Petri's Axioms of Concurrency

A selection of recent results

Overview

Speaker

Intro-duction
- Background/Motivation
- Basic concurrency axioms

Our results
- Classification
- Propagation rules
- Soundness/completness
- Structure of lines
- Orders and orientations
- Episode coherence
- Cycloids

Mark-Oliver Stehr

Olaf Kummer

Research related to Petri's Concurrency Theory

Concurrency axioms and D-continuous posets
Best, Merceron 84 P

D-continuous causal nets
Fernandez, Thiagarajan 84 P

A lattice theoretic view of K-density
Fernandez, Thiagarajan 85 P

Nonsequential Processes - A Petri net view
Best, Fernandez 88 P

Concurrency and Continuity
Petri, Smith 87 P(G)

Zur Bedeutung der Concurrency Theorie für den Aufbau hochverteilter Systeme
Smith 89 P

Nichtsequentielle Prozesse
Petri 77 P

Concurrency as a basis for systems thinking
Petri 79 G

Concurrency
Petri 80 G

State transition structures in physics and computation
Petri 82 P

Concurrency Theory
Petri 87 G

Concurrency Theory
Petri 88 G

Complete order of signals
Petri 89

Geschichte und Entwicklung der Concurrency-Theorie
Müller 93 G

Physically Motivated Axiomatic Concurrency Theory - A posetless approach
Stehr 93 G

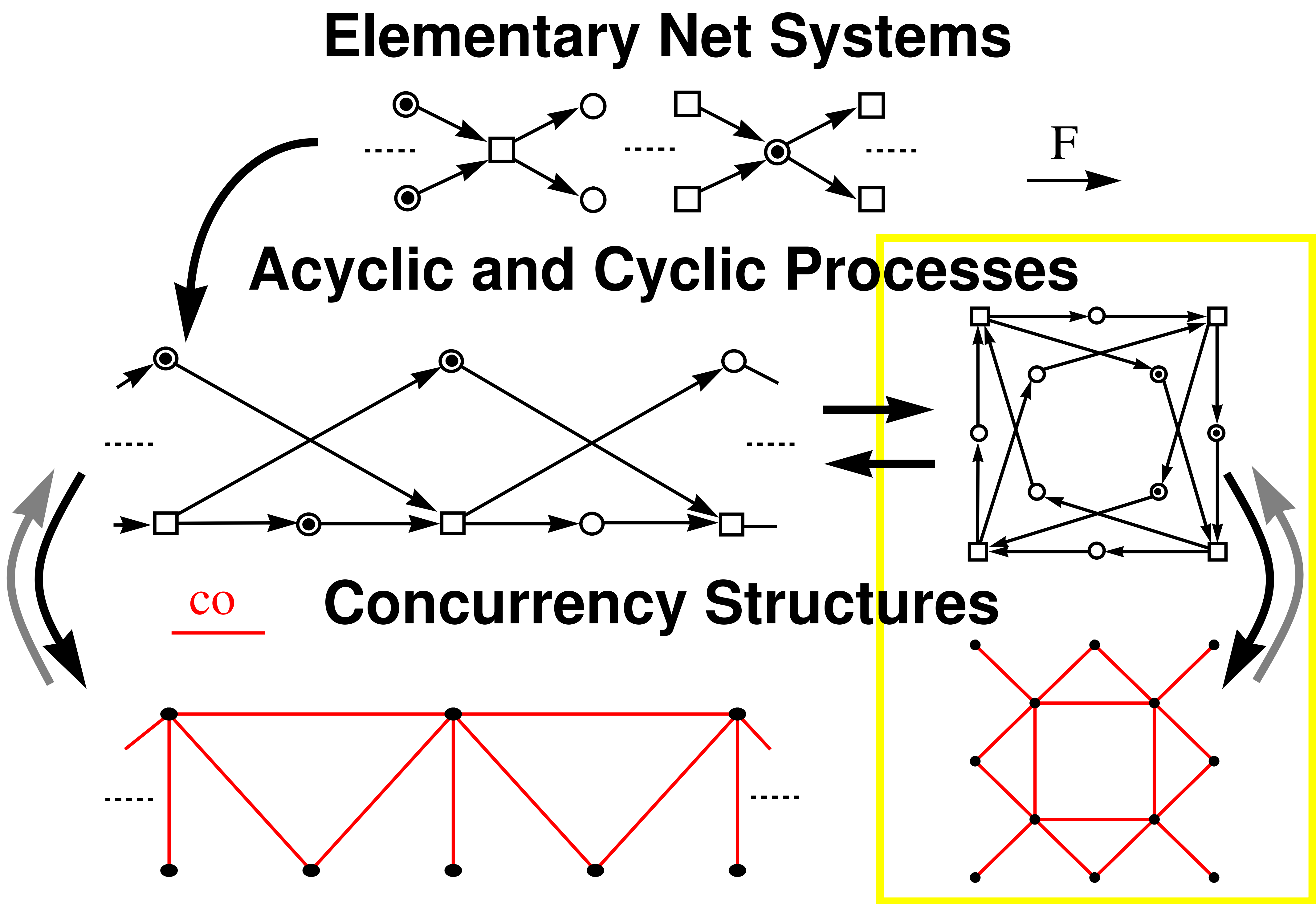
Axiomensysteme für die Theorie der Nebenläufigkeit
Kummer 96 G

Cyclic Orders
Stehr 96

Kausalität, Nebenläufigkeit und Konflikt in ENS aus top.rel. Sicht
Haar 97 G

Lecture Notes (University of Hamburg)

