FROM FUNDAMENTAL LAWS IN PHYSICS TO COORDINATION PRINCIPLES

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UNIVERSITY OF HAMBURG

ADVANCED TUTORIAL ON NETS, PHYSICS AND COORDINATION

PETRI NETS 2013, MILANO

Contents:

1. **Slowness: introduction and examples**
2. From Petri-coordinates to cycloids
3. Lorentz-transformation and slowness
4. Characterisation of cycloids
5. Slowness: looking back

Petri, Carl Adam: *Nets, Time and Space.* Theoretical Computer Science, 153(1–2):3–48, Januar 1996.

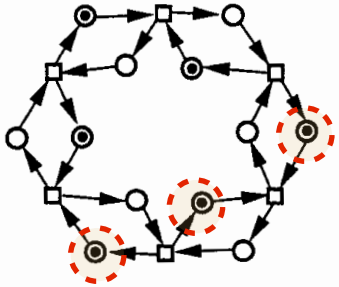


Fig. 18: (3,3,1,1) – Slowness = 0

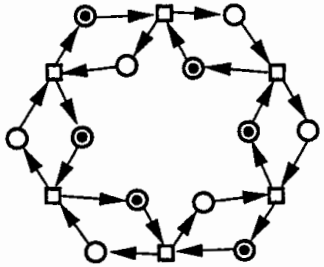


Fig. 19: (4,2,1,1) – Slowness = 1/3

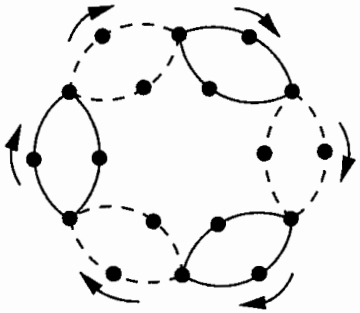
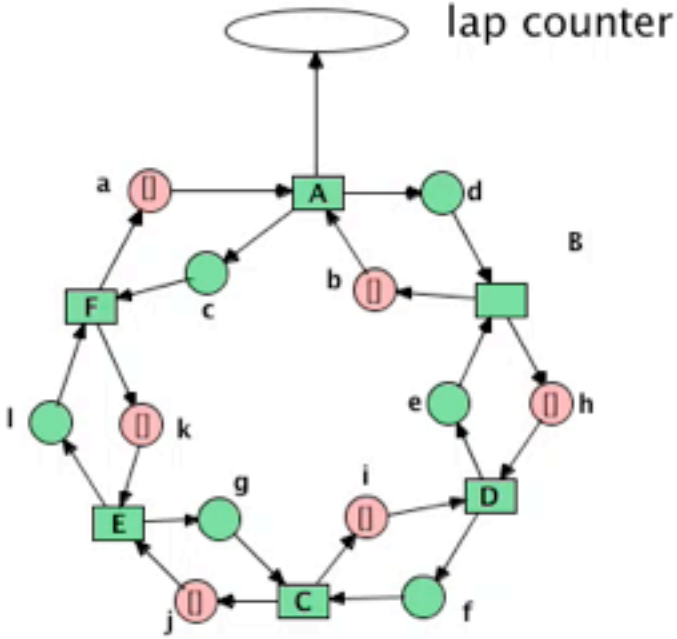


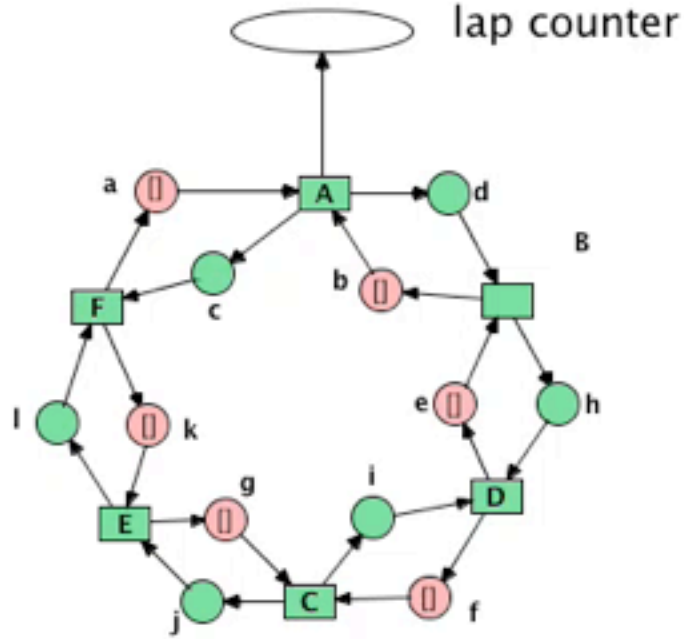
Fig. 20: Cyclic order realized by both systems

Given a CO and a realization the CO captures precisely the structure of basic circuits but in general the number of tokens on non-basic circuits remains unspecified leaving some freedom for different realizations. These realizations may differ, for instance, in their slowness, a notion defined in [12]. This situation can already be observed in small nets: For instance, the precycloid $(3,3,1,1)^{33}$ can be equipped with different s&l case classes realizing the same CO (see Figs. 18, 19 and 20). Intuitively, we recognize that the amount of parallelism in Fig. 18 is greater than in Fig. 19.

M.-O. Stehr: System Specification by Cyclic Causality Constraints. Bericht 210, FB Informatik, Univ. Hamburg, 1998

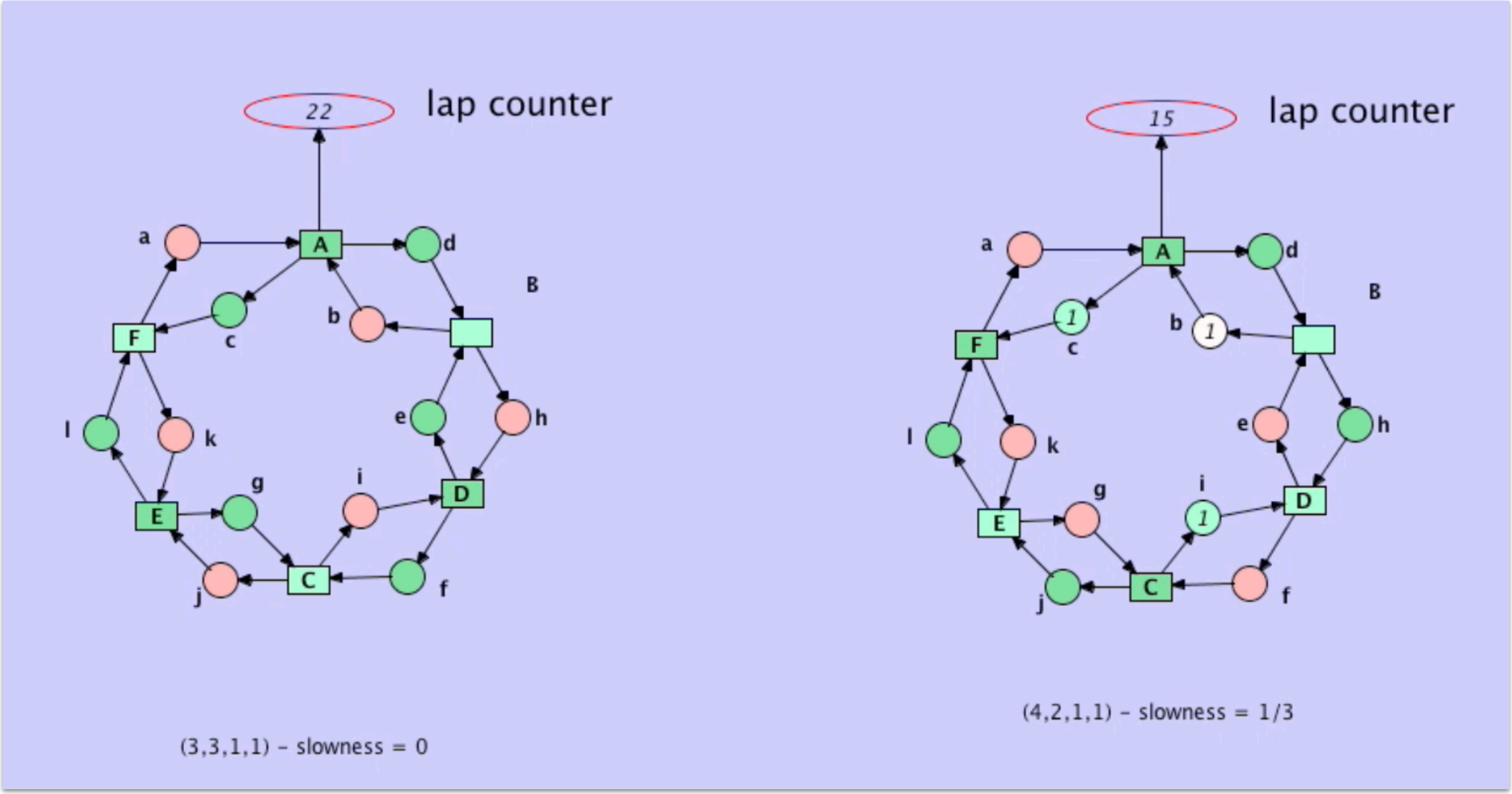


(3,3,1,1) - slowness = 0



(4,2,1,1) - slowness = 1/3

representation in **RENEW**



execution step in **RENEW**

Slowness Effects

This motion proceeds fastest if there is just one gap in front.

