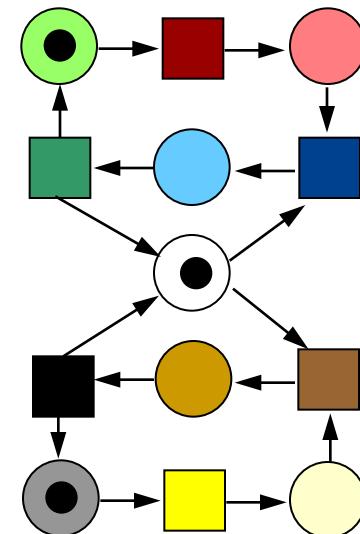
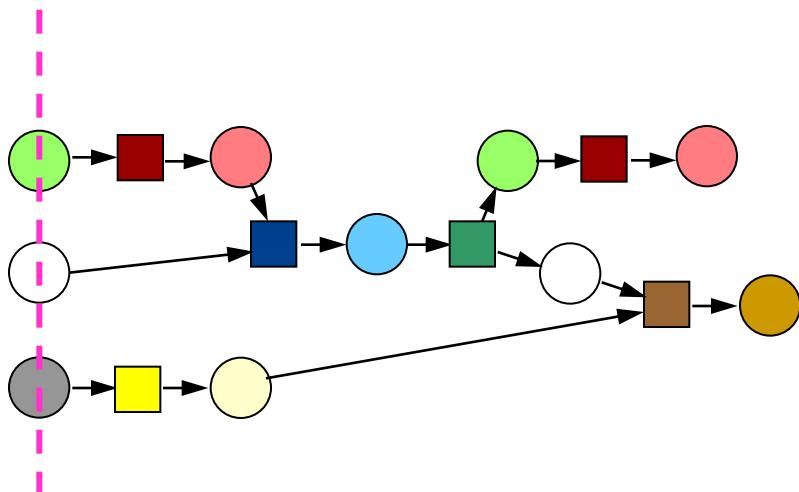


An occurrence net (B,E,K)

of a Petri net (S,T,F)

Lemma: finite cuts
(max. sets of mutually concurrent places)
map to

reachable markings

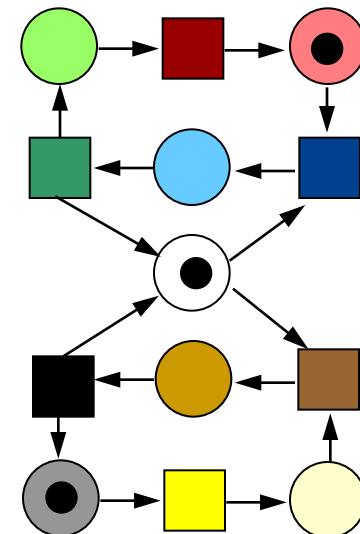
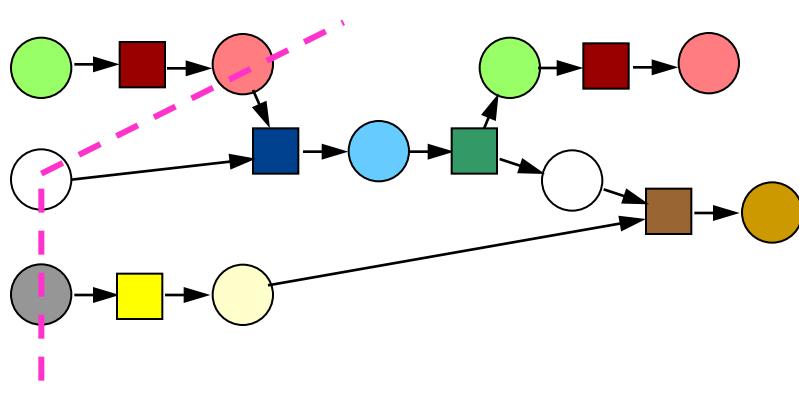


An **occurrence net** (B,E,K)

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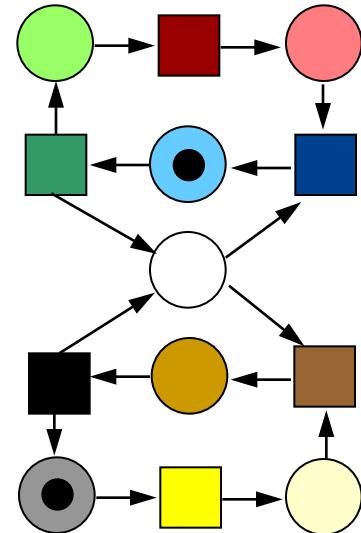
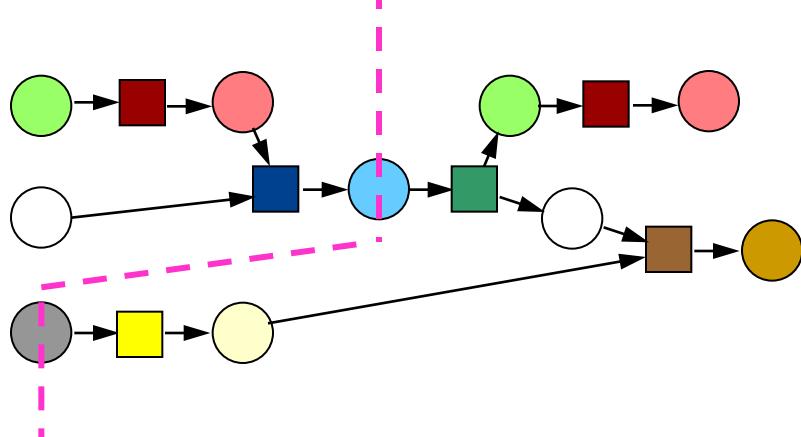


An **occurrence net** (B, E, K)

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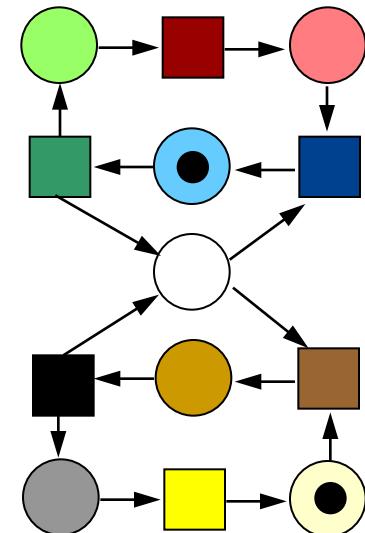
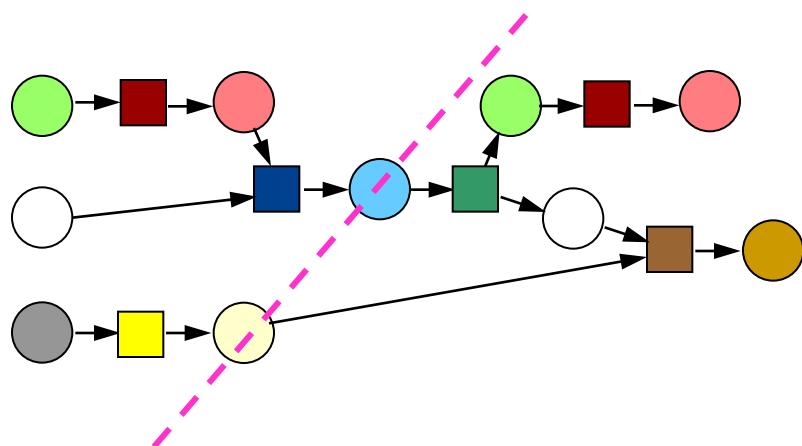


An **occurrence net** (B, E, K)

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Lemma: finite cuts
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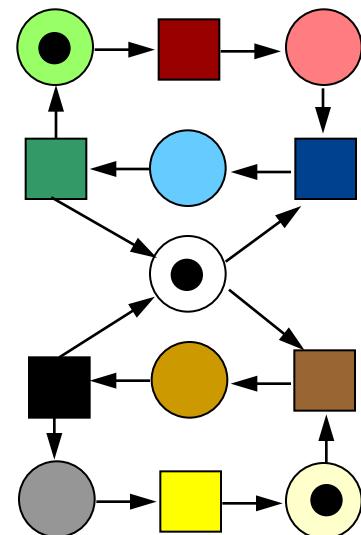
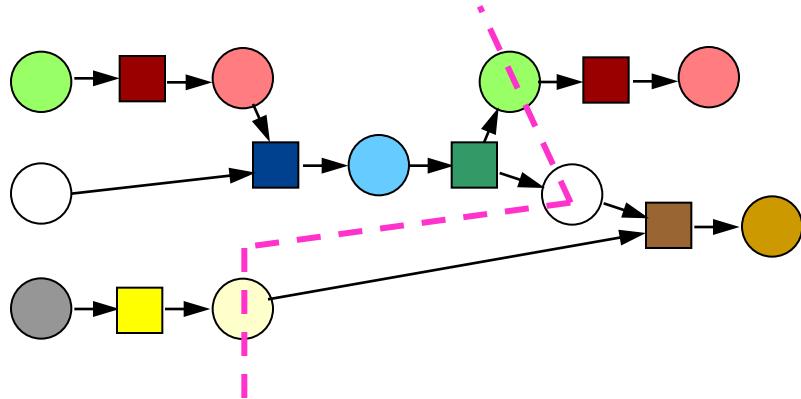


An **occurrence net** (B,E,K)

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Lemma: finite cuts
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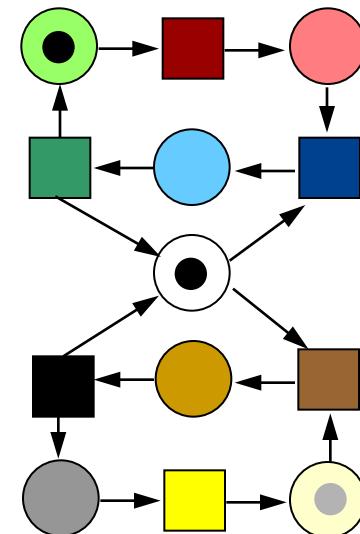
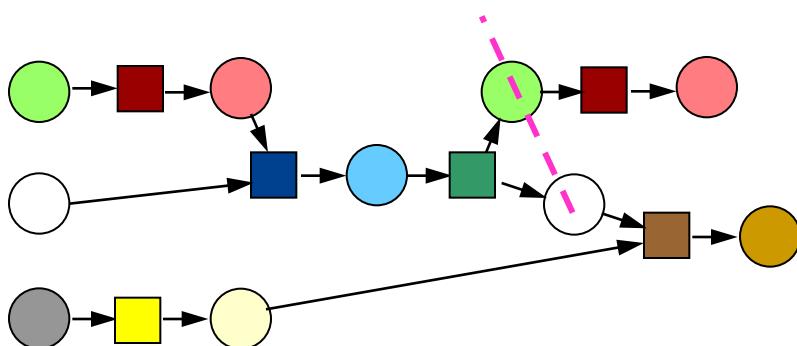


An **occurrence net** (B, E, K)

of a **Petri net** (S, T, F)

Corollary: finite co-sets
 (sets of mutually concurrent places)
 map to

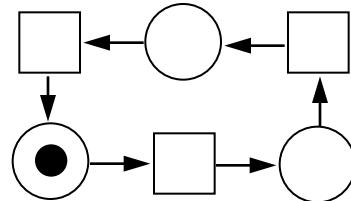
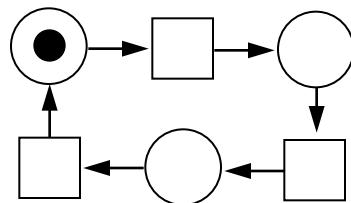
reachable sub-markings



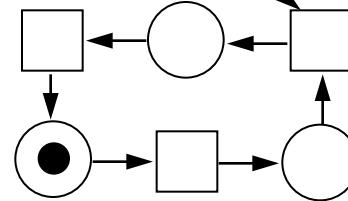
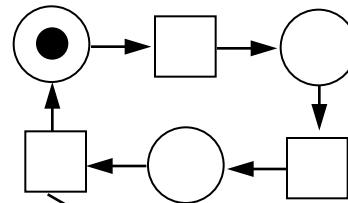
Theorem: each connected live and bounded Petri net is strongly connected