P = partial order based CT   G = general CT

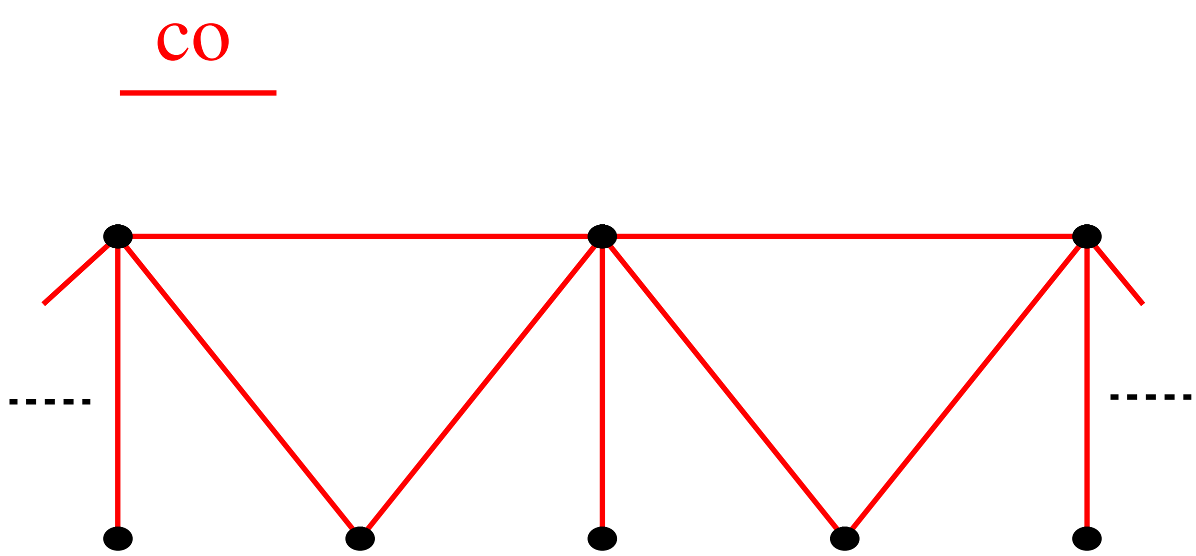


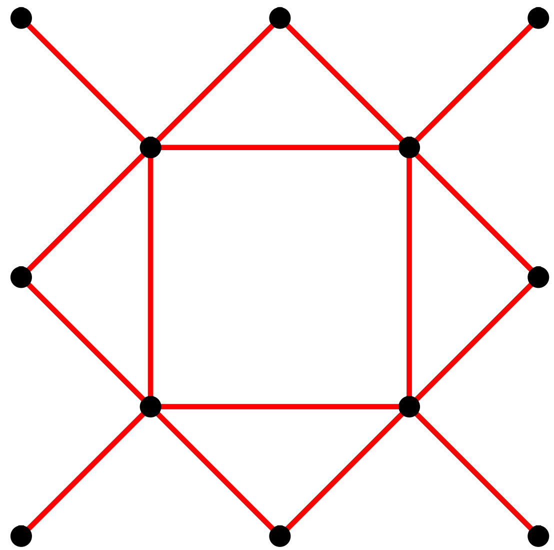
Elementary Net Systems
F
Acyclic and Cyclic Processes
co
Concurrency Structures

Concurrency Structures

DEFINITION Let X be a set with $|X| > 1$ and co,li $\subseteq X \times X$.
(X,co,li) is a *concurrency structure* iff

$$\mathrm{li} \cap \mathrm{id} = \mathrm{co} \cap \mathrm{id} = \emptyset,$$
$$\mathrm{co}^{-1} = \mathrm{co},\ \mathrm{li}^{-1} = \mathrm{li},$$
$$\mathrm{co} \cap \mathrm{li} = \emptyset,\ \mathrm{co} \cup \mathrm{li} \cup \mathrm{id} = X \times X,$$





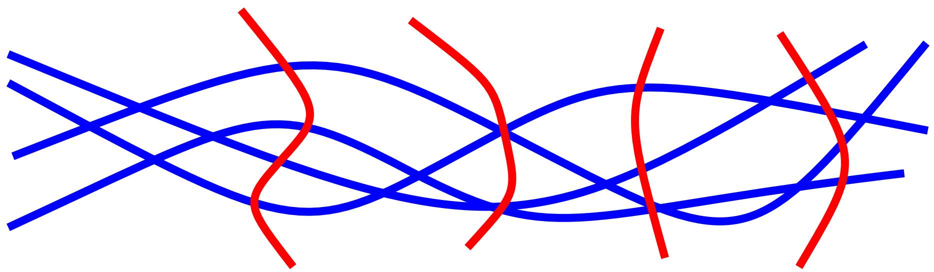
AXIOMS *li- and co-Irreducibility*
li- and co-Coherence