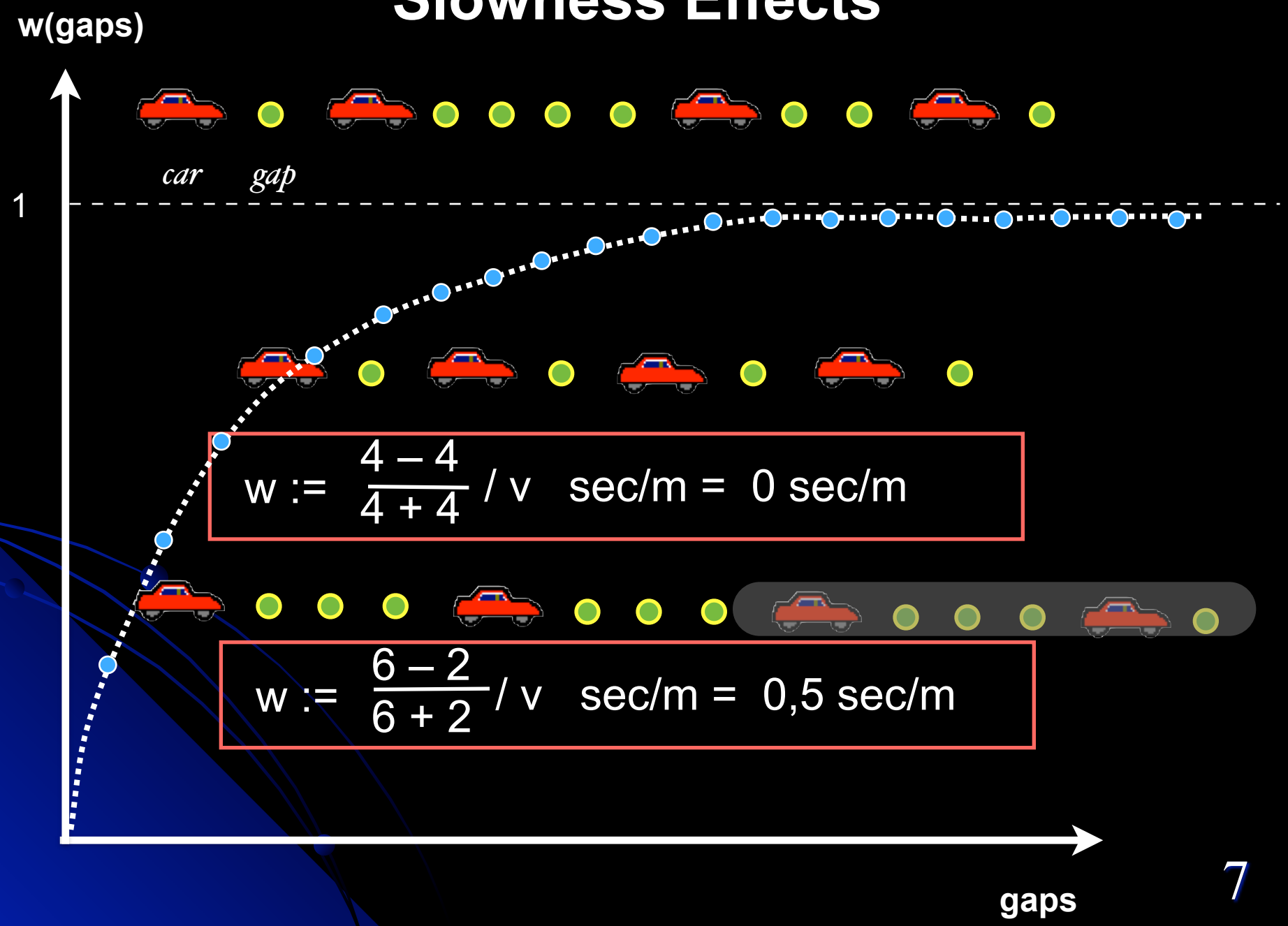
Otherwise, we define SLOWNESS w as the quotient

$$w := \frac{\text{gaps} - \text{cars}}{\text{gaps} + \text{cars}} / v\ (\text{cars}) \qquad \text{sec/m}$$

$$w := \frac{8 - 4}{8 + 4} / v \quad \text{sec/m} = 1/3\ \text{sec/m}$$

The concept of slowness is a key to understanding repetitive GROUP behaviour. It can be applied to Organization, to Work Flow (**Just-in-time Production**), and to Physical Systems.

Slowness Effects



Little's law: *population* = *throughput* × *sojourn time*

$$L = \lambda \times W$$

$$L = \lambda \cdot W$$

The long-term average number of customers in a stable system L

is equal to

the long-term average effective arrival rate, λ,

multiplied by

the average time a customer spends in the system, W;

$$w := \frac{\text{gaps} - \text{cars}}{\text{gaps} + \text{cars}}$$

Little's law: *population* = *throughput* × *sojourn time*

$$L = \lambda \times W$$

$$\lambda = \frac{L}{W} = \frac{1}{m+1} \Rightarrow m = \frac{1}{\lambda} - 1$$

for m gaps we have:

$$w = \frac{m-1}{m+1} = \frac{\frac{1}{\lambda} - 1 - 1}{\frac{1}{\lambda} - 1 + 1} = \frac{\frac{1}{\lambda} - 2}{\frac{1}{\lambda}} = 1 - 2 \cdot \lambda$$