Theorem: each **connected** live and bounded Petri net is **strongly connected**

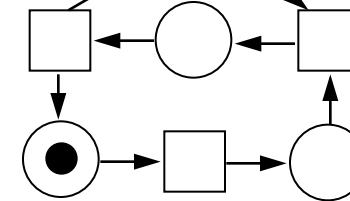
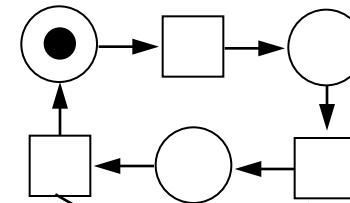
not connected



connected



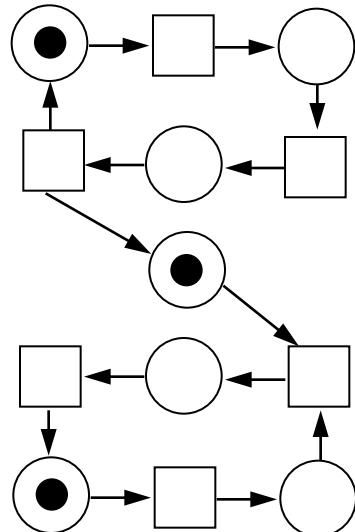
strongly connected



Theorem: each connected **live** and **bounded** Petri net is strongly connected

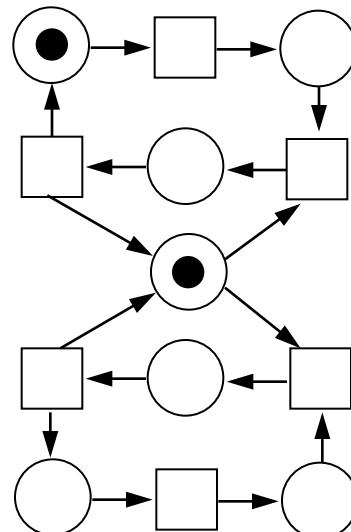
live, not bounded

each transition can
always occur again

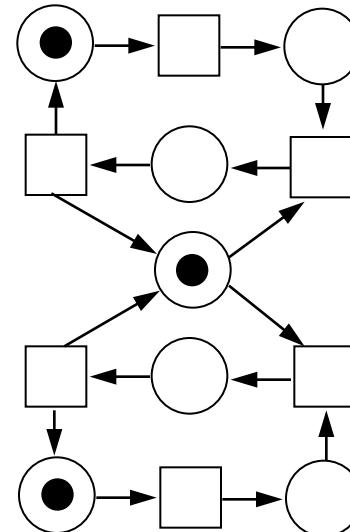


bounded, not live

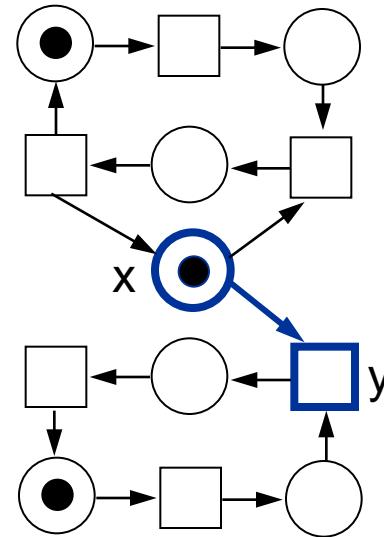
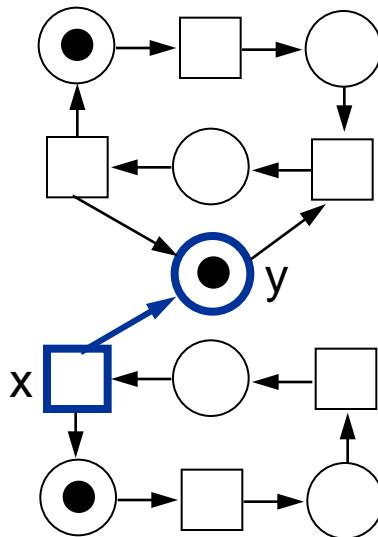
each place has a bound
(maximal number of tokens)



live and bounded



Theorem: each connected live and bounded Petri net is strongly connected



Lemma: if a net is connected but not strongly connected then
for some arc (x,y) there is no directed path from y to x

Corollary: if, in a connected net, for each arc (x,y)
there is a path from y to x , then the net is strongly connected

Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

We will show that there is a path from y to x .

Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).



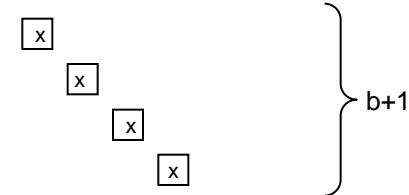
Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b + 1$ occurrences of the transition x .
(exists, because the net is live).



Theorem: each connected live and bounded Petri net is strongly connected

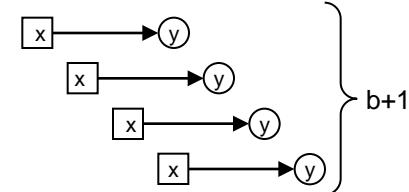
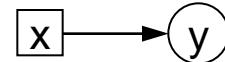
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(exists, because the net is live).

Since postsets of the occurrences of x are respected,
each occurrence of x has an occurrence of y in its postset.



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

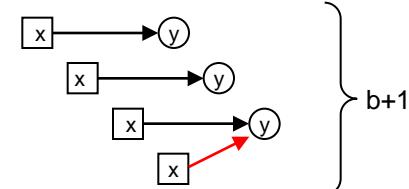
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Since postsets of the occurrences of x are respected,
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Since places in occurrence nets are not branched,
all these occurrences of y are distinct.



Theorem: each connected live and bounded Petri net is strongly connected

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Case 1: x is a transition and y is a place.

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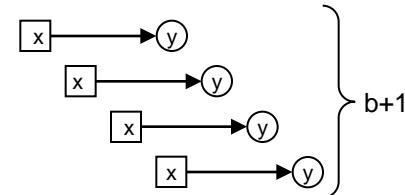
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Since places in occurrence nets are not branched,
 all these occurrences of y are distinct.

Since b is its bound, the place y never carries more than b tokens.

Hence no co-set contains all $b+1$ occurrences of y .



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

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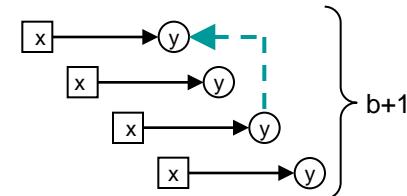
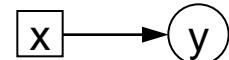
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Hence no co-set contains all $b+1$ occurrences of y .

So at least two of these occurrences are connected by a path.



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

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Since postsets of the occurrences of x are respected,
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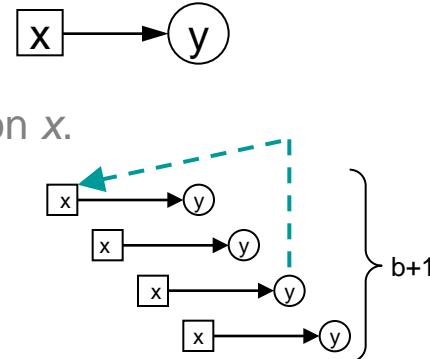
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Since b is its bound, the place y never carries more than b tokens.
 Hence no co-set contains all $b+1$ occurrences of y .

So at least two of these occurrences are connected by a path.

Again since places in occurrence nets are not branched,
 this path goes through an occurrence of x .

So there is a path from an occurrence of y to an occurrence of x .



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 1: x is a transition and y is a place.

Let b be the bound of y (exists, because the net is bounded).

Assume an occurrence net with $b+1$ occurrences of the transition x .
 (exists, because the net is live).

Since postsets of the occurrences of x are respected,
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Since places in occurrence nets are not branched,
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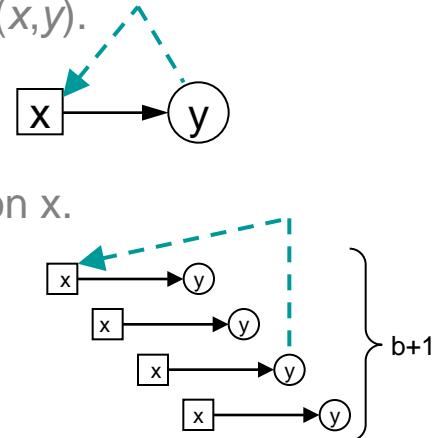
Since b is its bound, the place y never carries more than b tokens.
 Hence no co-set contains all $b+1$ occurrences of y .

So at least two of these occurrences are connected by a path.

Again since places in occurrence nets are not branched,
 this path goes through an occurrence of x .

So there is a path from an occurrence of y to an occurrence of x .

Since paths are mapped to paths, there is a path from y to x .



Theorem: each connected live and bounded Petri net is strongly connected

Proof: Consider a live and bounded connected net and an arbitrary arc (x,y) .

Case 2: x is a **place** and y is a **transition**.

Let b be the bound of x (exists, because the net is bounded).

Assume an occurrence net with $b+1$ occurrences of the transition y .
 (exists, because the net is live).

Since **presets** of the occurrences of y are respected,
 each occurrence of y has an occurrence of x in its **preset**.

Since places in occurrence nets are not branched,
 all these occurrences of x are distinct.

Since b is its bound, the place x never carries more than b tokens.

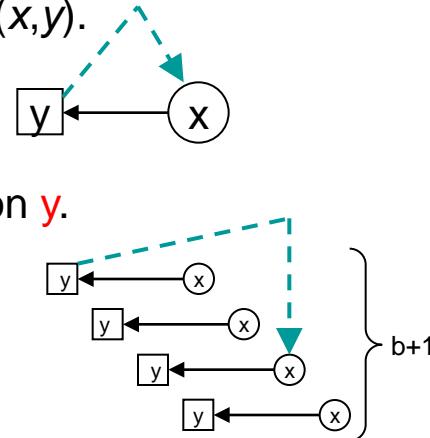
Hence no co-set contains all $b+1$ occurrences of x .

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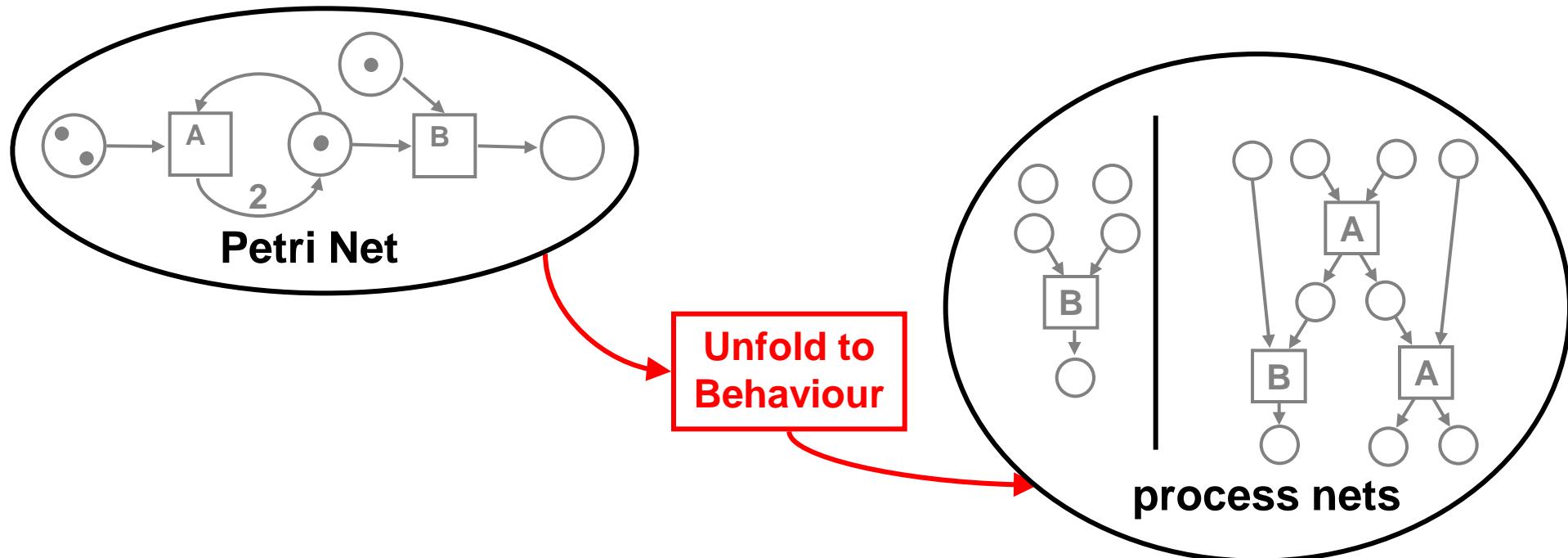
PART II:

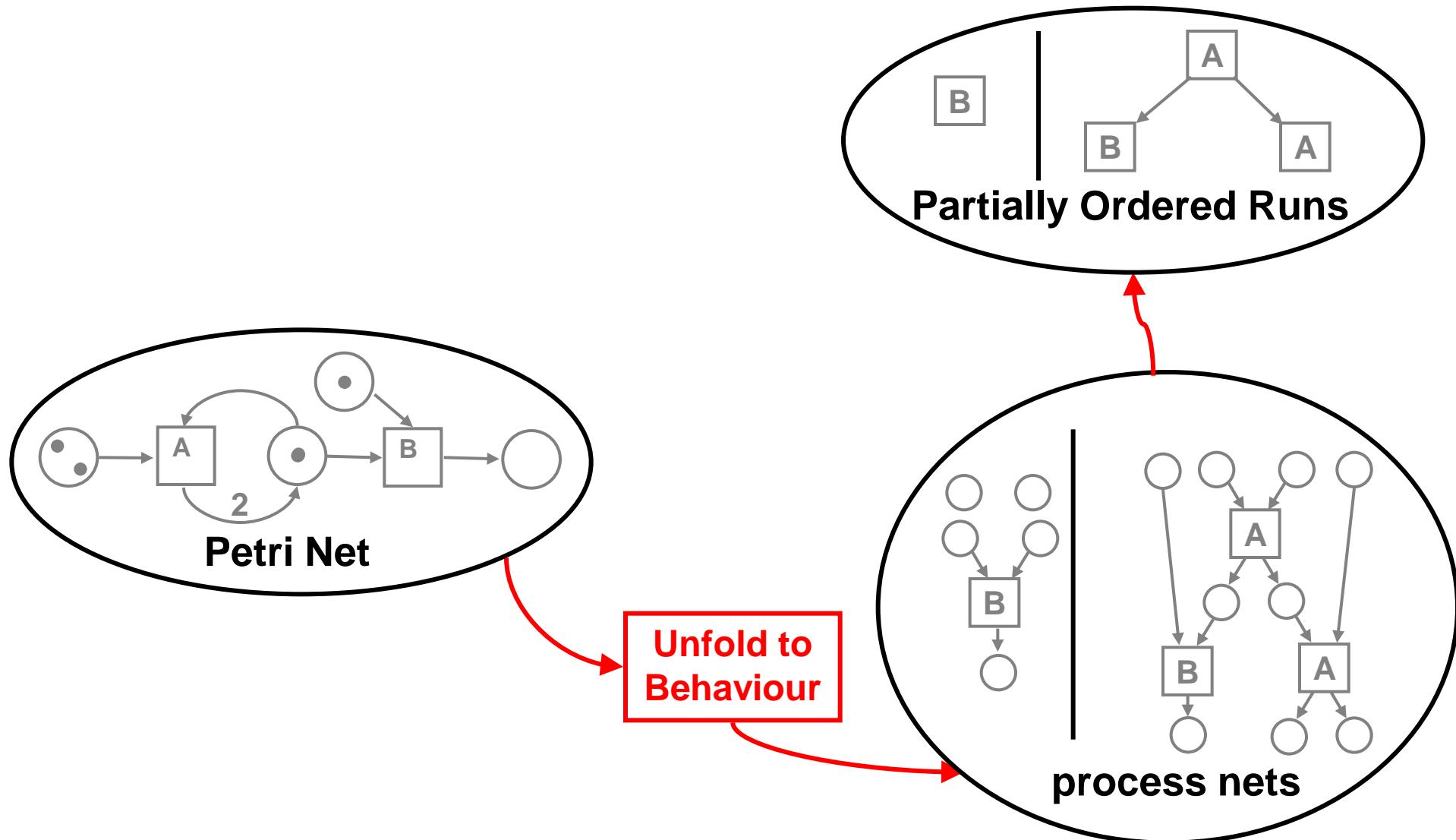
Process Model Synthesis From Partial Orders (ViPtool)