Cuts and Lines

DEFINITION

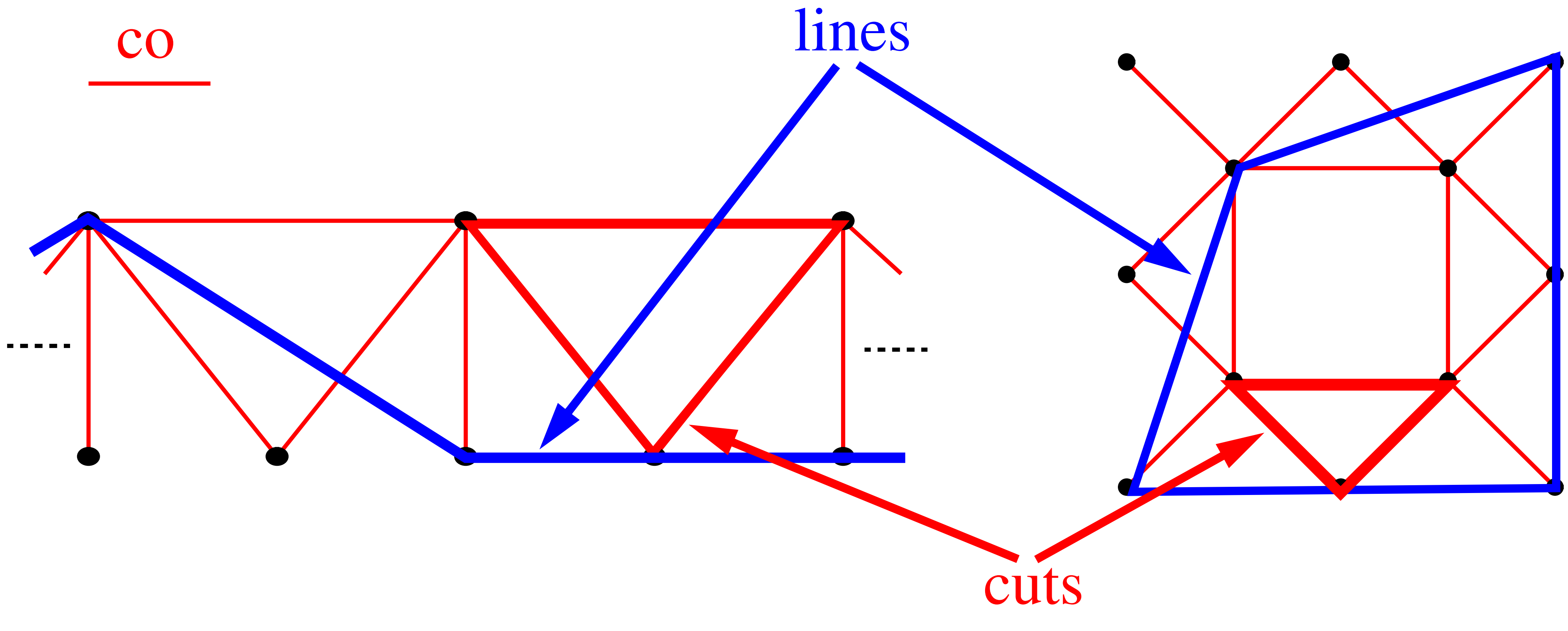
Lines := Kens(li)

Cuts := Kens(co)

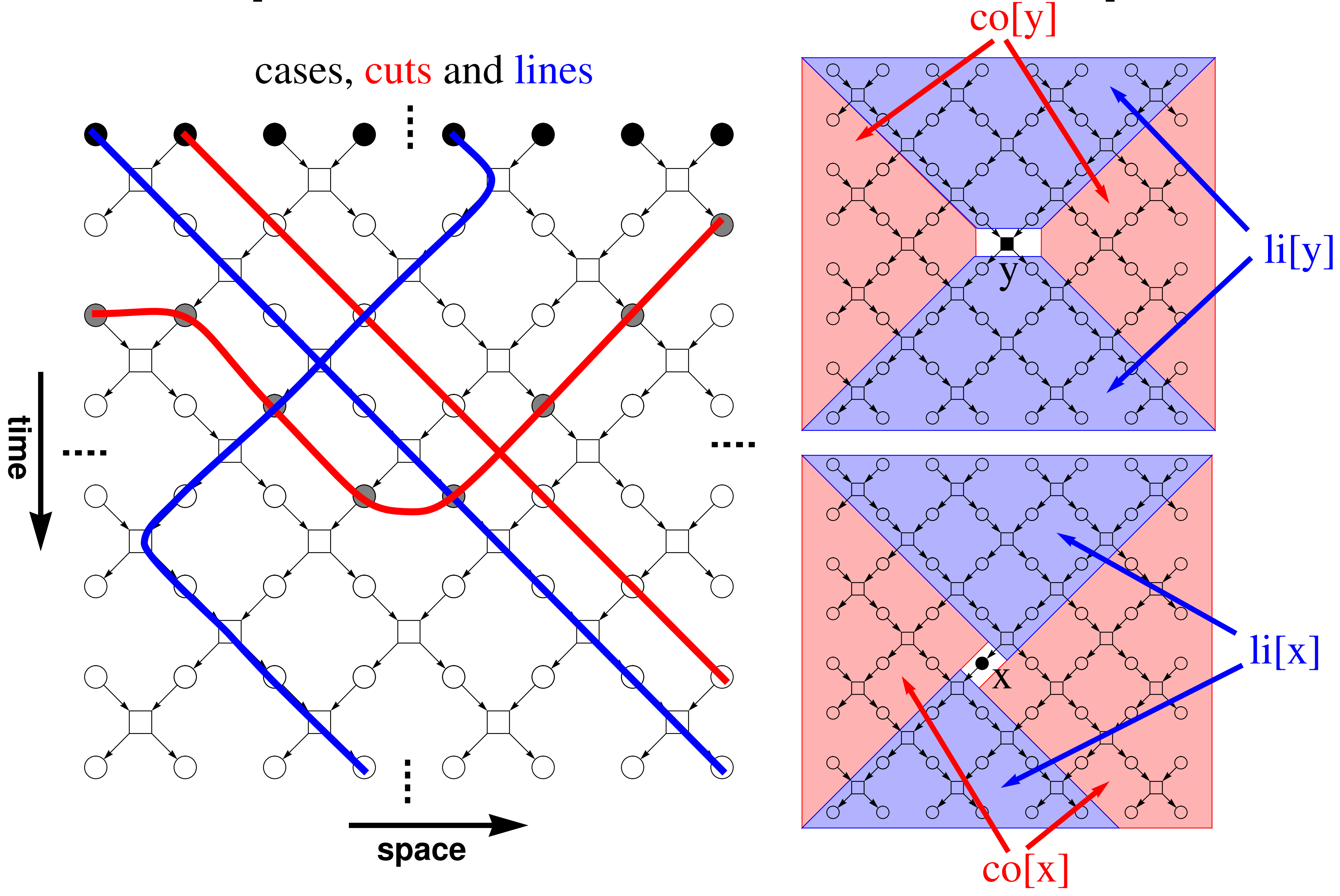


AXIOM

$$\forall L \in \text{Lines}\ \forall C \in \text{Cuts} : \ |L \cap C| = 1 \qquad \textit{(K-density)}$$