$$= 1 - \frac{2 \cdot L}{W}$$

$$W = \frac{2 \cdot L}{1 - w}$$

growing slowness w
implies
growing sojourn time W

$$w := \frac{\text{gaps} - \text{cars}}{\text{gaps} + \text{cars}}$$

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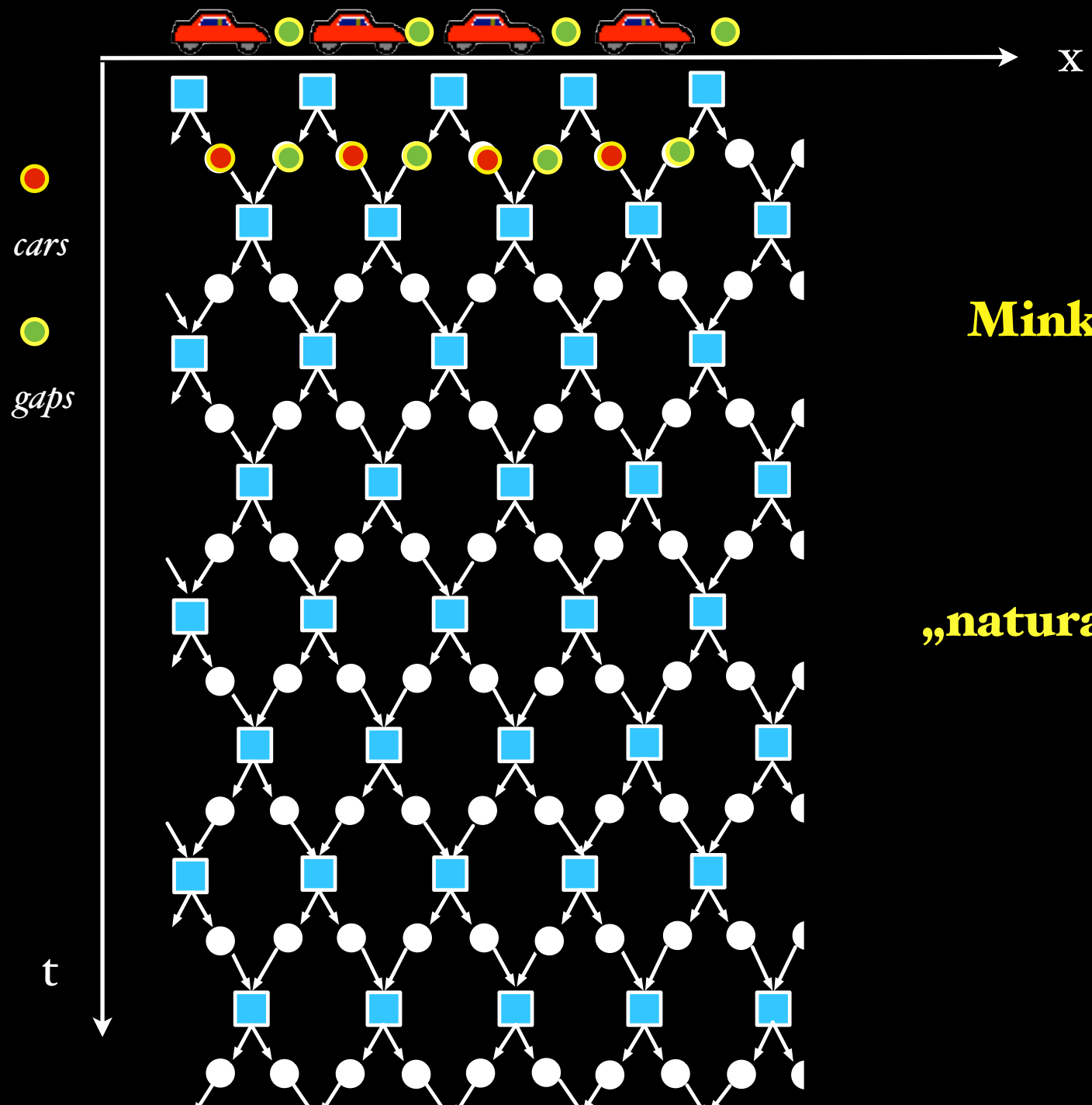