PART II:

Process Model Synthesis From Partial Orders (VIPtool)

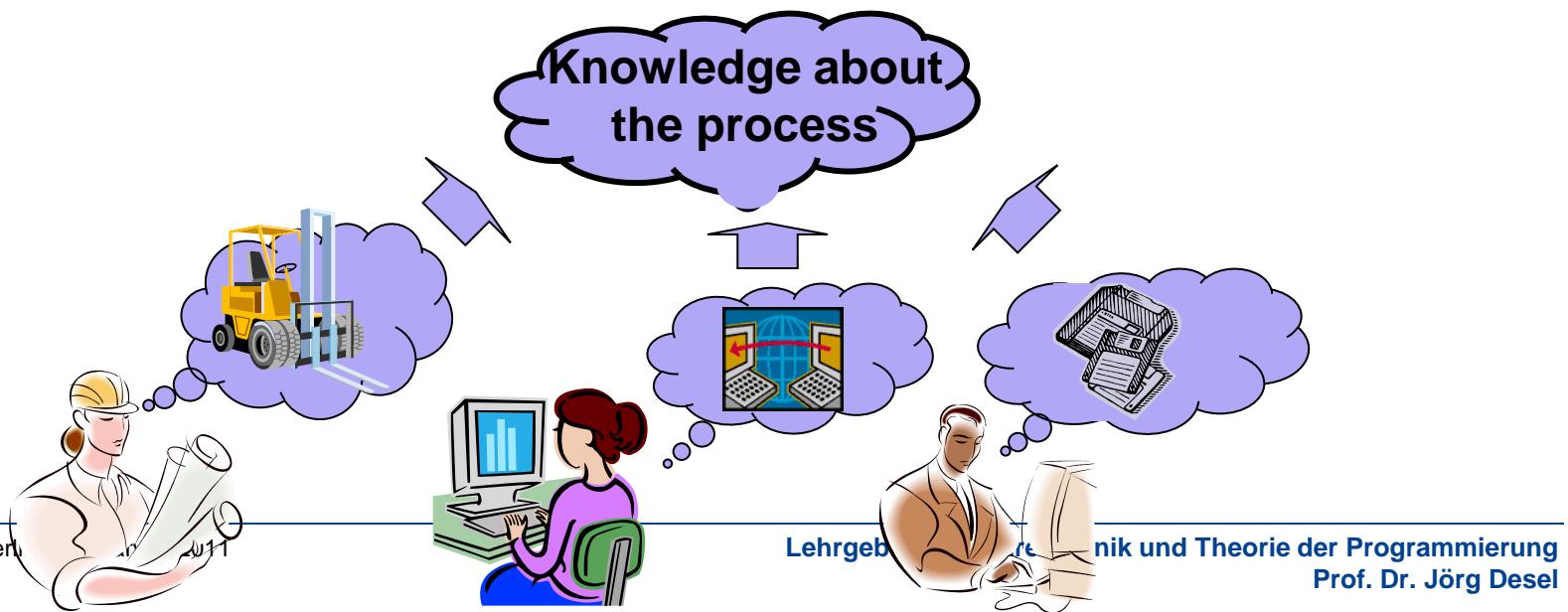
**VIPtool was originally created in 1996 -1998
by my group and the group of Andreas Oberweis,
Institute AIFB, University of Karlsruhe
(Carl-Adam-Petri award !!)**