Net Representation of Minkowski Space



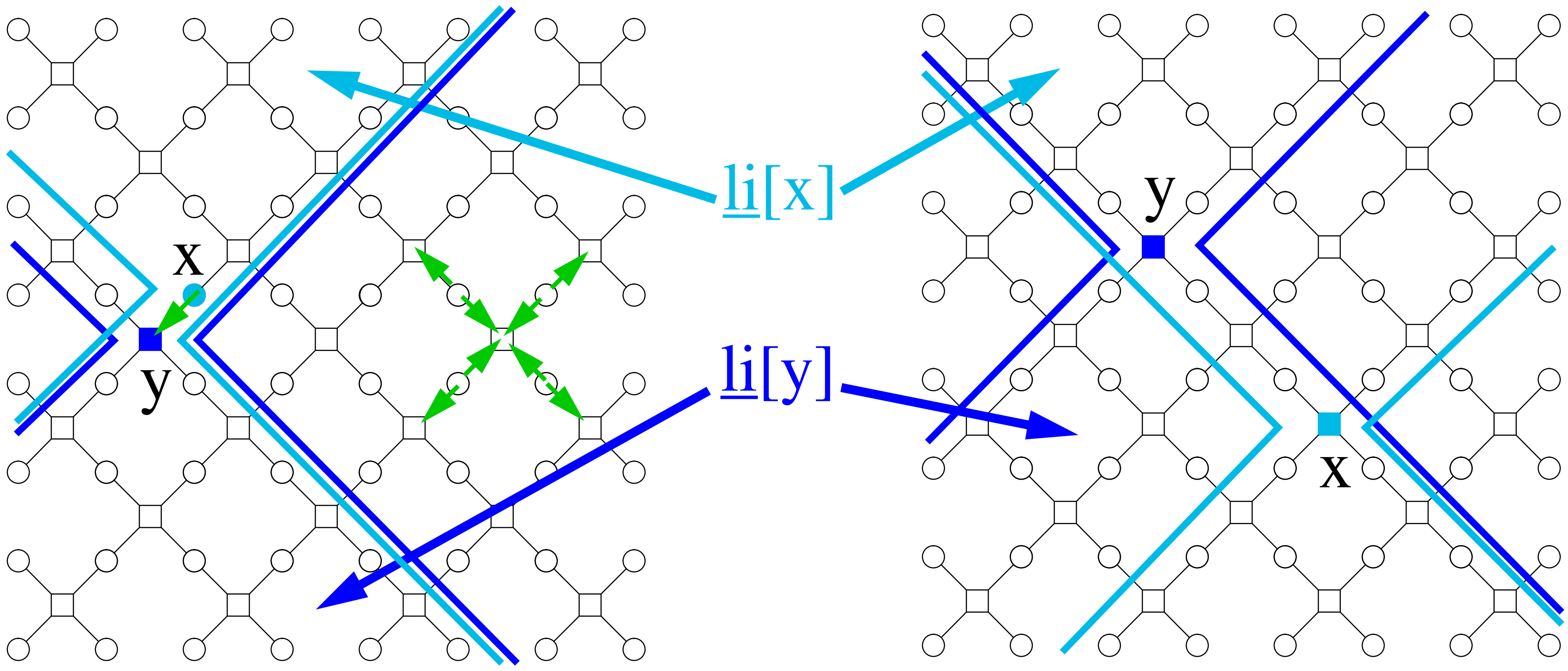
Recovering Locality

DEFINITION

$x \; P \; y \; :\Leftrightarrow \underline{li}[x] \subset \underline{li}[y]$ *(proximity)*