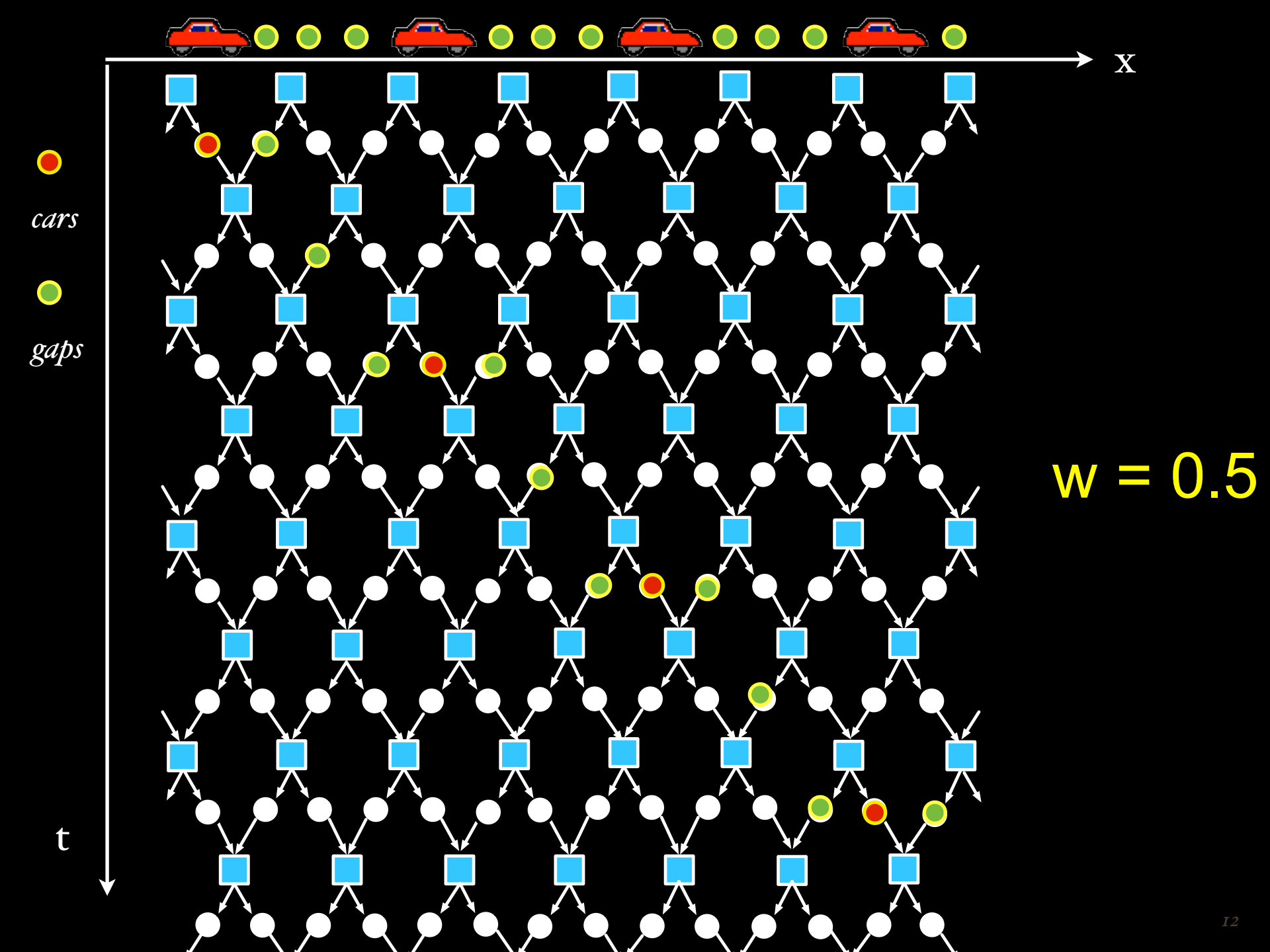
from

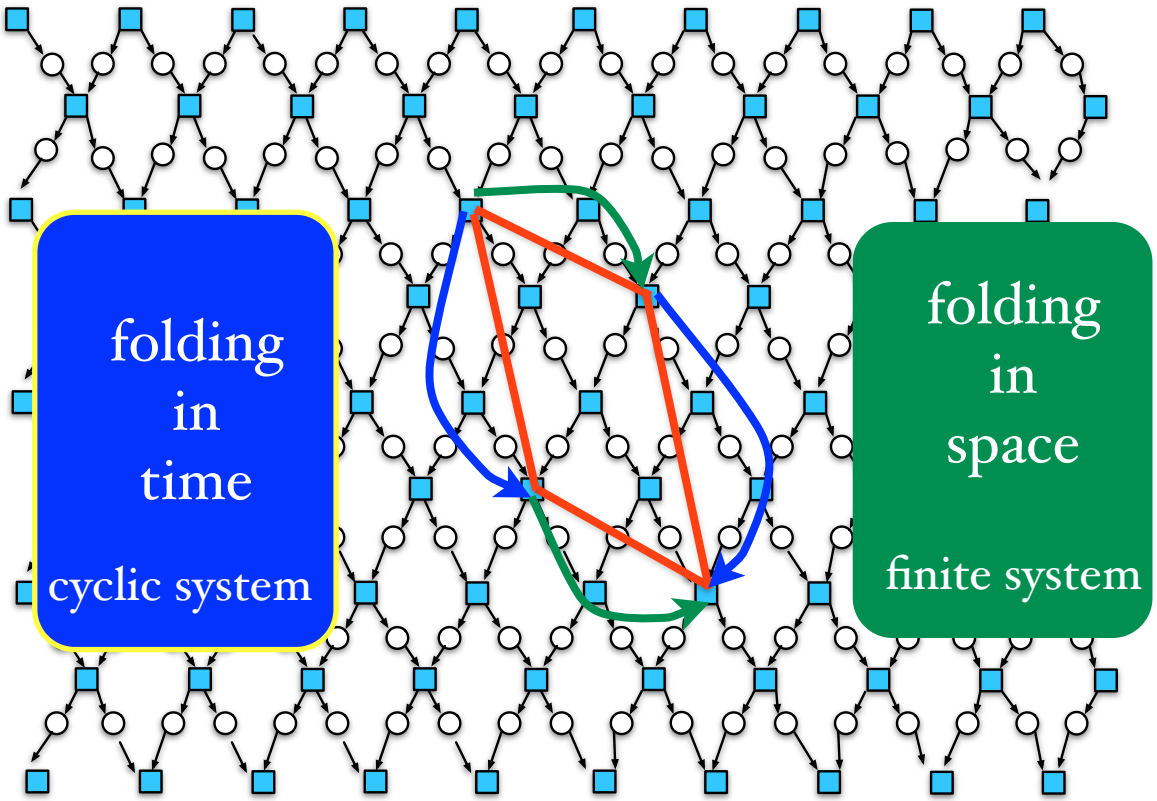
Minkowski Space

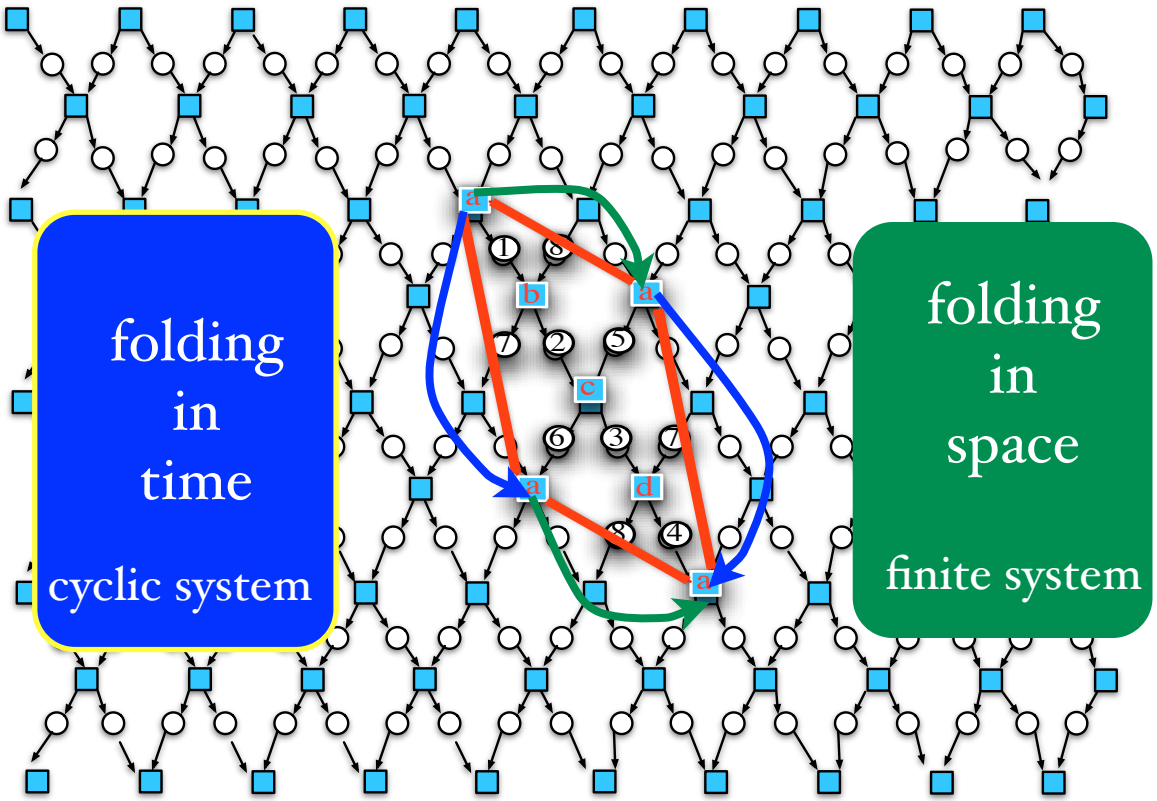
to

Petri's
„natural coordinates“

w = 0


x
cars
gaps
t
w = 0.5


folding
in
time
cyclic system
folding
in
space
finite system


folding
in
time
cyclic system
folding
in
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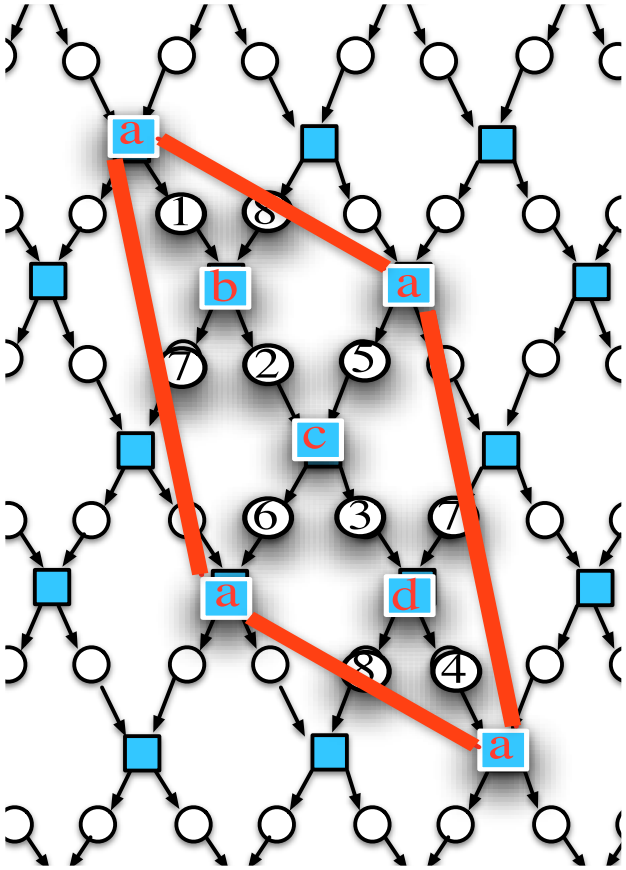
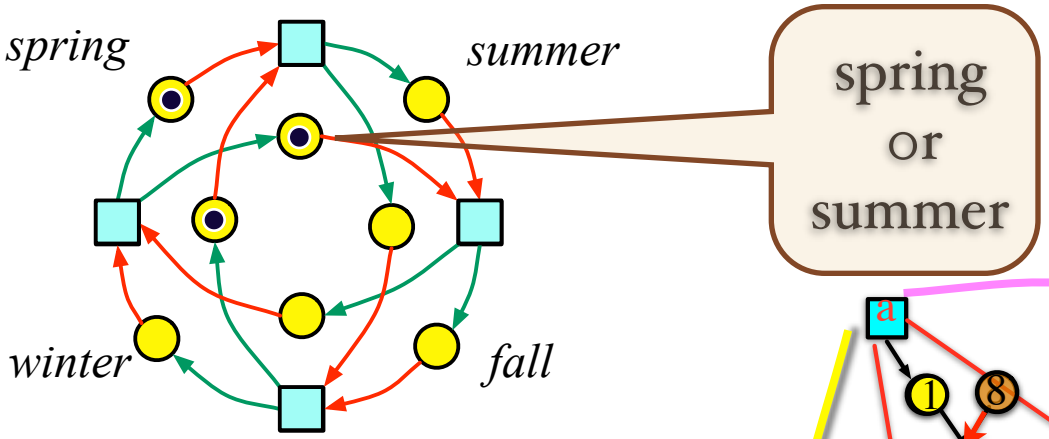
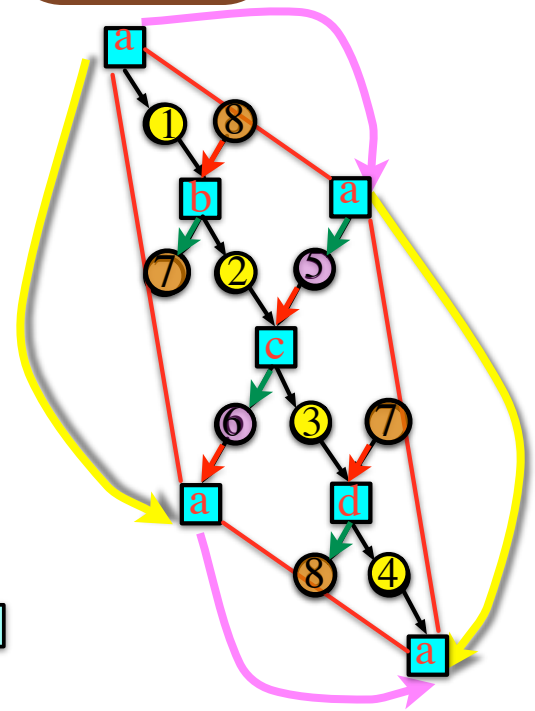
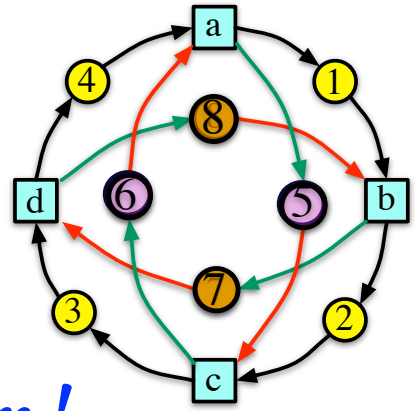
finite system