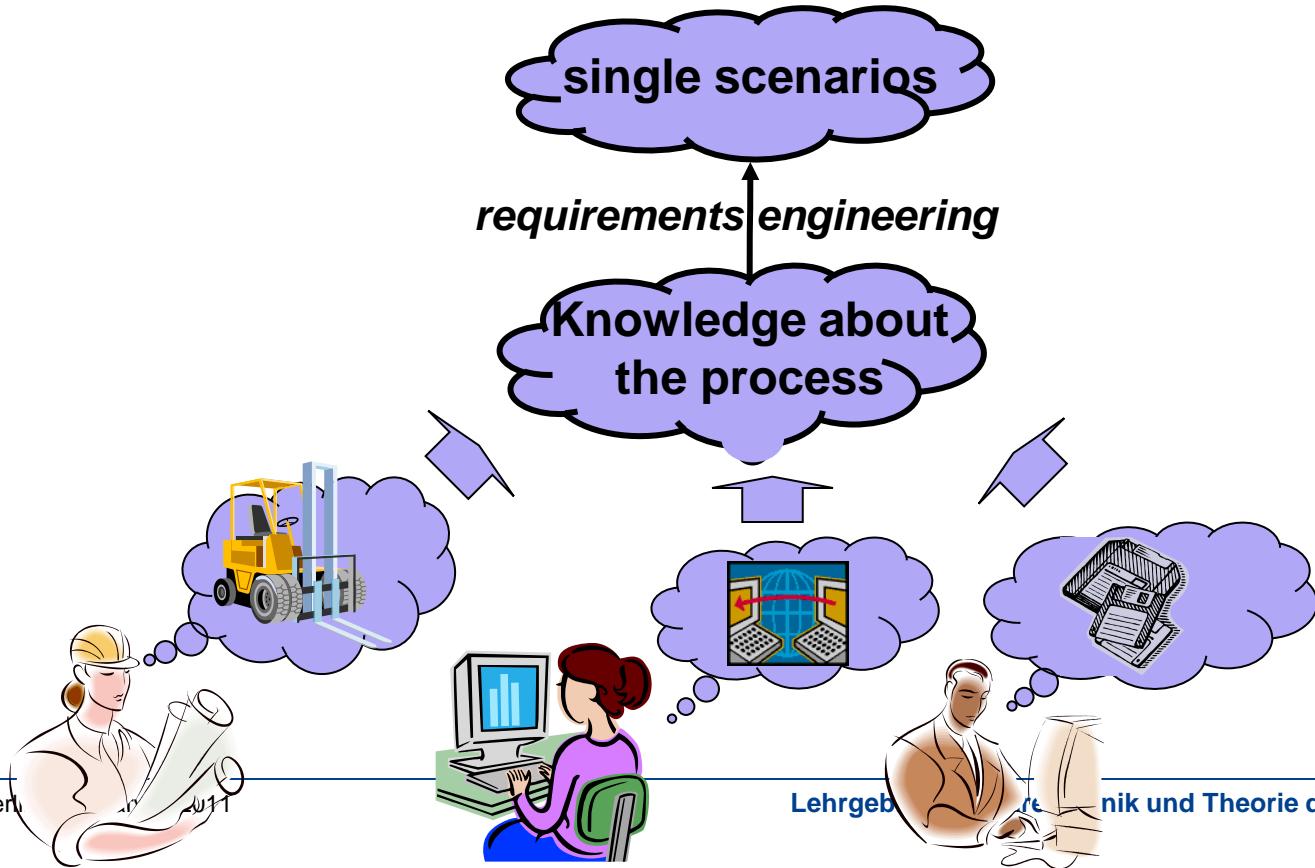


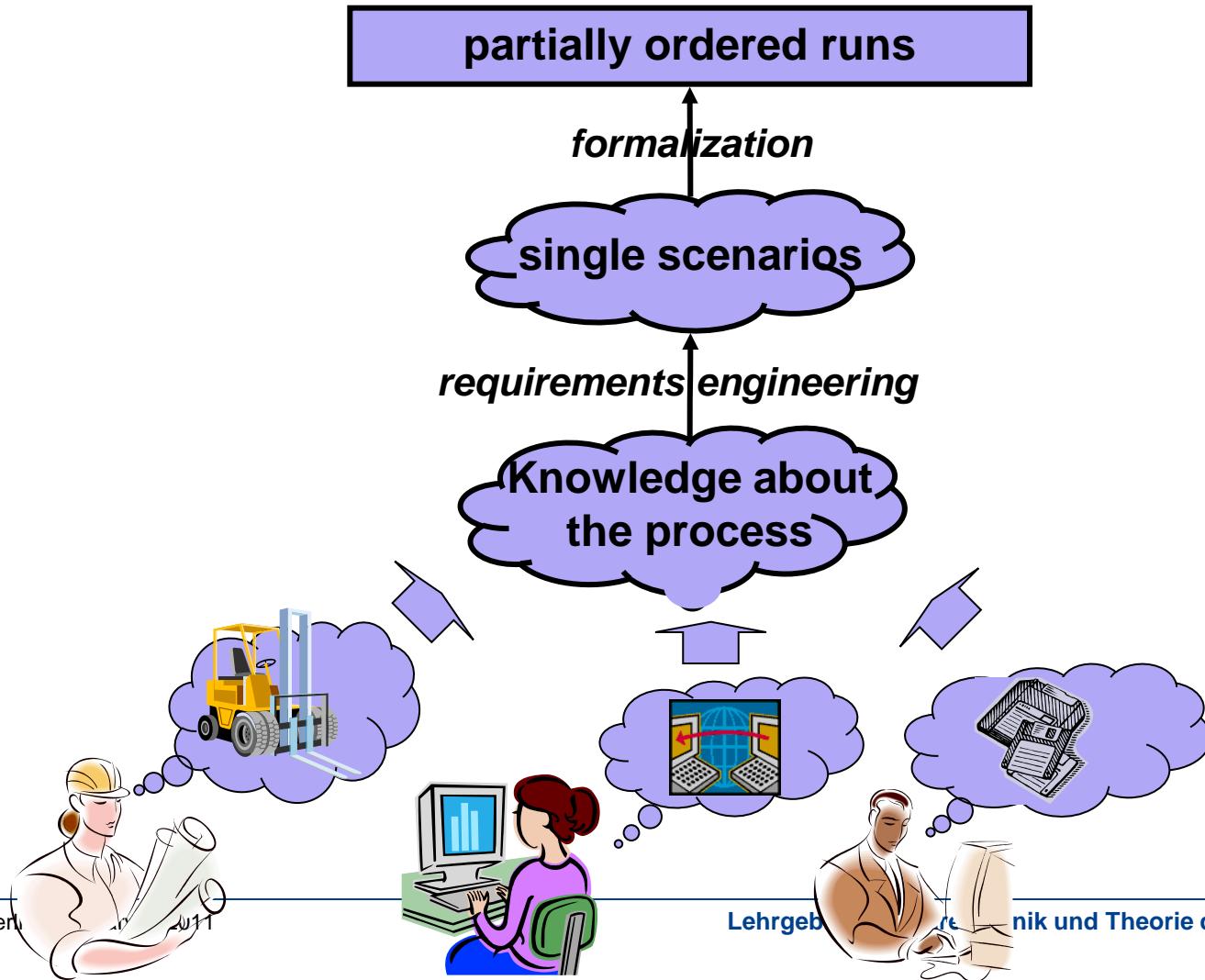


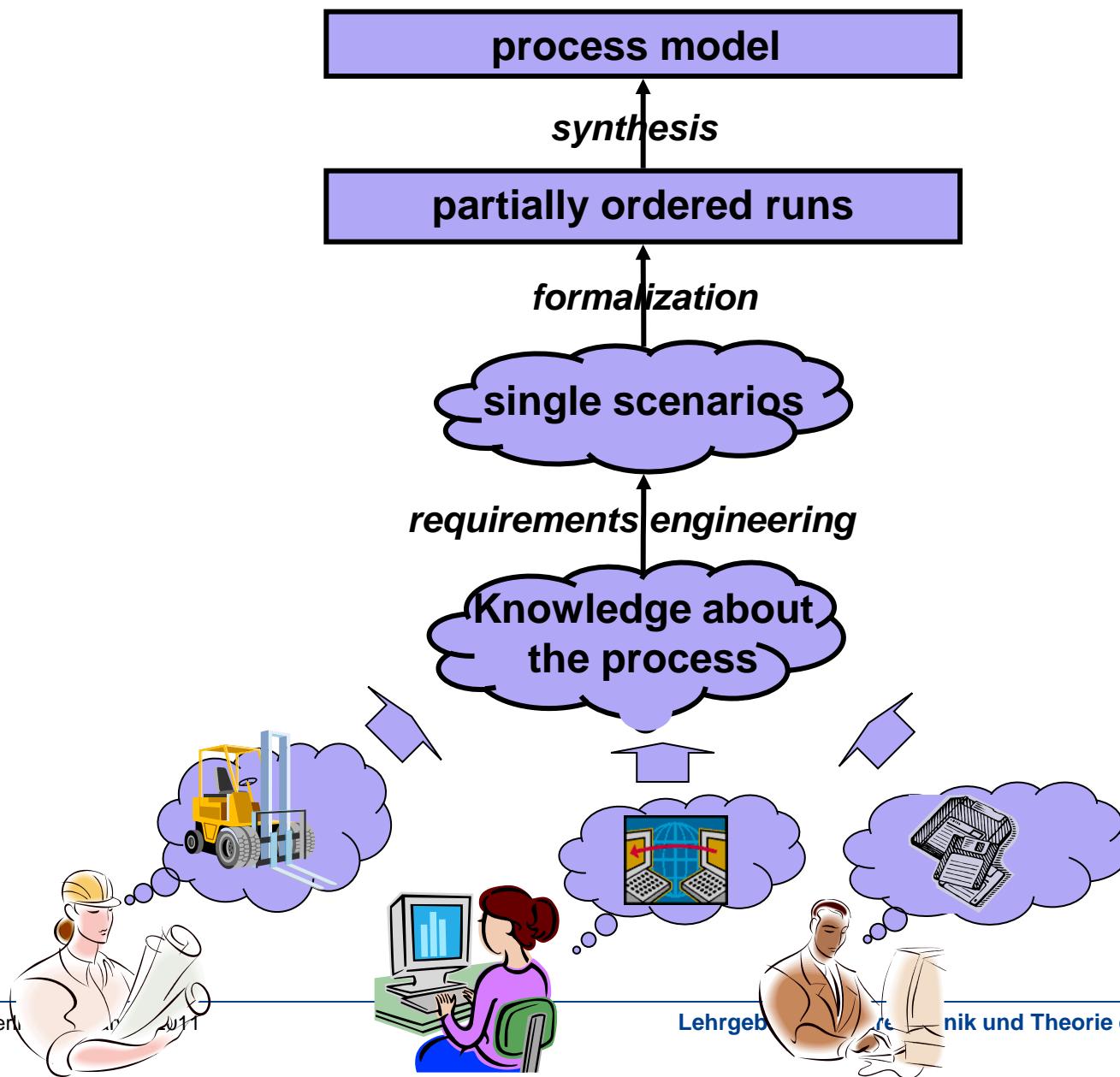
Initial Situation:

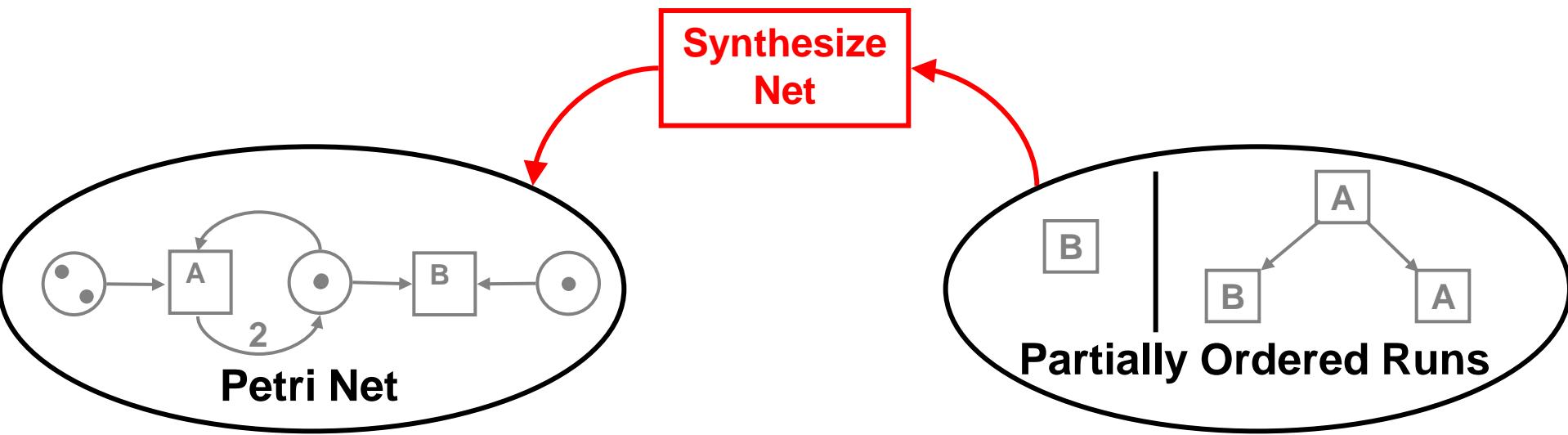
Knowledge about a process is distributed in several peoples' mind in an informal environment