$im := P \cup P^{-1}$ *(immediate causality)*

$S := \mathrm{dom}\; P,\; T := \mathrm{ran}\; P$ *(states and transitions)*



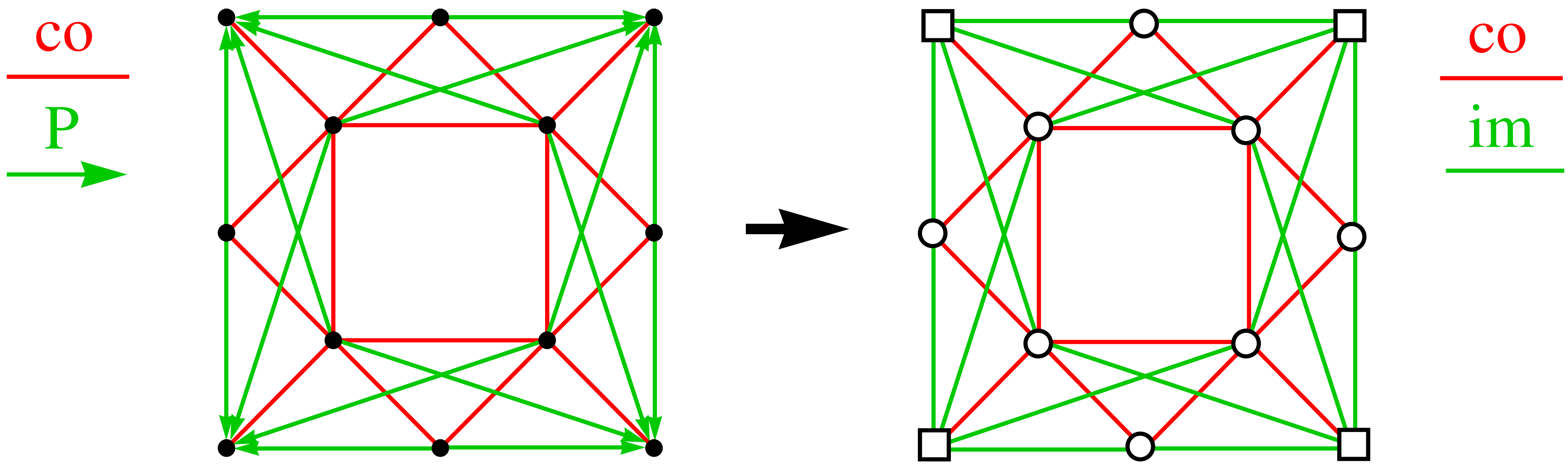
Proximity and Immediate Causality

DEFINITION

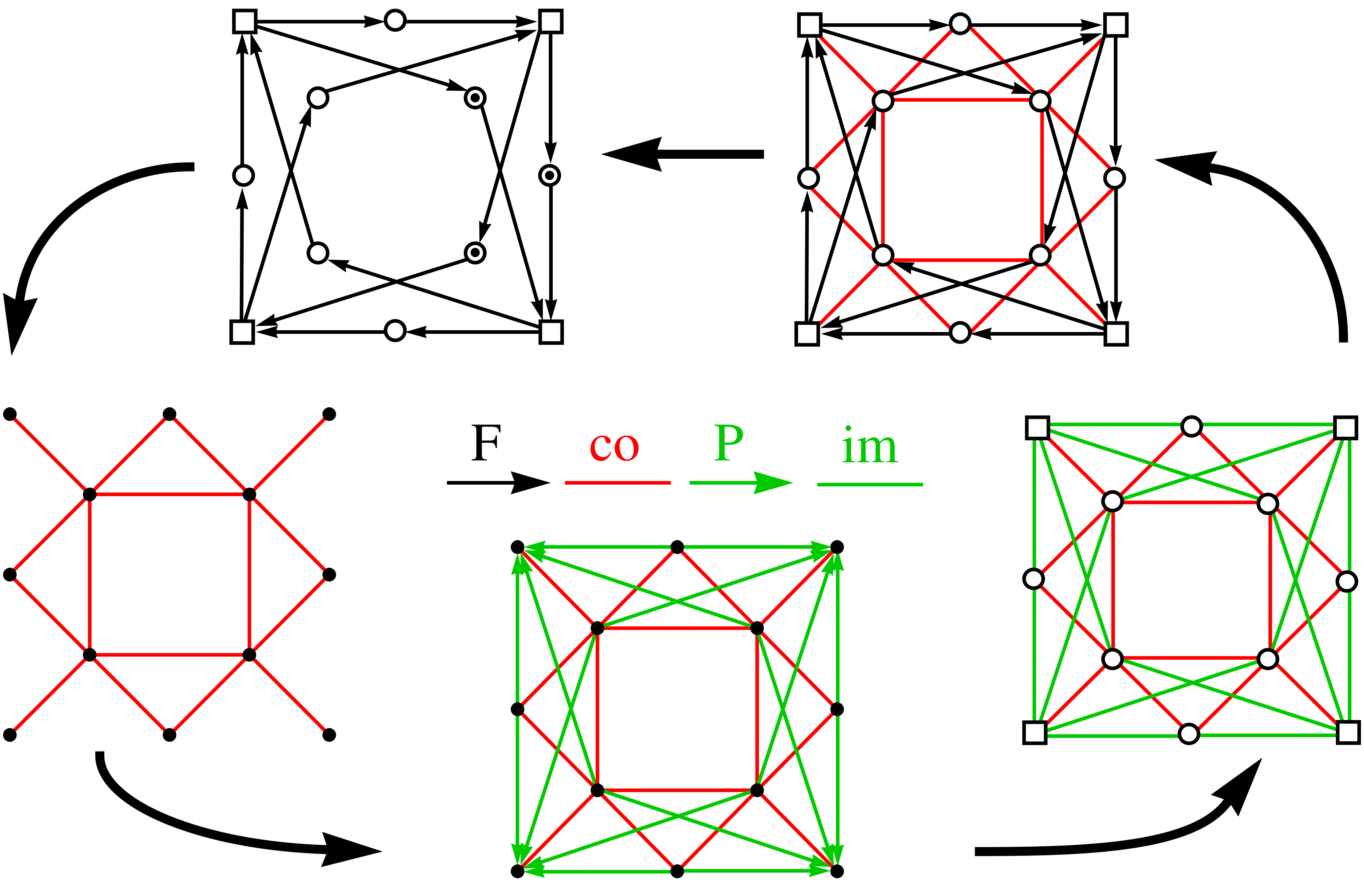
$x \; P \; y \; :\Leftrightarrow \; li[x] \subset li[y]$	*(proximity)*
$im := P \cup P^{-1}$	*(immediate causality)*
$S := \mathrm{dom}\, P, \; T := \mathrm{ran}\, P$	*(states and transitions)*