a
1
8
b
a
7
2
5
c
6
3
7
a
d
8
4
a

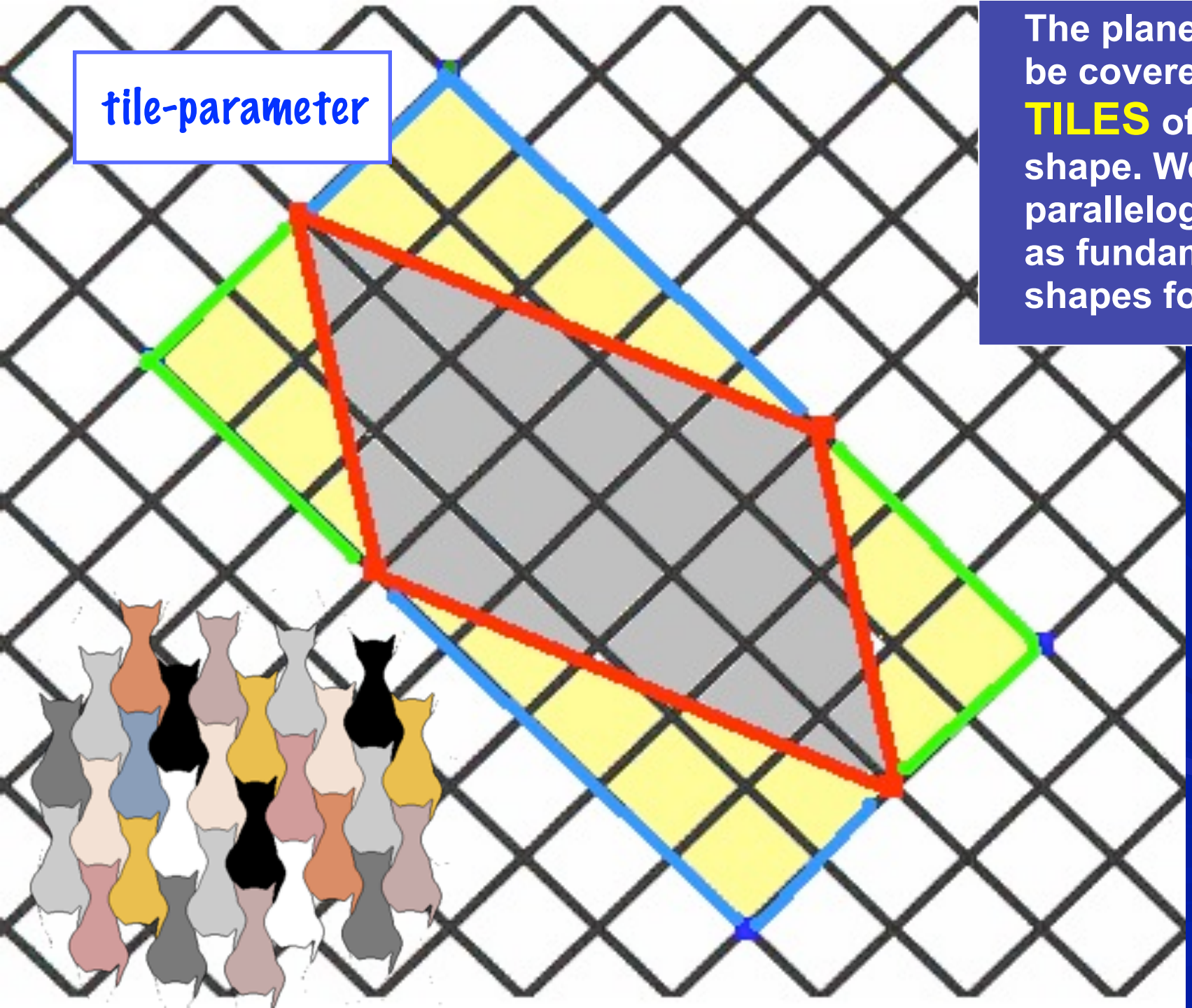


4 seasons



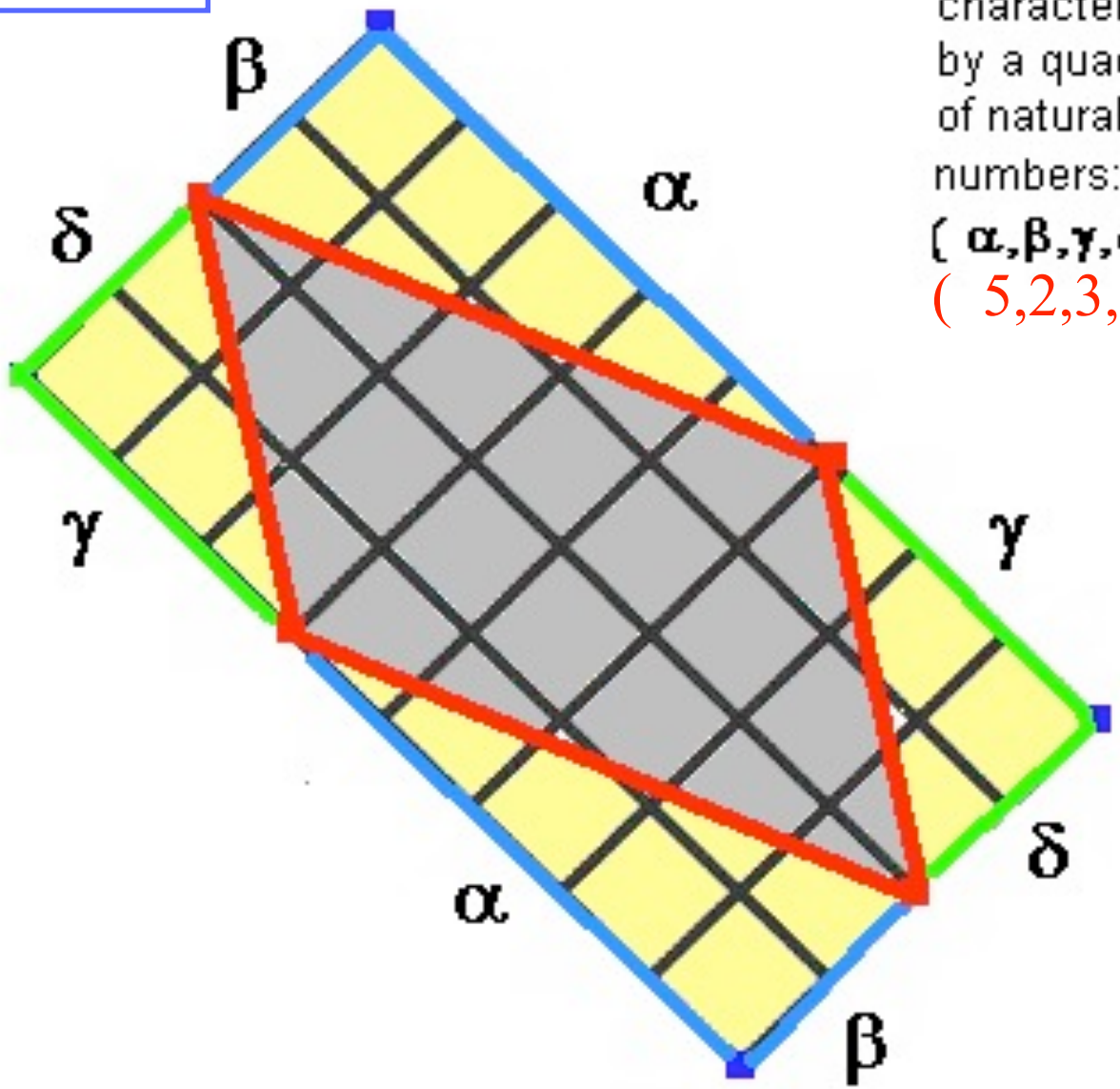


Oscillator !