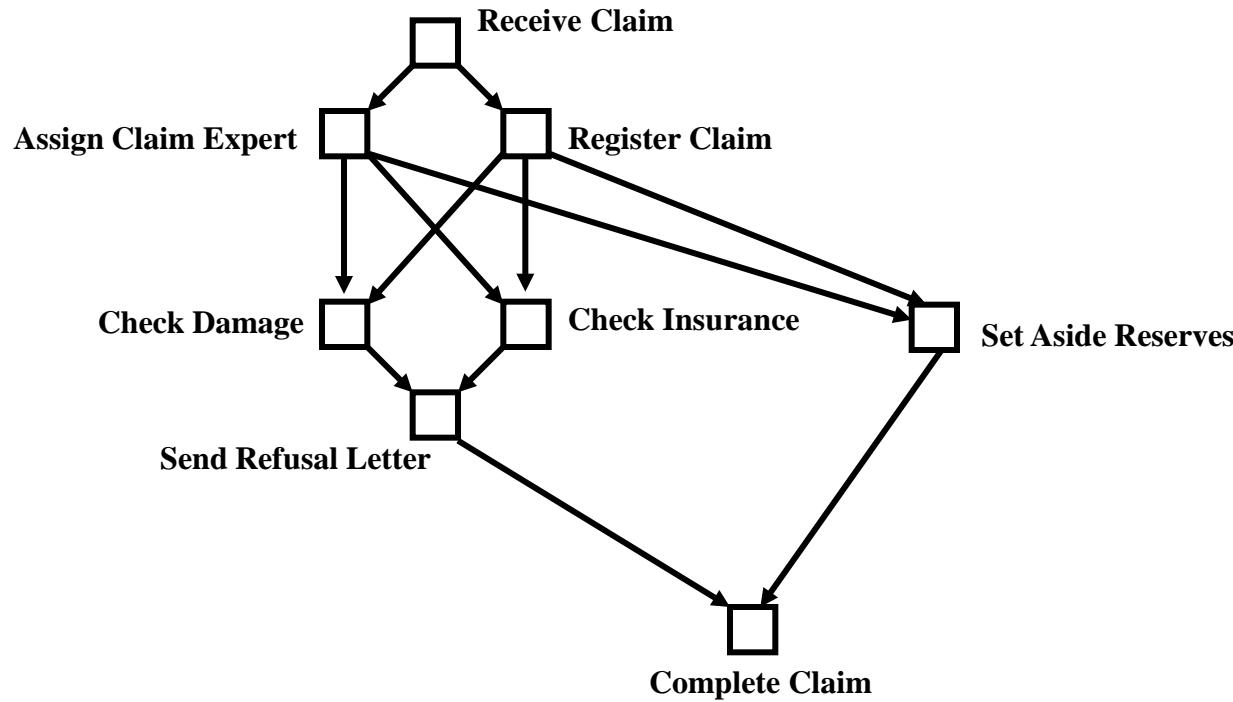




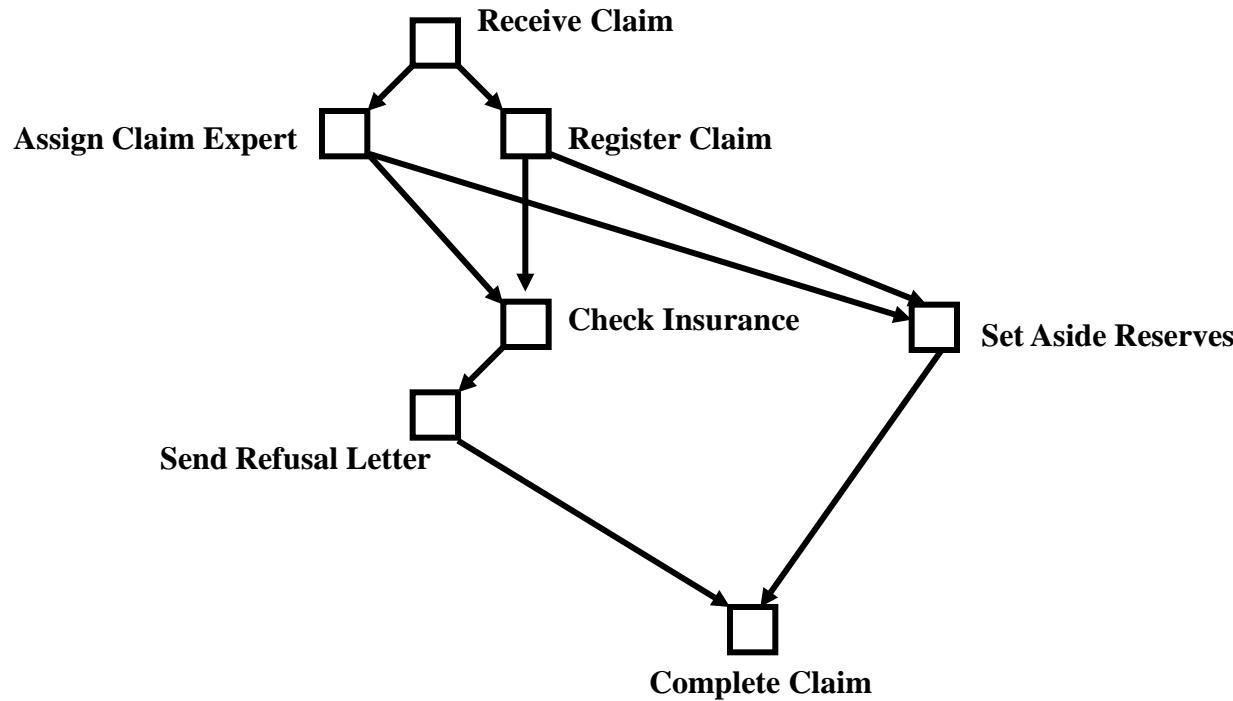




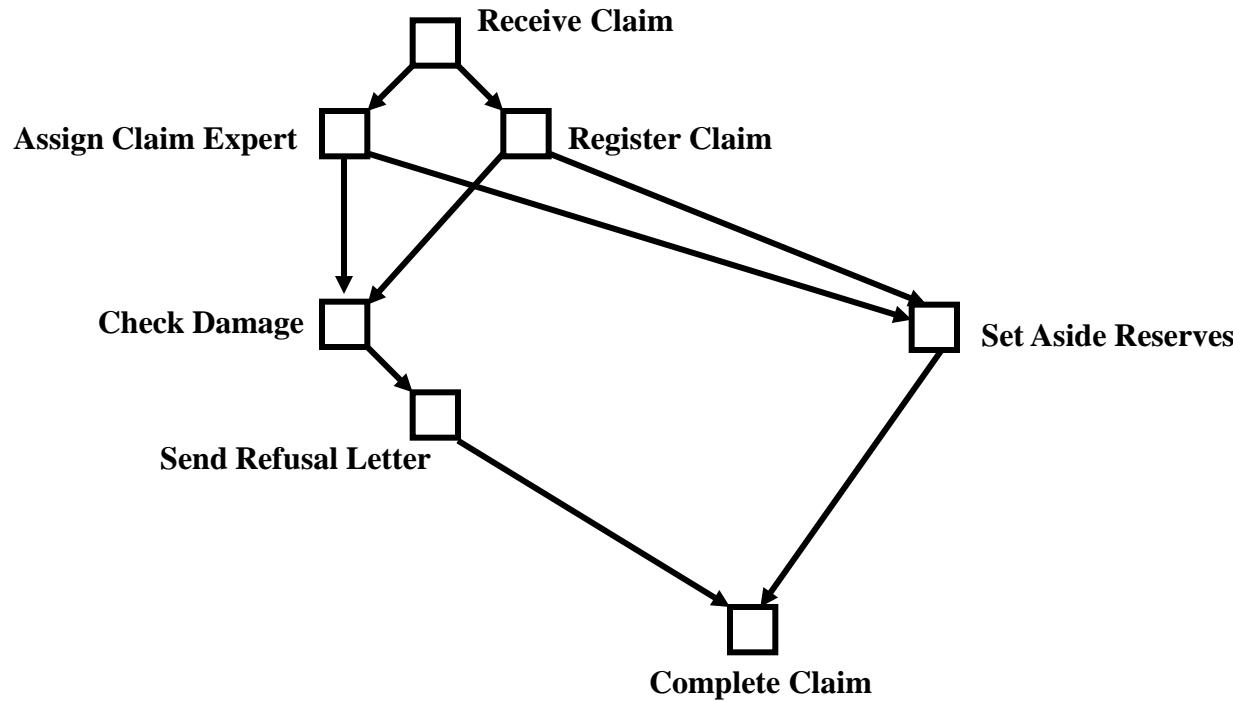




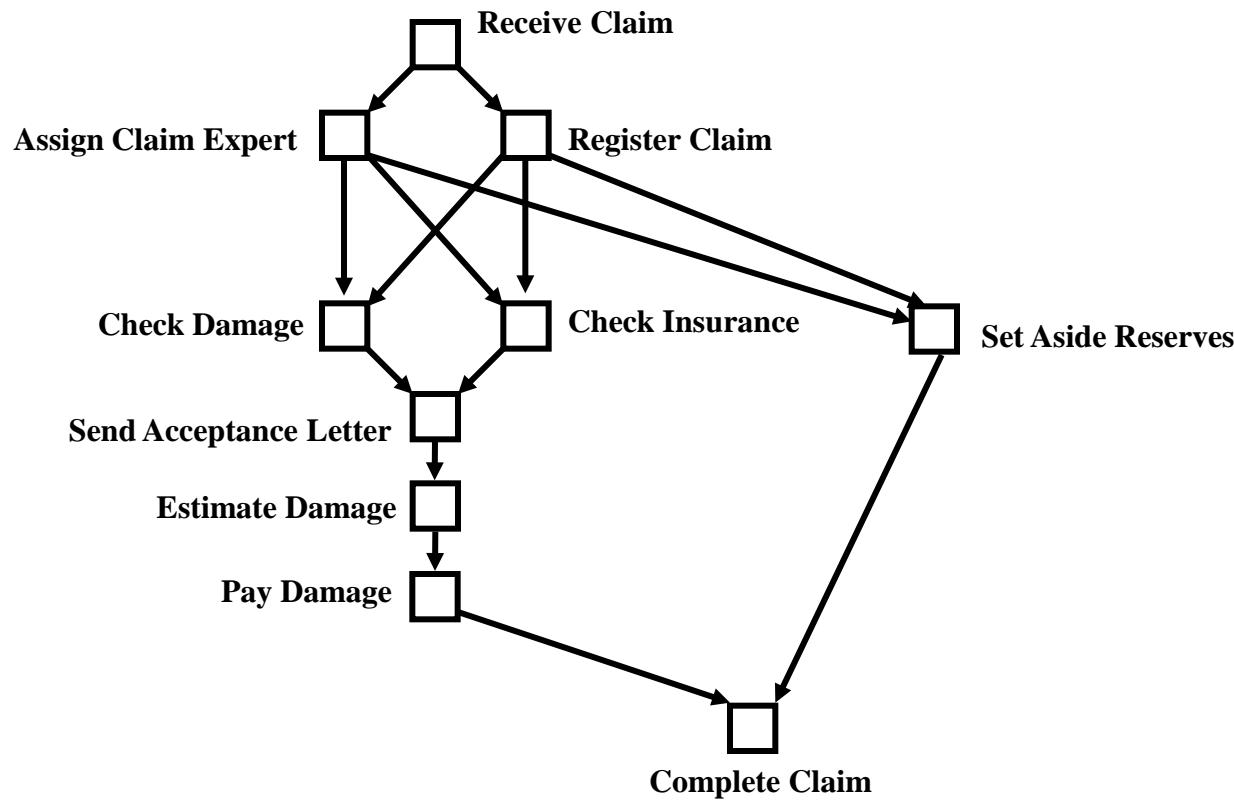
partially ordered runs



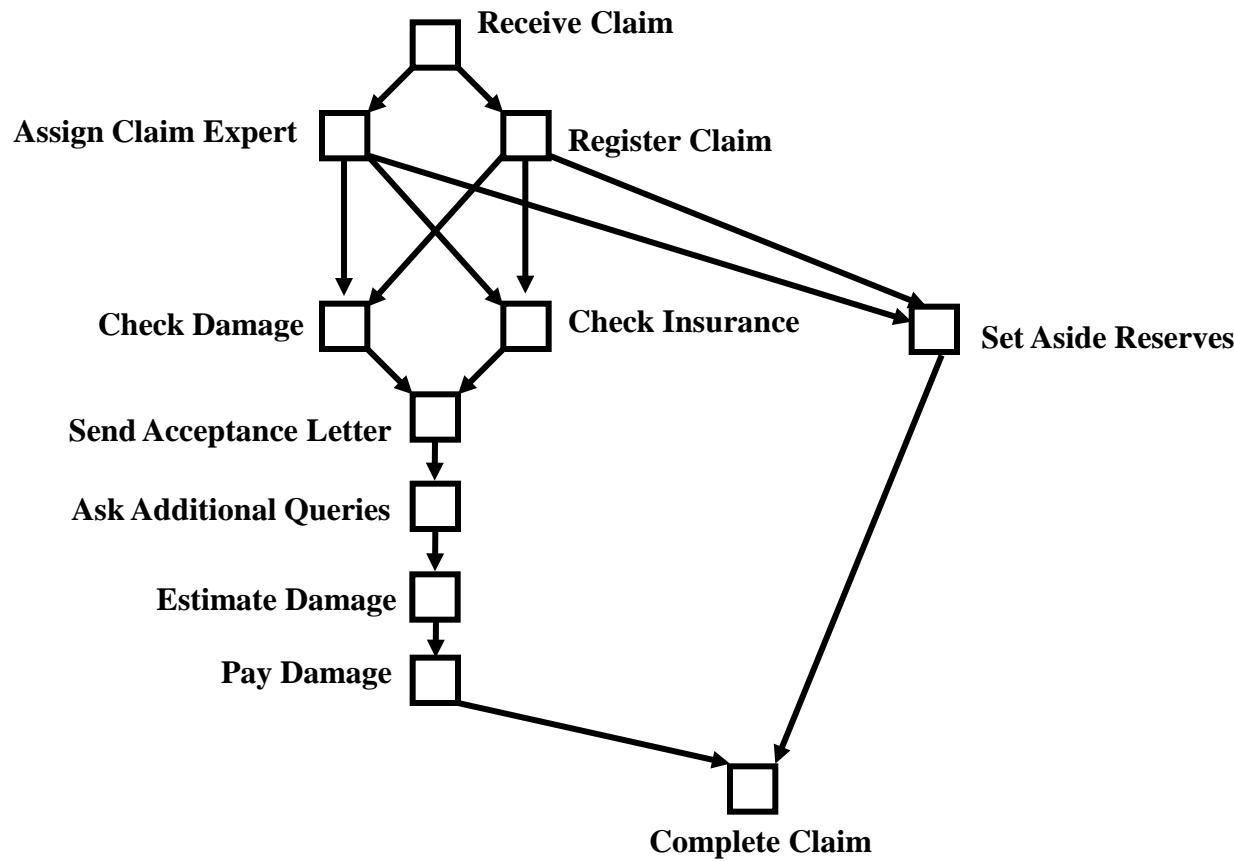
partially ordered runs



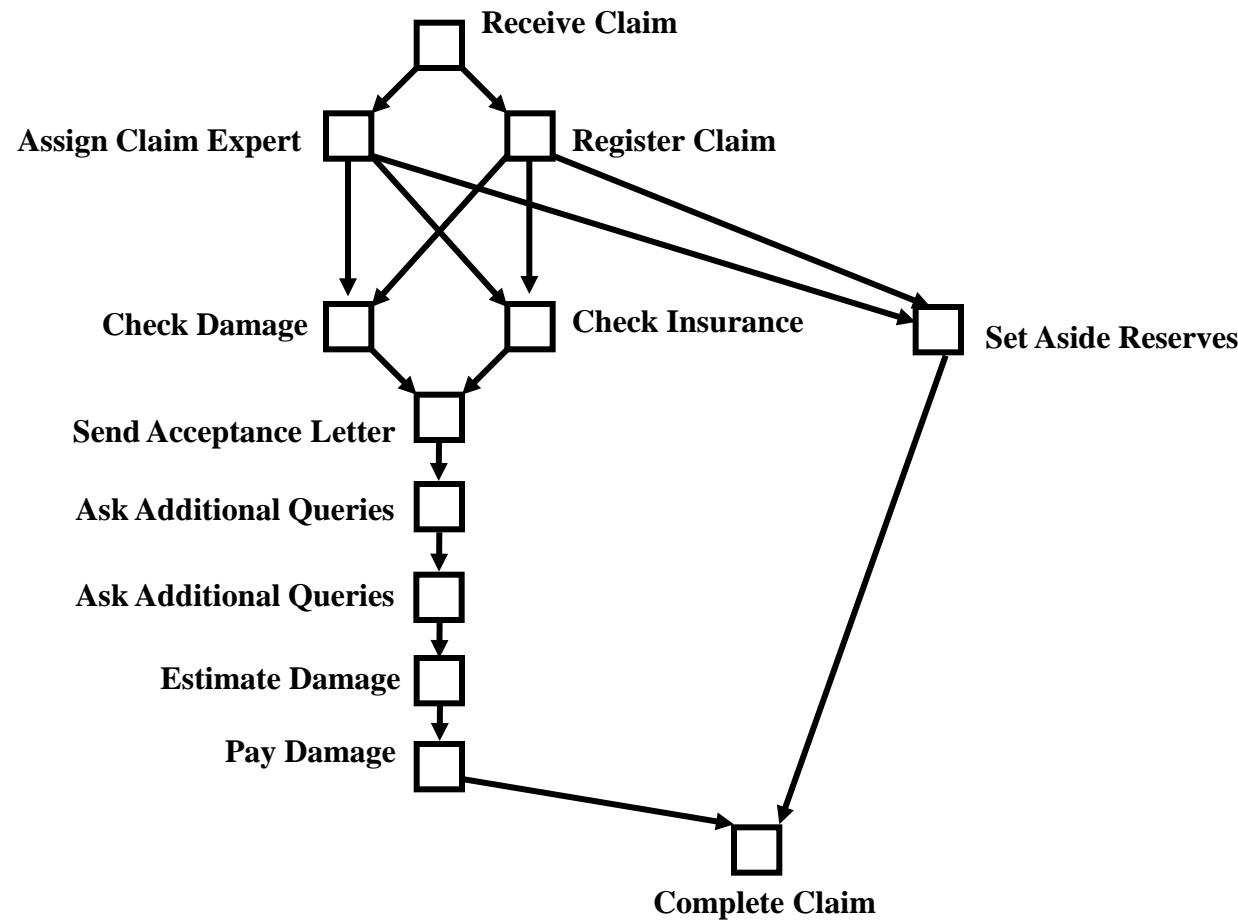
partially ordered runs



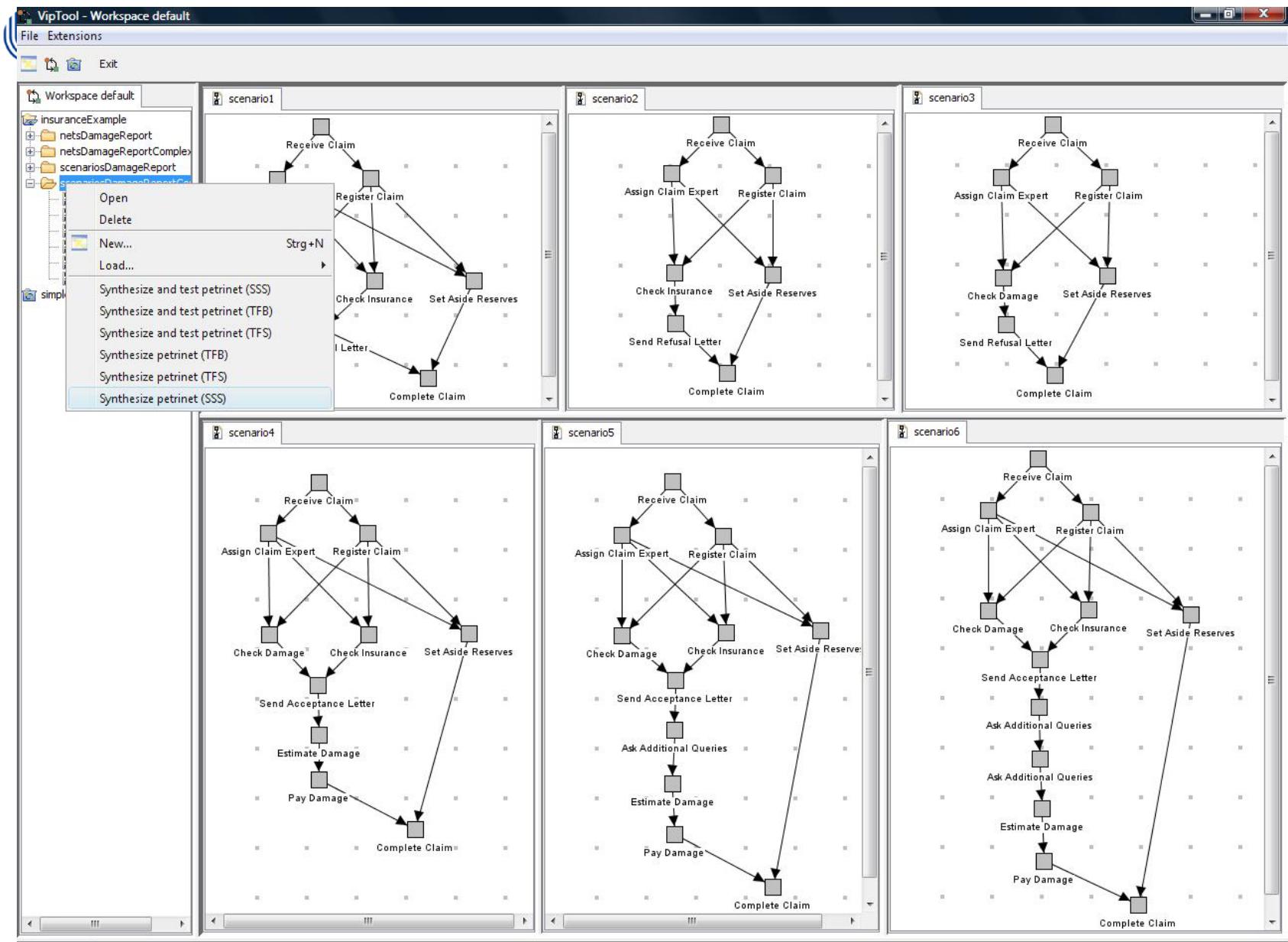