AXIOMS

$S \cap T = \varnothing$	*(two-sortedness)*
$im^* = X \times X$	*(im-coherence)*



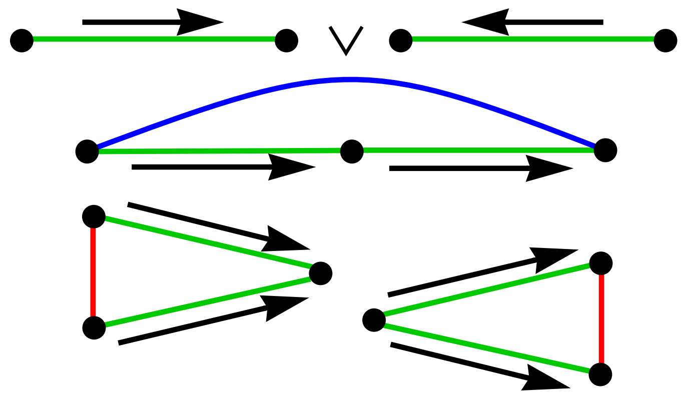
Processes and Concurrency Structures

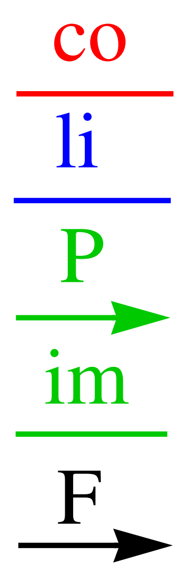


Consistent Orientability

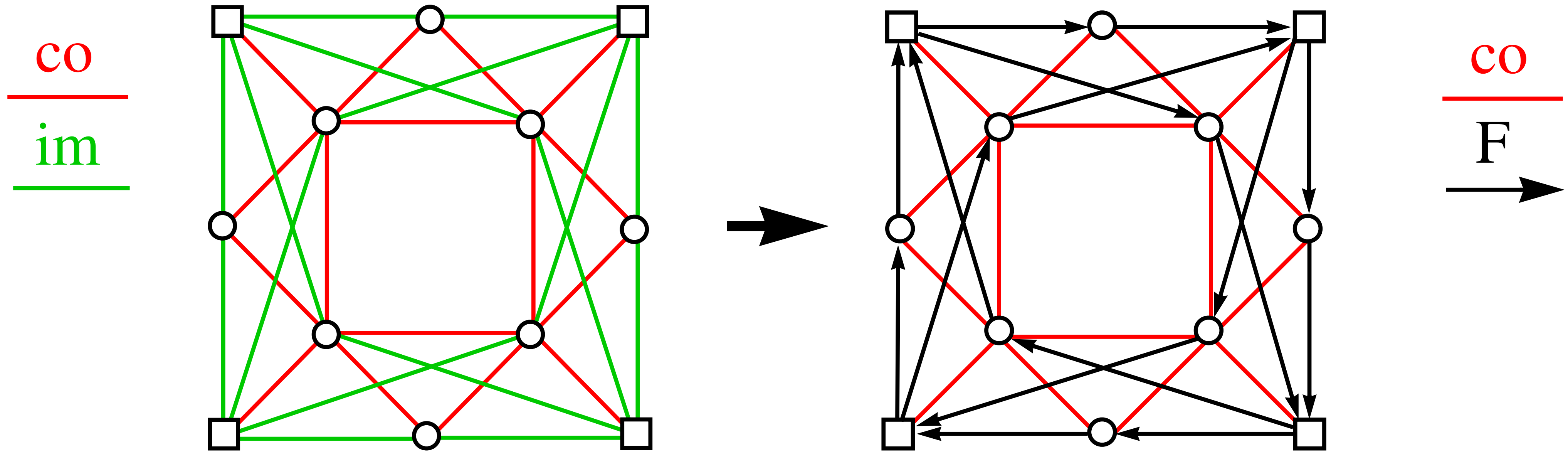
DEFINITION F is a *consistent orientation* iff

$$F \cup F^{-1} = \mathrm{im},$$
$$F \circ F \subseteq \mathrm{li},$$
$$F^{-1} \circ F \subseteq \underline{\mathrm{co}},$$
$$F \circ F^{-1} \subseteq \underline{\mathrm{co}}.$$





AXIOM There is a consistent orientation F. *(cons.orientability)*



PROPOSITION (S,T,F) is a net without branching states.

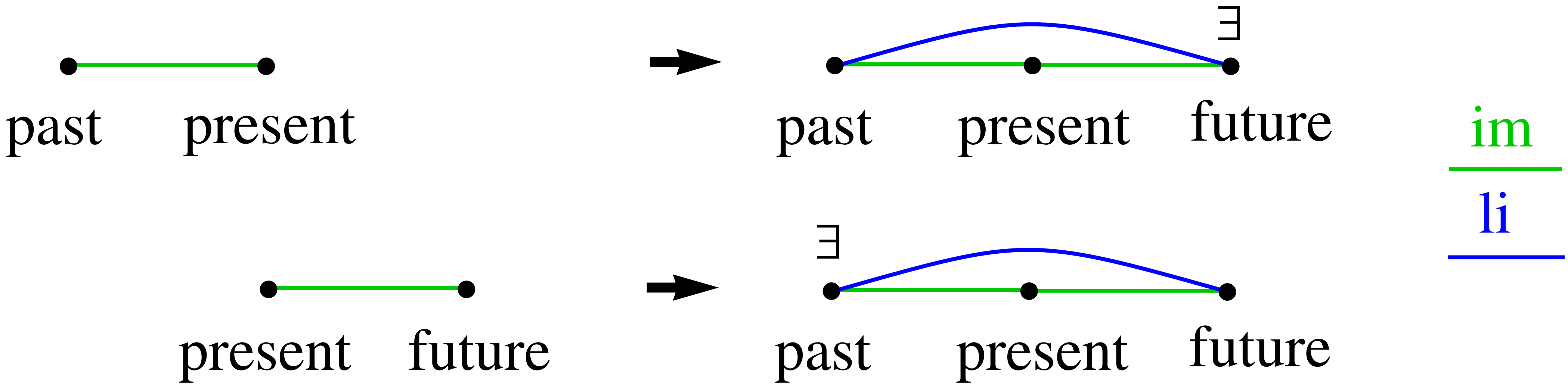
Local Axioms

AXIOMS *local co-transitivity* and *local-orientability*

THEOREM These axioms are necessary
but **not** sufficient for consistent orientability.

PROOF By computer verified model construction.