The plane can be covered by **TILES** of equal shape. We use parallelograms as fundamental shapes for tiling.

tile-parameter



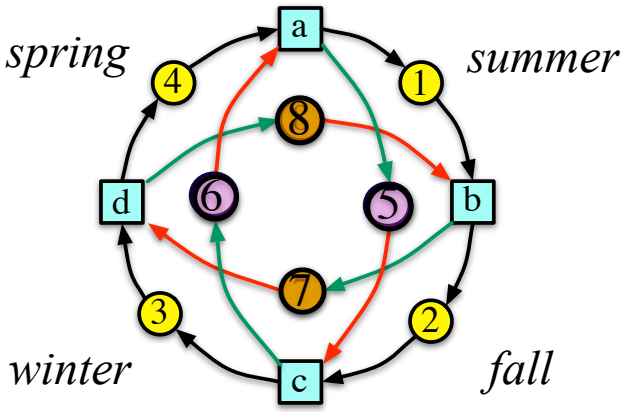
Repetitive Group Behaviour is

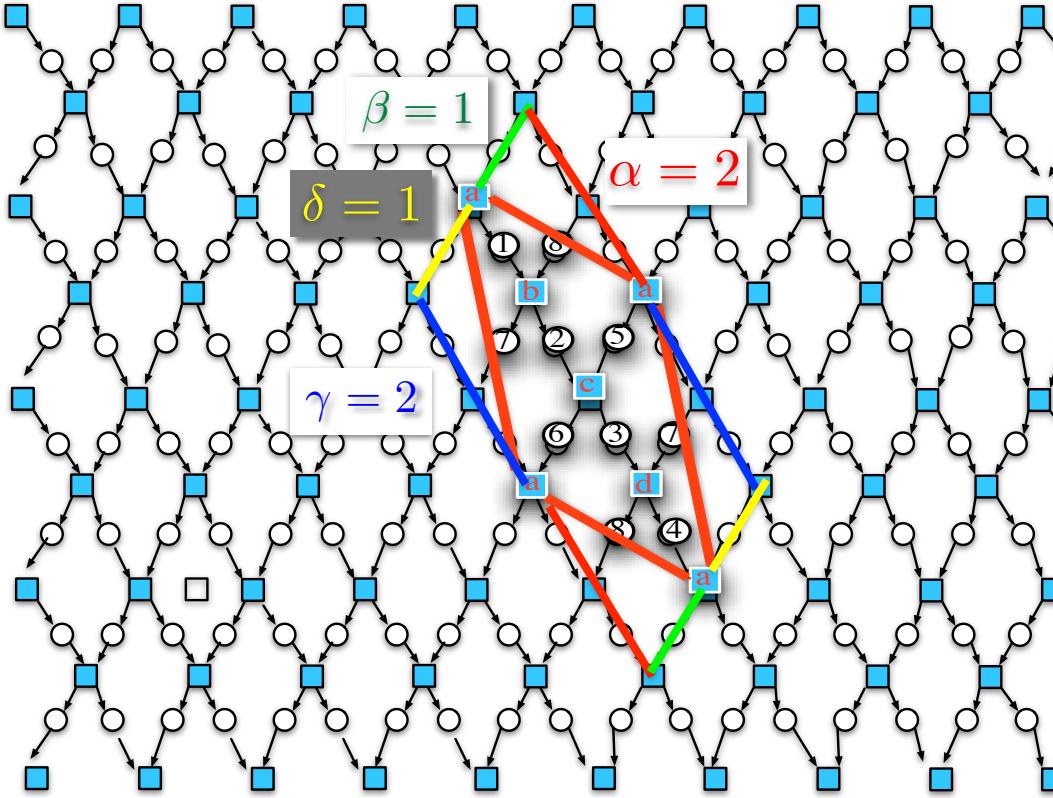
characterized by a quadruple of natural numbers:

(α,β,γ,δ)

(5,2,3,2)

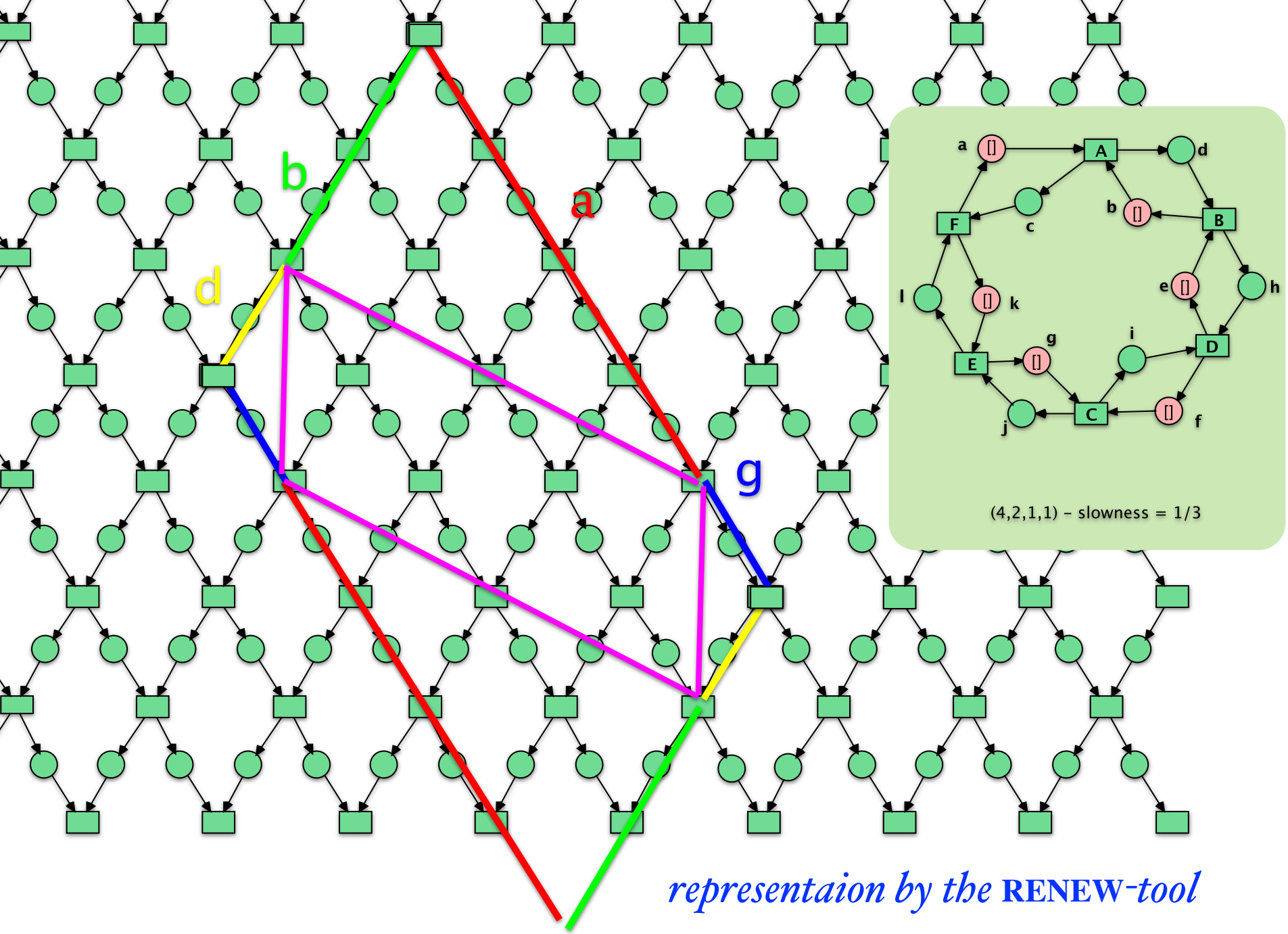
(2,1,2,1)