partially ordered runs



partially ordered runs

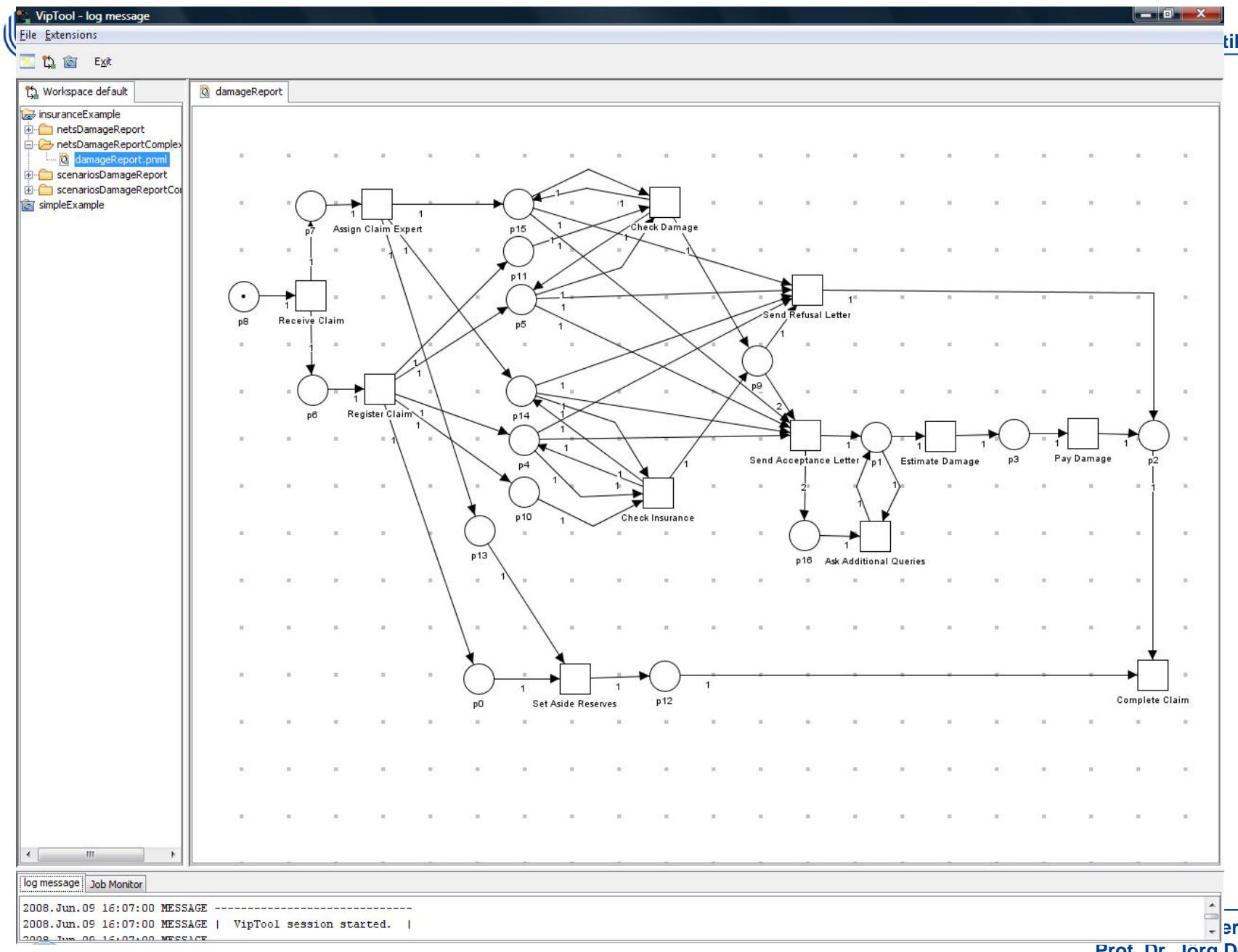


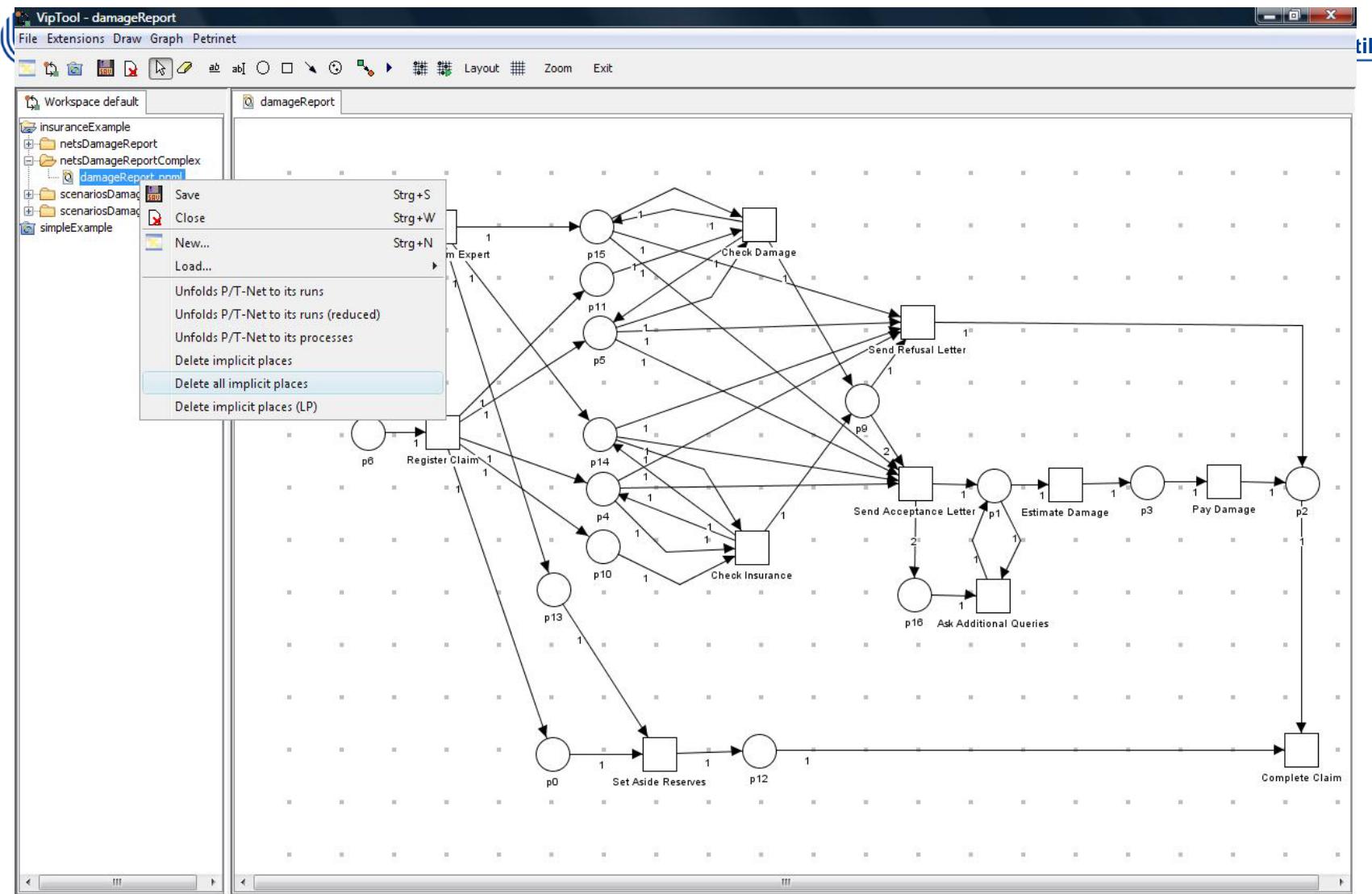
partially ordered runs



log message

2008.Jun.09 16:07:00 MESSAGE -----



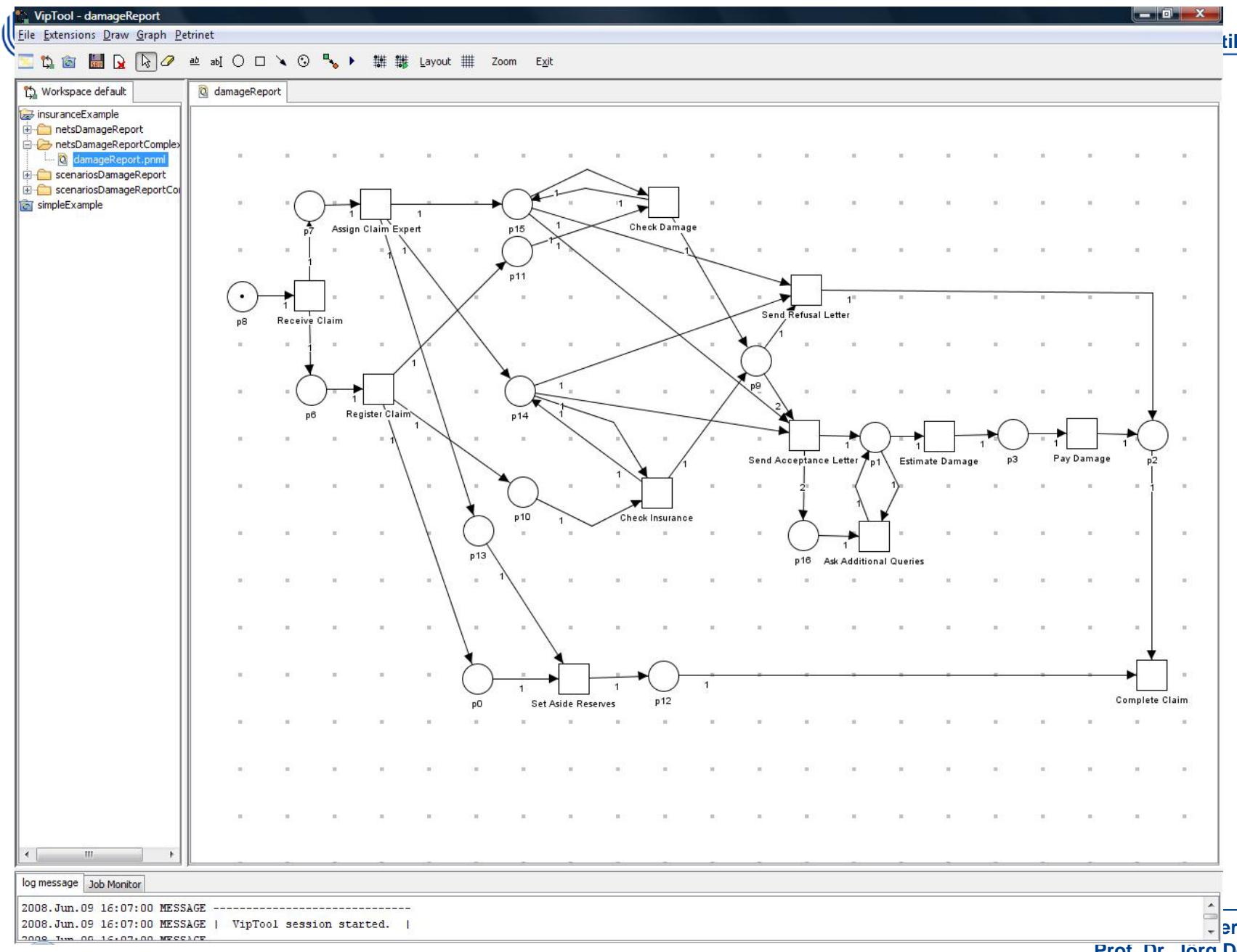


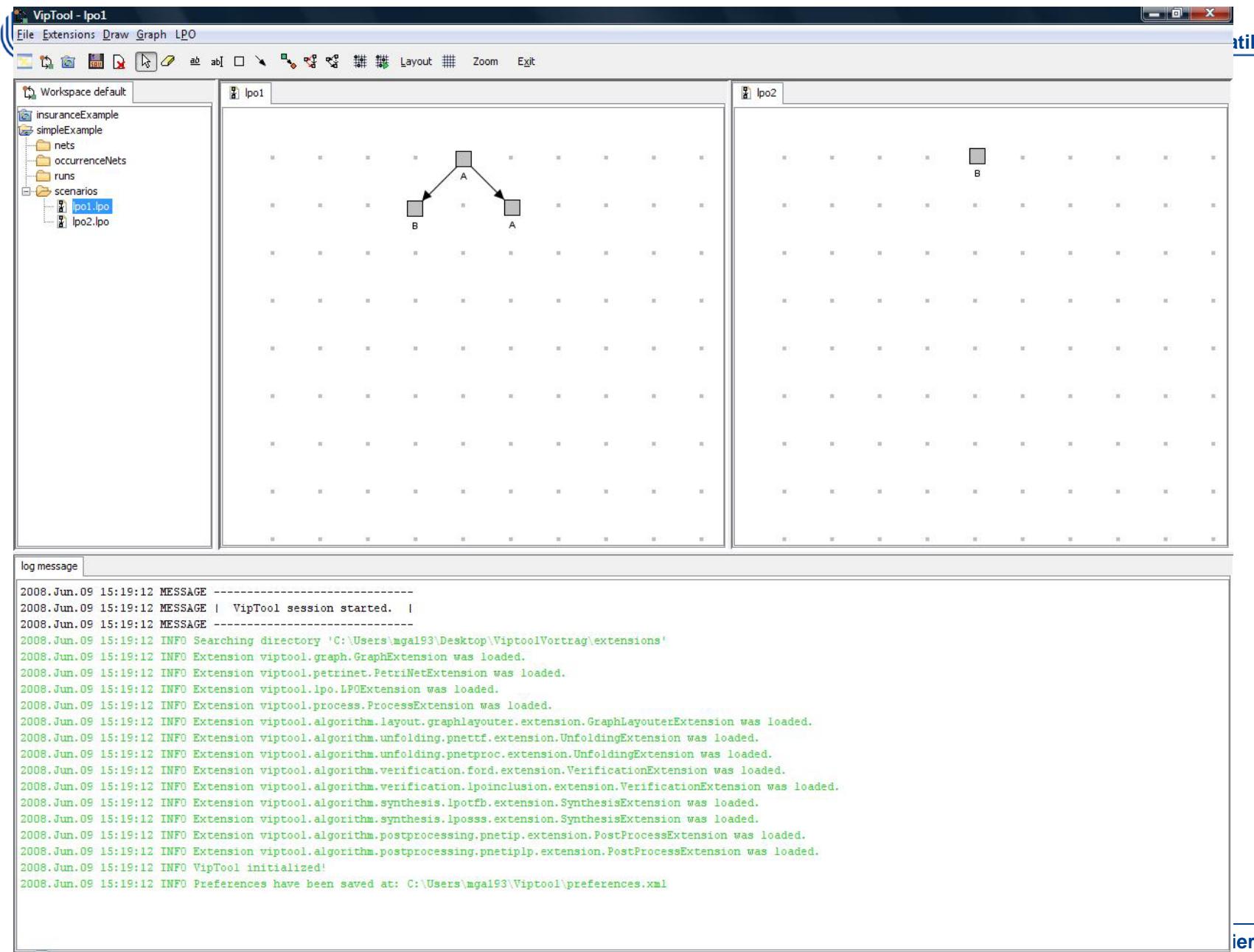
log message

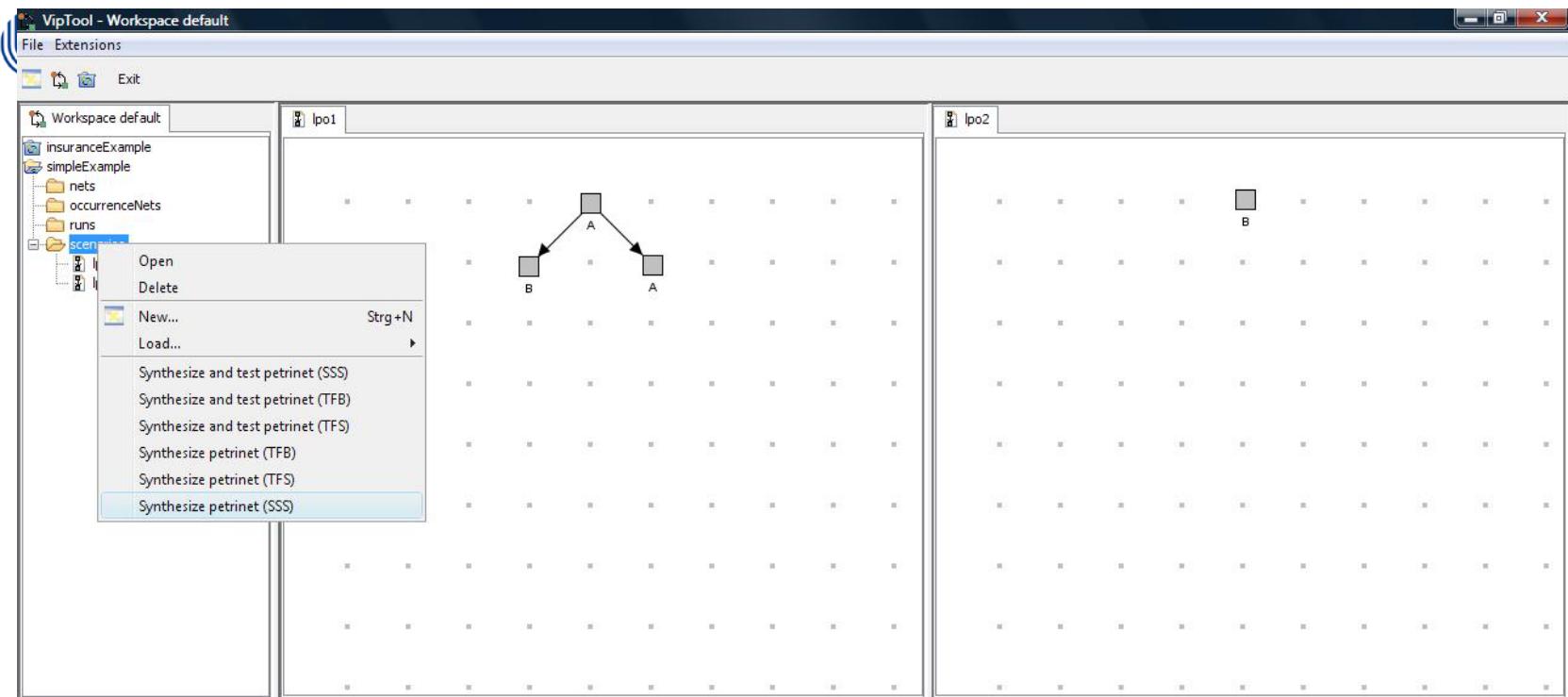
```

2008.Jun.16 10:20:50 MESSAGE -----
2008.Jun.16 10:20:50 MESSAGE | VipTool session started. |
2008.Jun.16 10:20:50 MESSAGE -----
2008.Jun.16 10:20:52 INFO Searching directory 'C:\Users\mgal93\Desktop\VipTool\extensions'
2008.Jun.16 10:20:52 INFO Extension viptool.graph.GraphExtension was loaded.
2008.Jun.16 10:20:52 INFO Extension viptool.petrinet.PetriNetExtension was loaded.