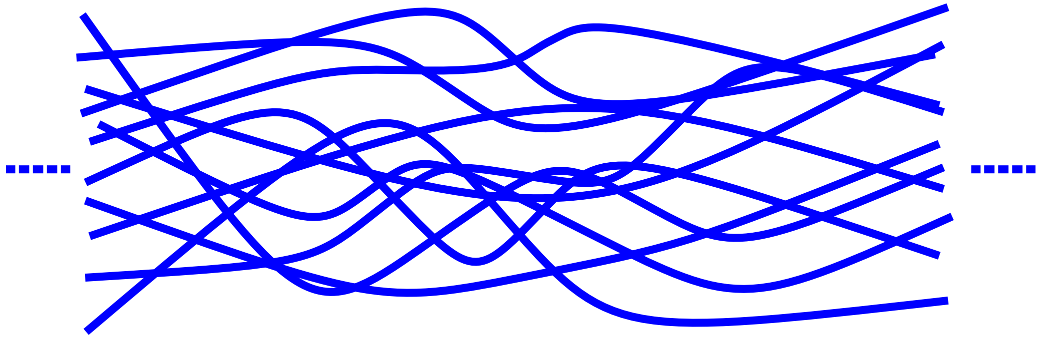
AXIOM *local extensibility*



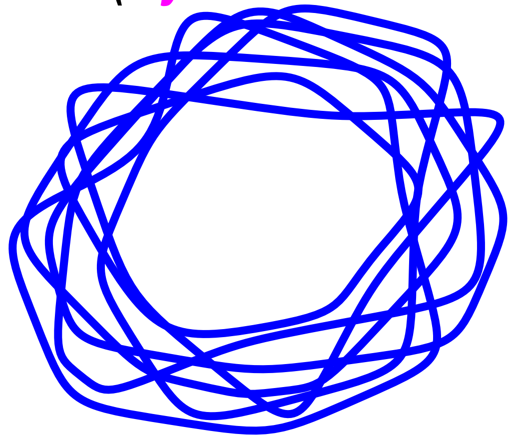
Now all *basic concurrency axioms* have been introduced.
They will be assumed for the following results.

Classifying Concurrency Structures

AXIOM All lines are infinite
(acyclic structures)

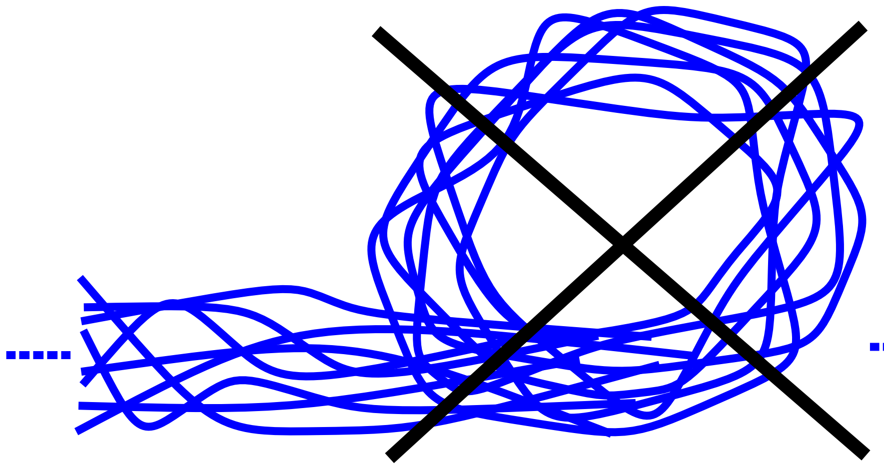


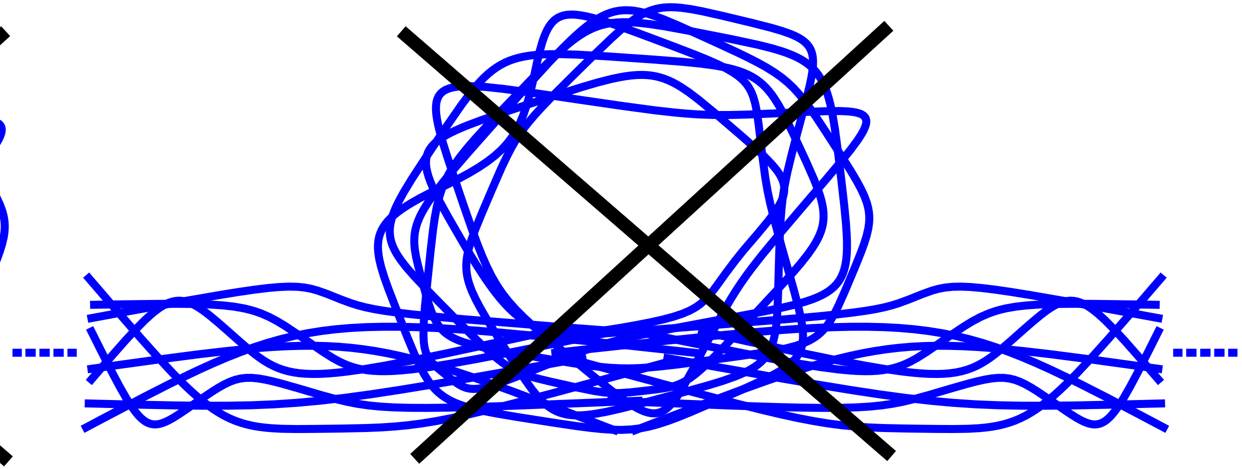
AXIOM All lines are finite
(cyclic structures)



AXIOM $\forall\, x \in X :$ co[x] is finite *(concurrency cone finiteness)*

THEOREM Concurrency structures are either acyclic or cyclic.