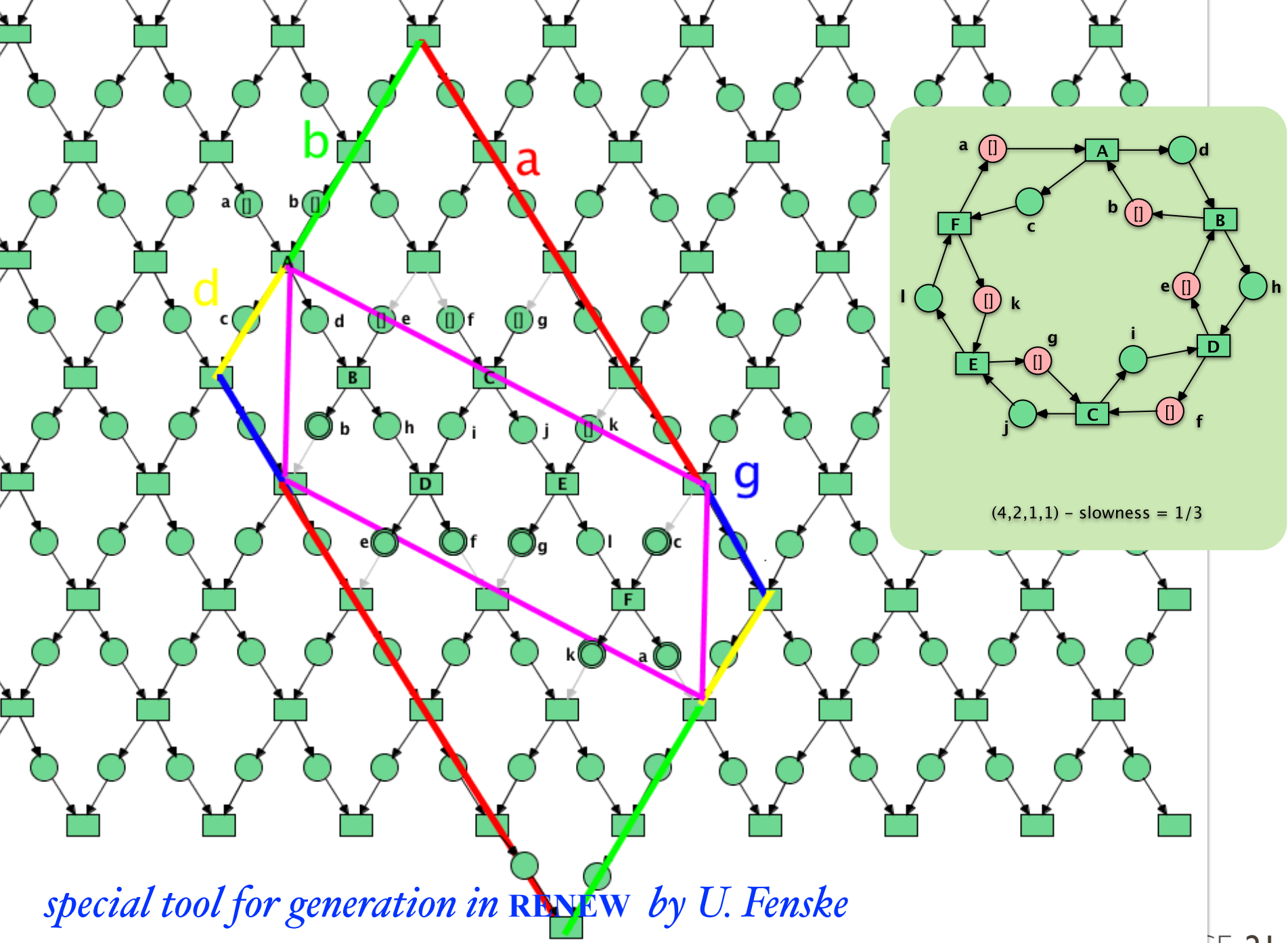


4 seasons

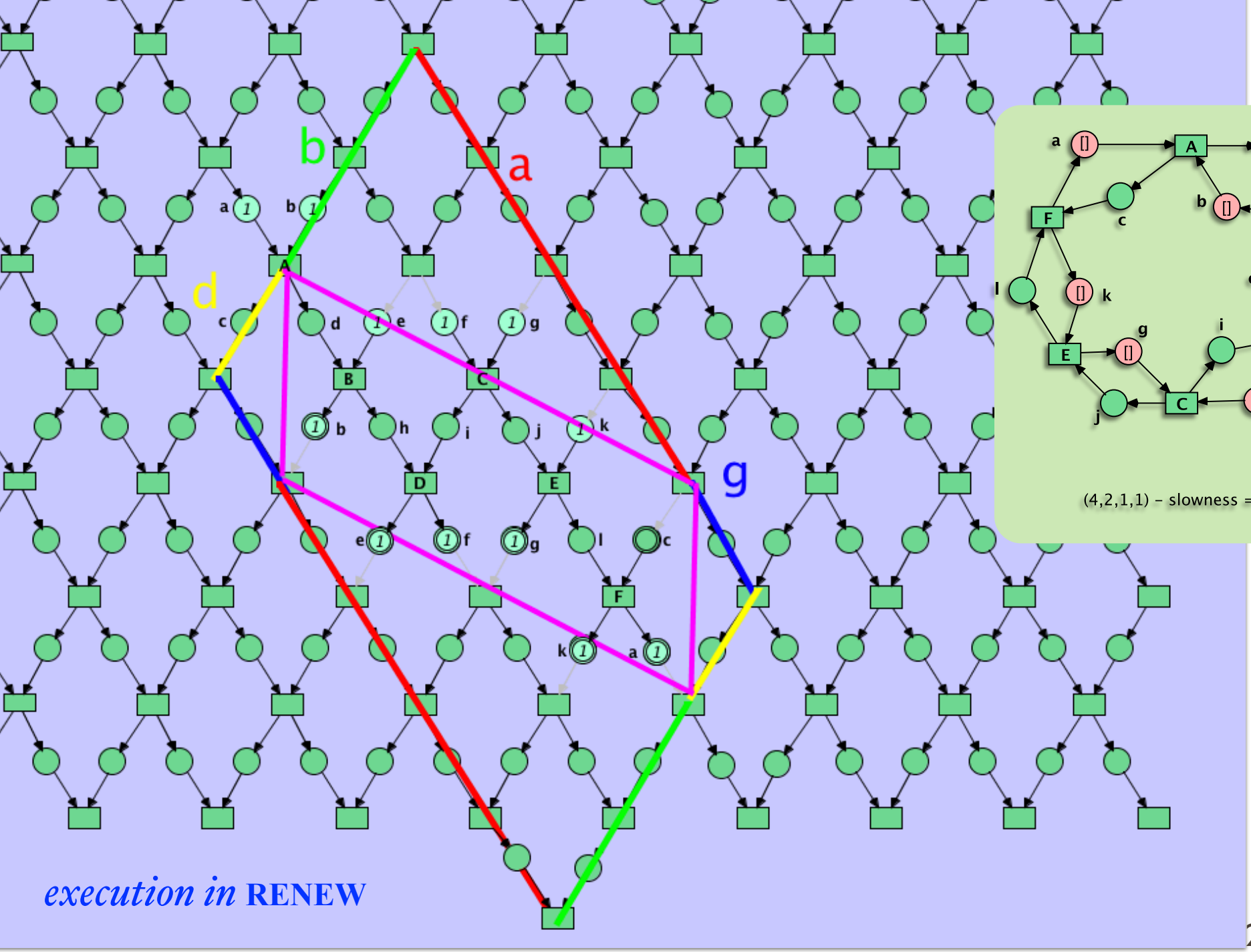
Oscillator !