```





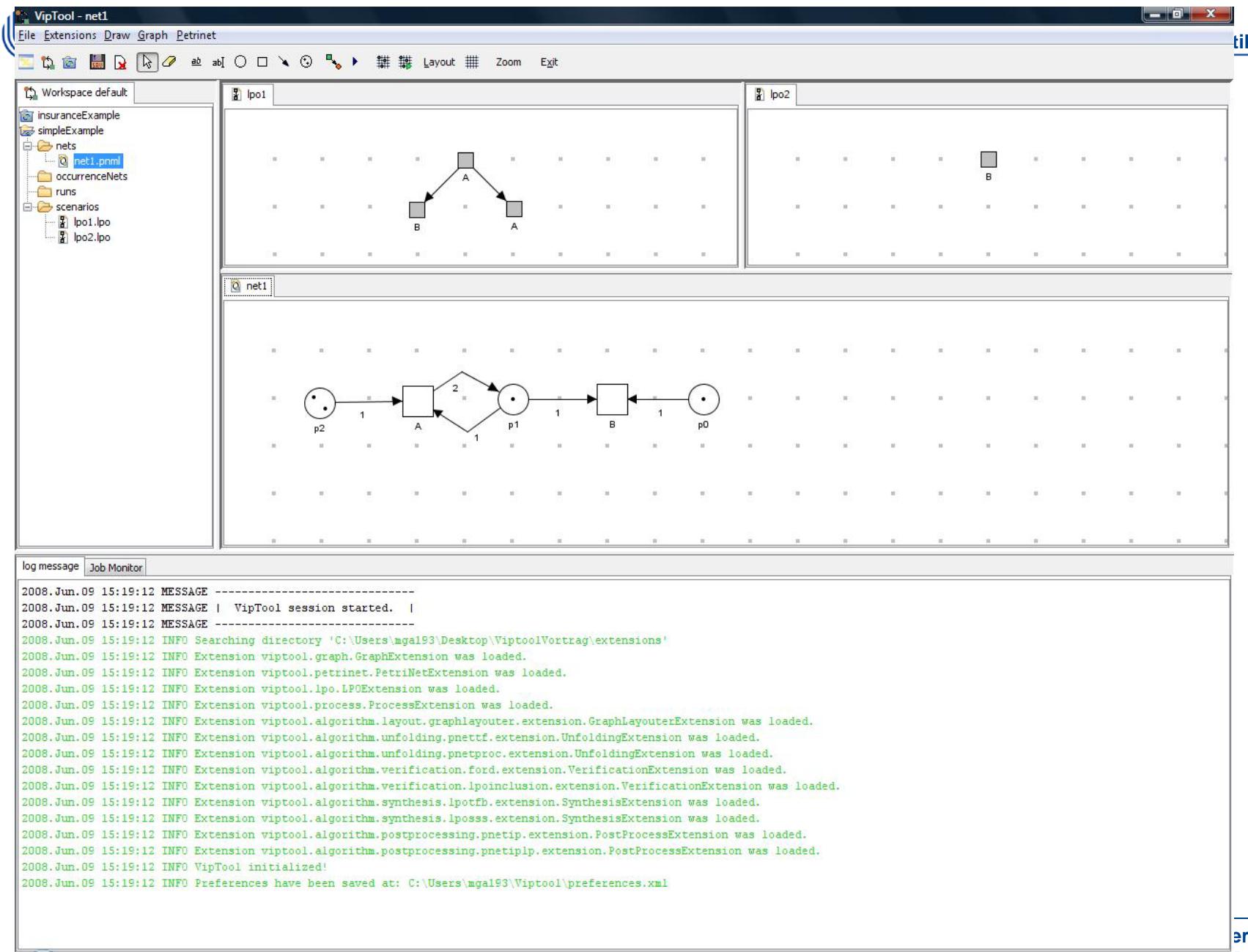


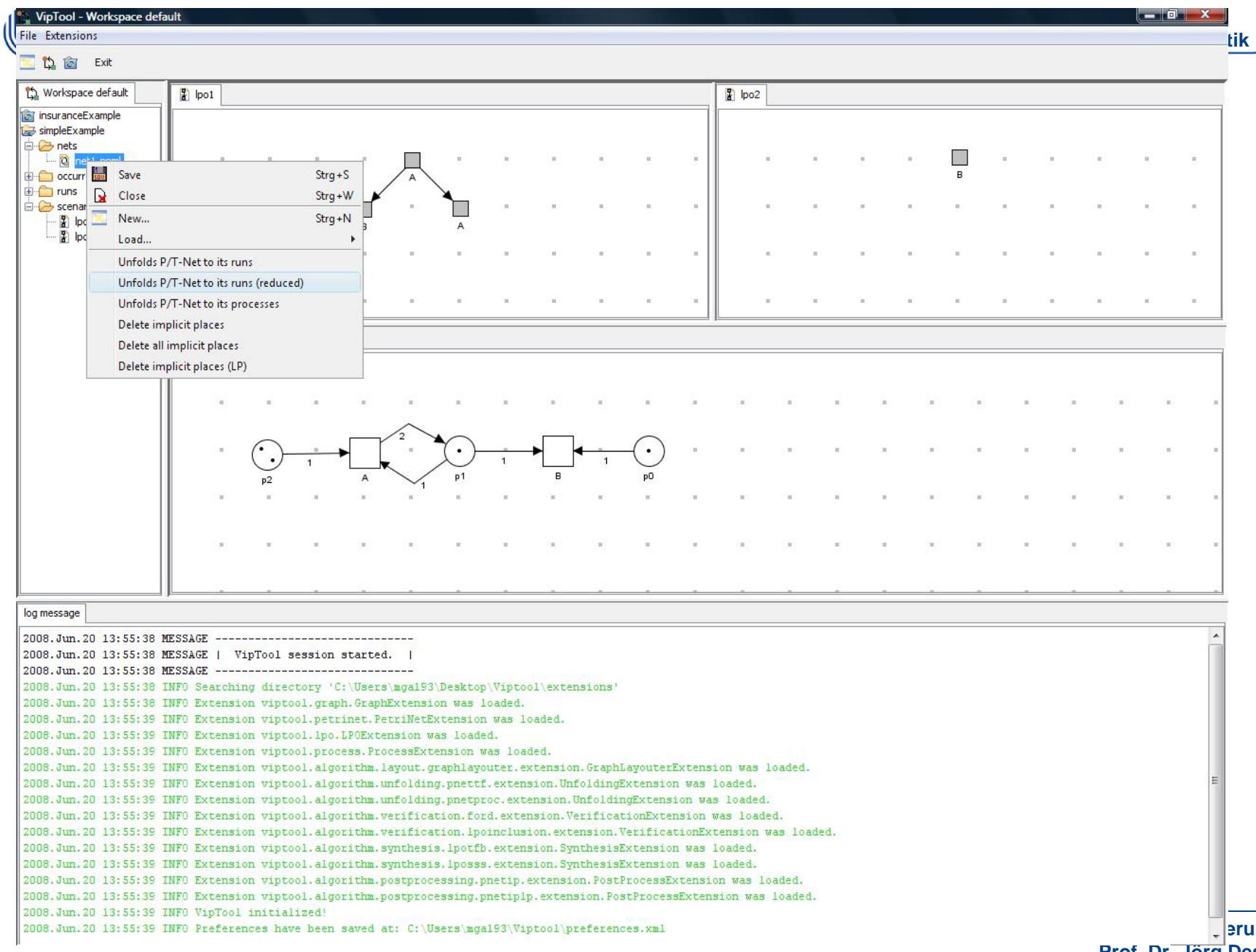
log message

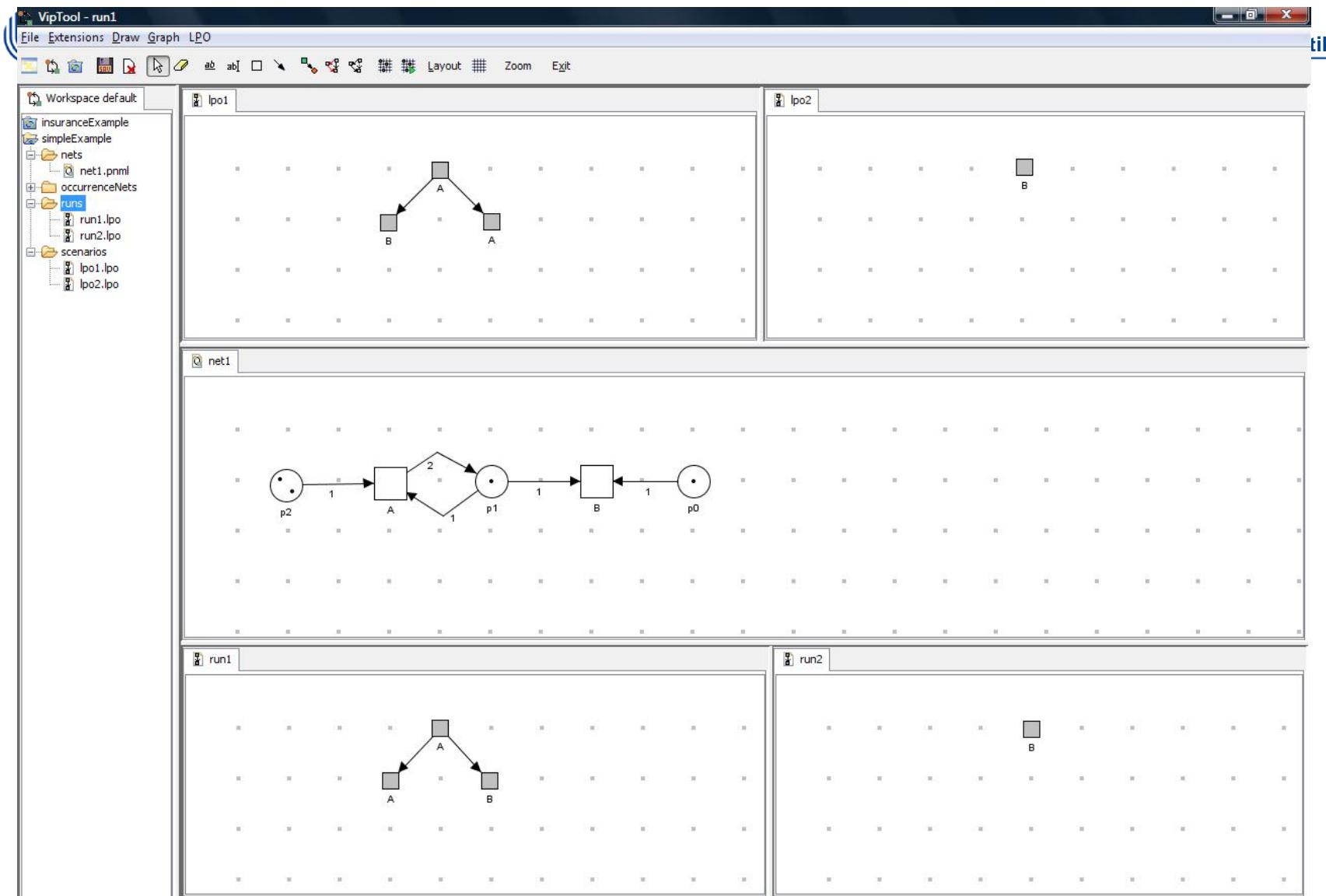
```

2008.Jun.09 15:19:12 MESSAGE -----
2008.Jun.09 15:19:12 MESSAGE | VipTool session started. |
2008.Jun.09 15:19:12 MESSAGE -----
2008.Jun.09 15:19:12 INFO Searching directory 'C:\Users\mgal93\Desktop\ViptoolVortrag\extensions'
2008.Jun.09 15:19:12 INFO Extension viptool.graph.GraphExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.petrinet.PetriNetExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.lpo.LPOExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.process.ProcessExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.layout.graphlayouter.extension.GraphLayouterExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.unfolding.pnettf.extension.UnfoldingExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.unfolding.pnetproc.extension.UnfoldingExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.verification.ford.extension.VerificationExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.verification.lpotfb.extension.VerificationExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.synthesis.lposss.extension.SynthesisExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.synthesis.lpotfb.extension.SynthesisExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.postprocessing.pnetip.extension.PostProcessExtension was loaded.
2008.Jun.09 15:19:12 INFO Extension viptool.algorithm.postprocessing.pnetiplp.extension.PostProcessExtension was loaded.
2008.Jun.09 15:19:12 INFO VipTool initialized!
2008.Jun.09 15:19:12 INFO Preferences have been saved at: C:\Users\mgal93\Viptool\preferences.xml

```





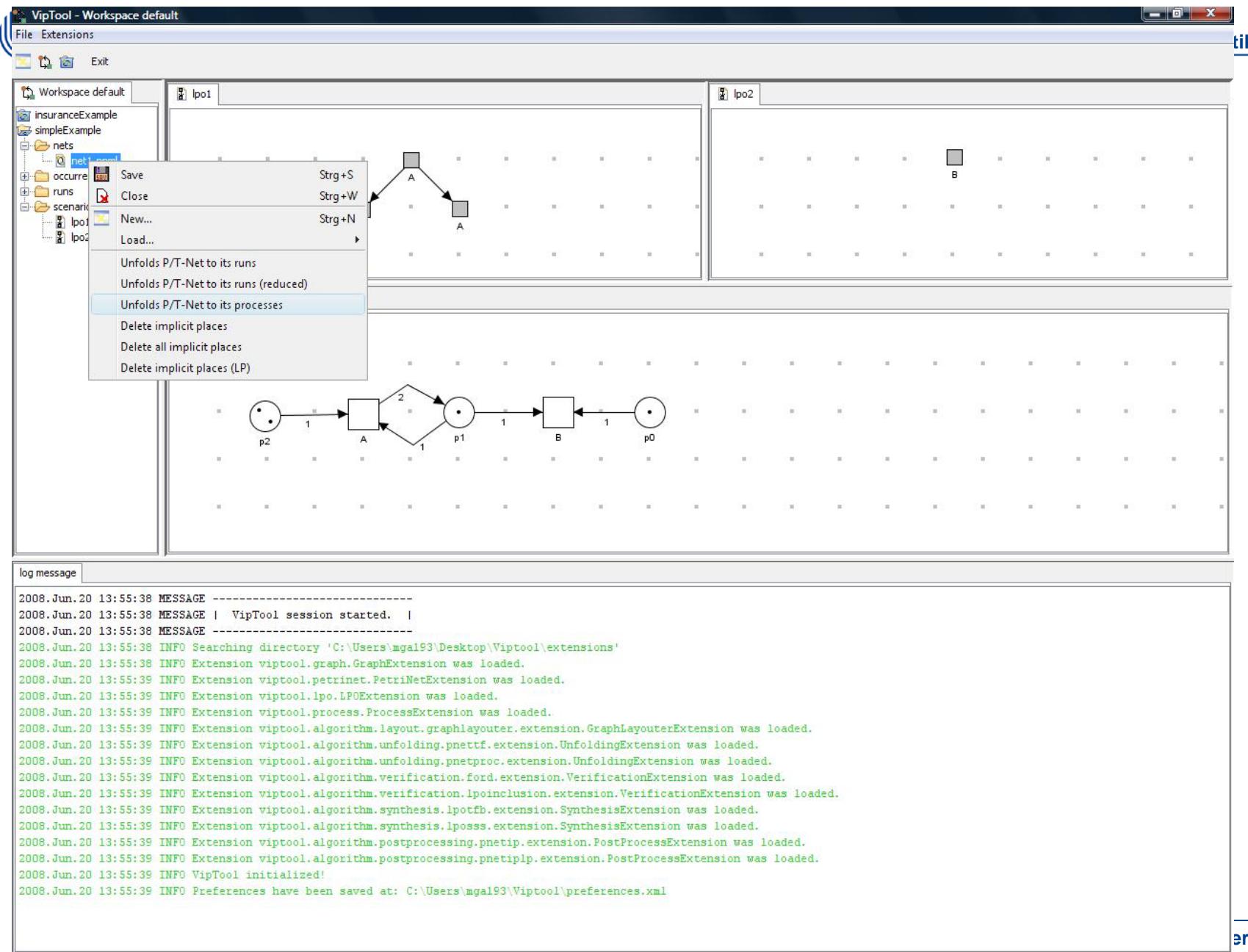


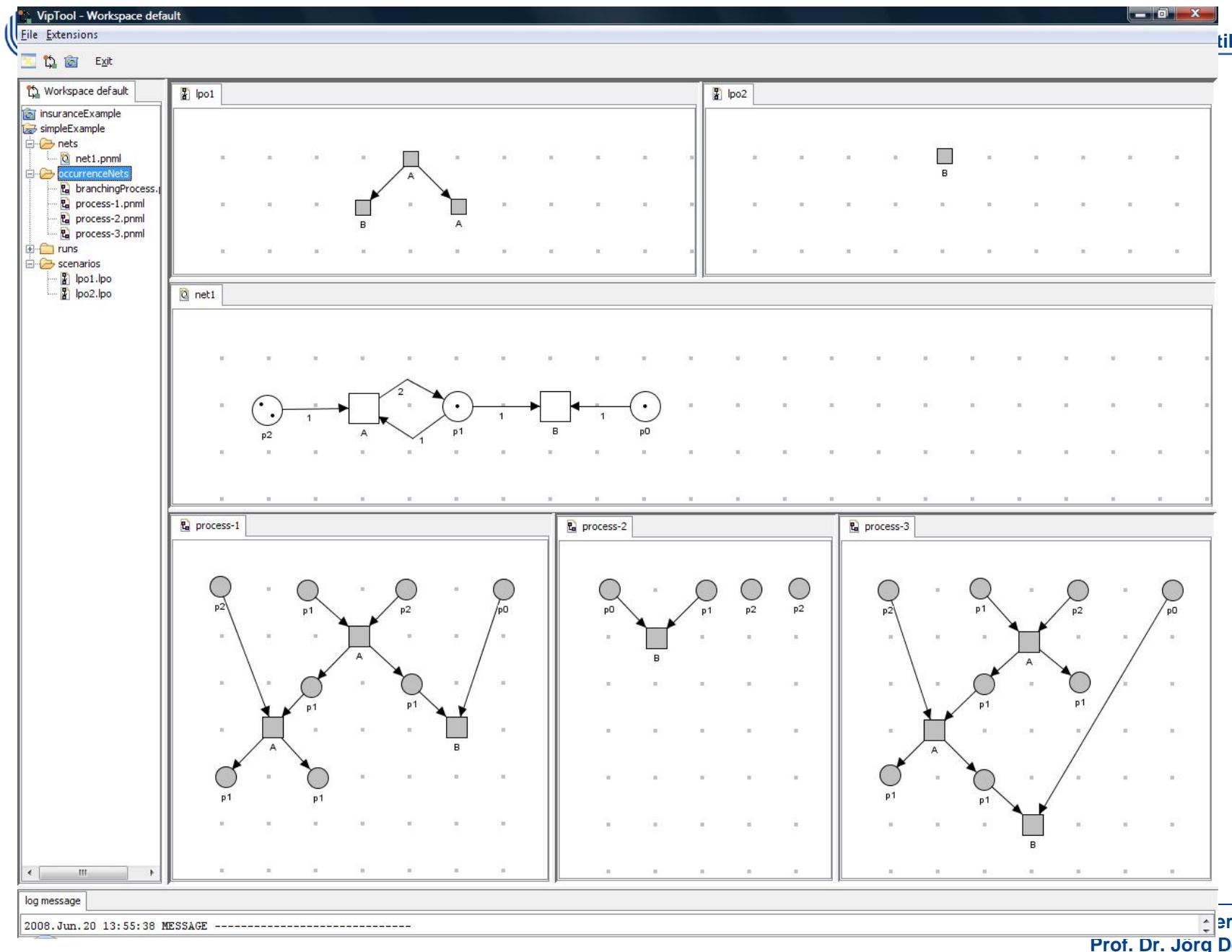
log message

```

2008.Jun.20 13:55:38 MESSAGE -----
2008.Jun.20 13:55:38 MESSAGE | VipTool session started. |
2008.Jun.20 13:55:38 MESSAGE -----
2008.Jun.20 13:55:38 INFO Searching directory 'C:\Users\mgal93\Desktop\Viptool\extensions'

```







Partial Orders Fit For Work

Jörg Desel

FernUniversität in Hagen