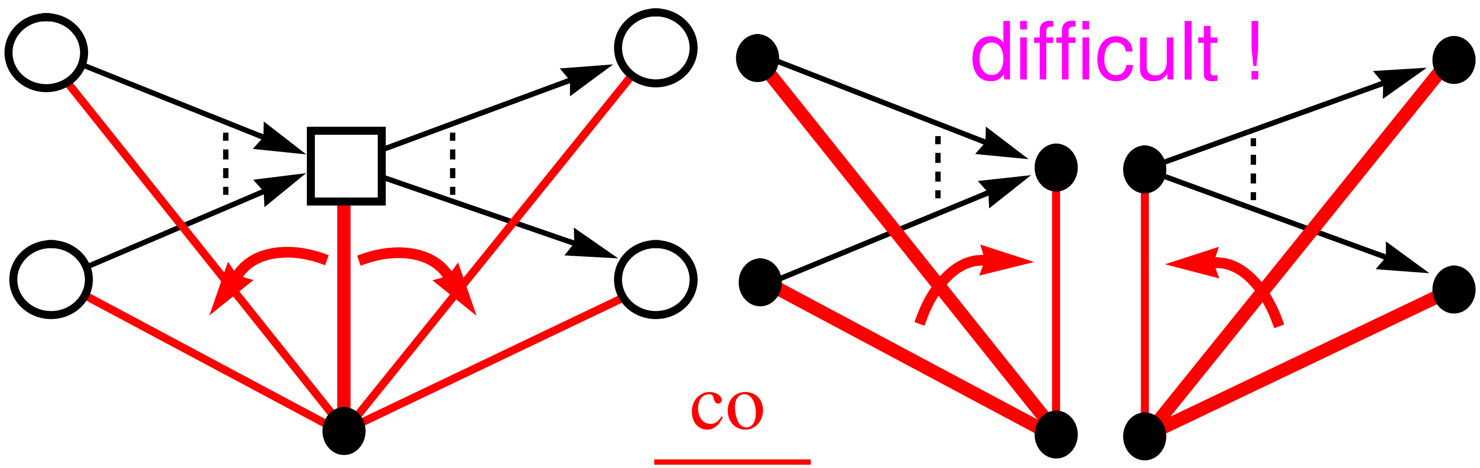




Propagating Concurrency and Causality

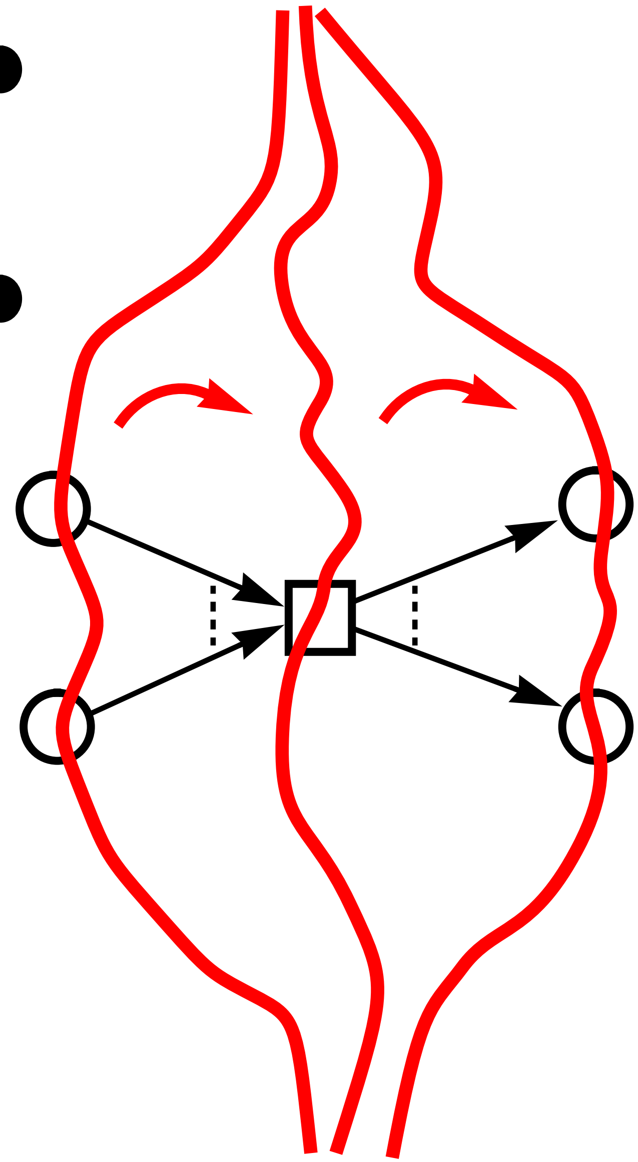
LEMMA

co propagation rules



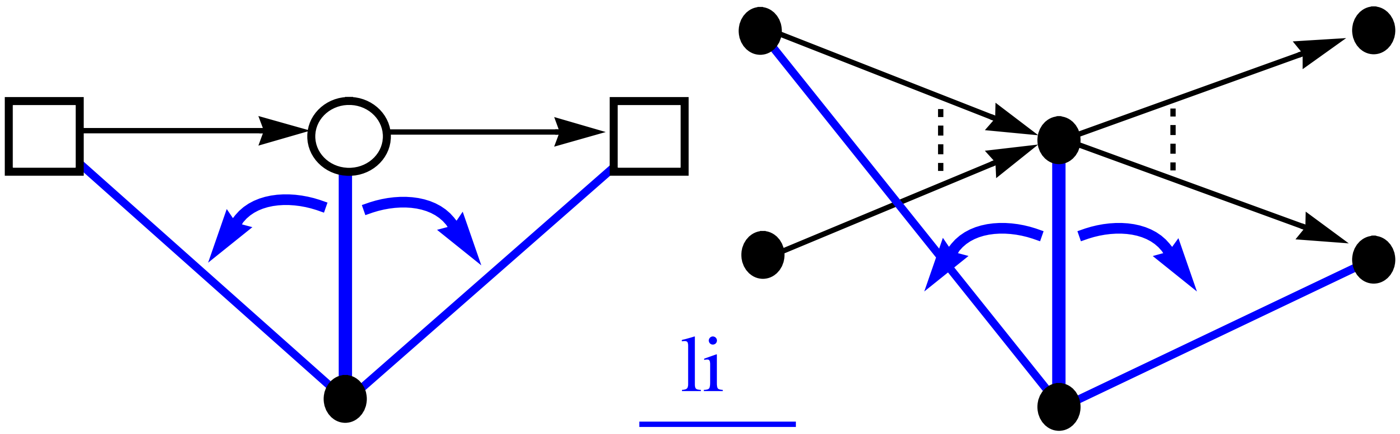
THEOREM

cut propagation rule



LEMMA

li propagation rules



Soundness and Completeness of the Token Game

Think of S-Cuts as *valid states*
and of the *token game* as an *inference rule*.

COROLLARY We cannot leave the set of S-cuts
by the token game. *(soundness)*

AXIOMS The concurrency structure has
"sufficient length",
"more coherence" and
"higher density".

LEMMA Every S-Cut is reachable from every other S-Cut
by the token game. *(completeness)*

THEOREM The set of S-cuts constitutes a single case class.