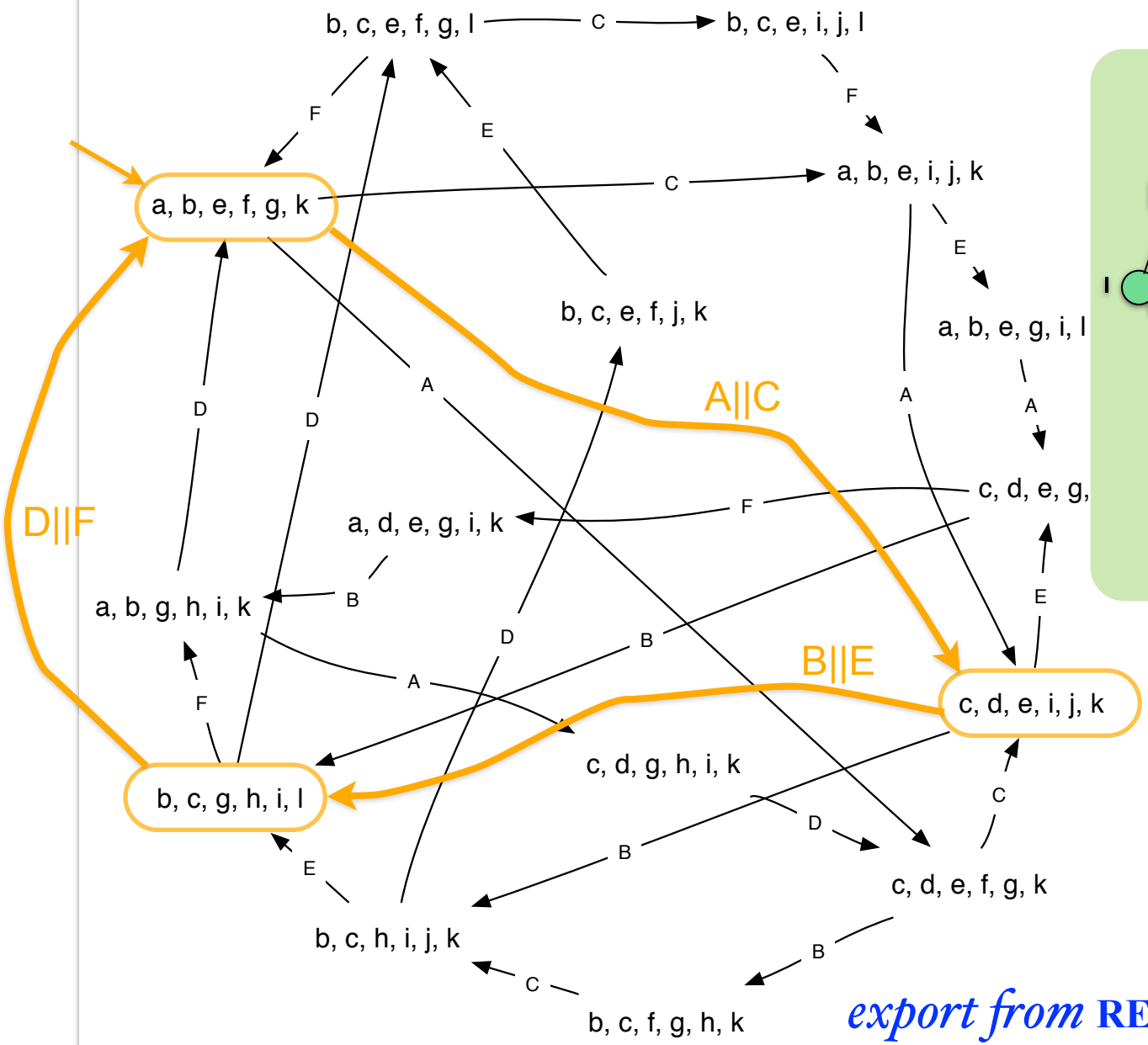
representaion by the **RENEW**-*tool*

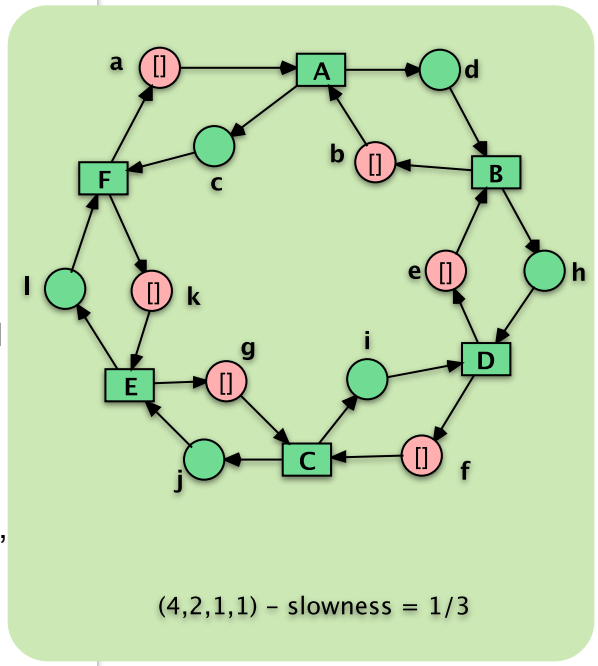


special tool for generation in **RENEW** *by U. Fenske*



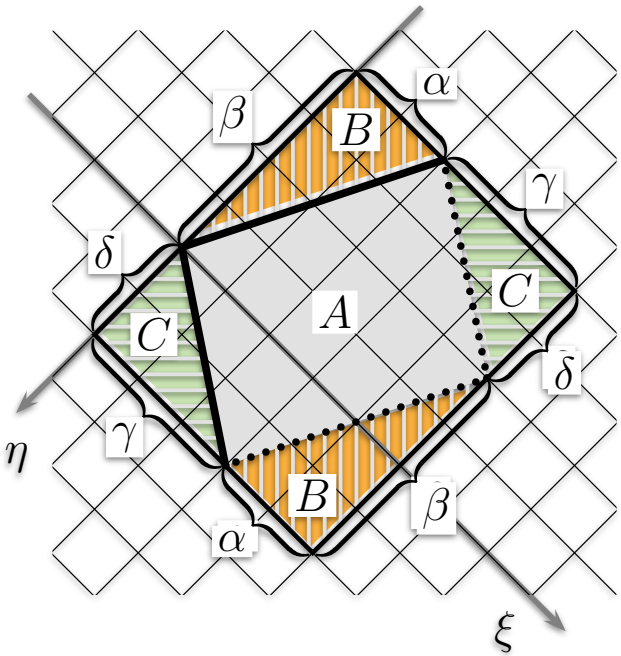
execution in **RENEW**





export from **RENEW** *to the* **Lola**-*tool*

computation of cycloid aera



$$
\begin{aligned}
A &= (\alpha+\gamma)(\beta+\delta) - 2B - 2C \\
&= (\alpha\beta + \alpha\delta + \beta\gamma + \gamma\delta) - 2(\alpha\beta/2) - 2(\gamma\delta/2) \\
&= \alpha\delta + \beta\gamma
\end{aligned}
$$

$$A = 2 \cdot 2 + 4 \cdot 3 = 16$$

Uwe Fenske: Petri's Cycloids and its Extensions, Diploma Thesis 2008

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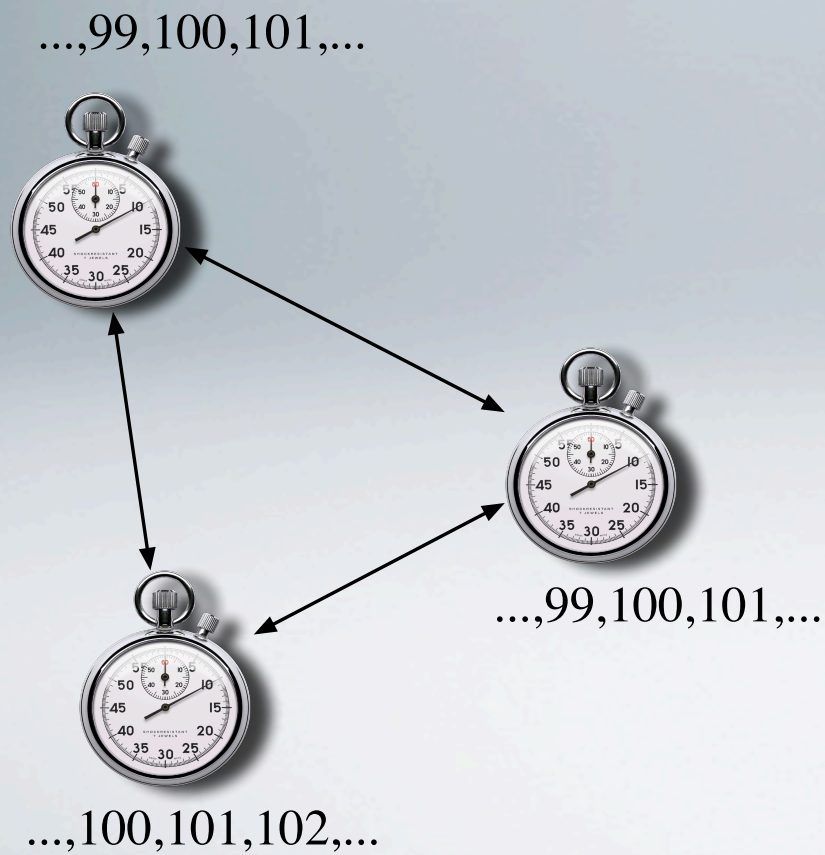
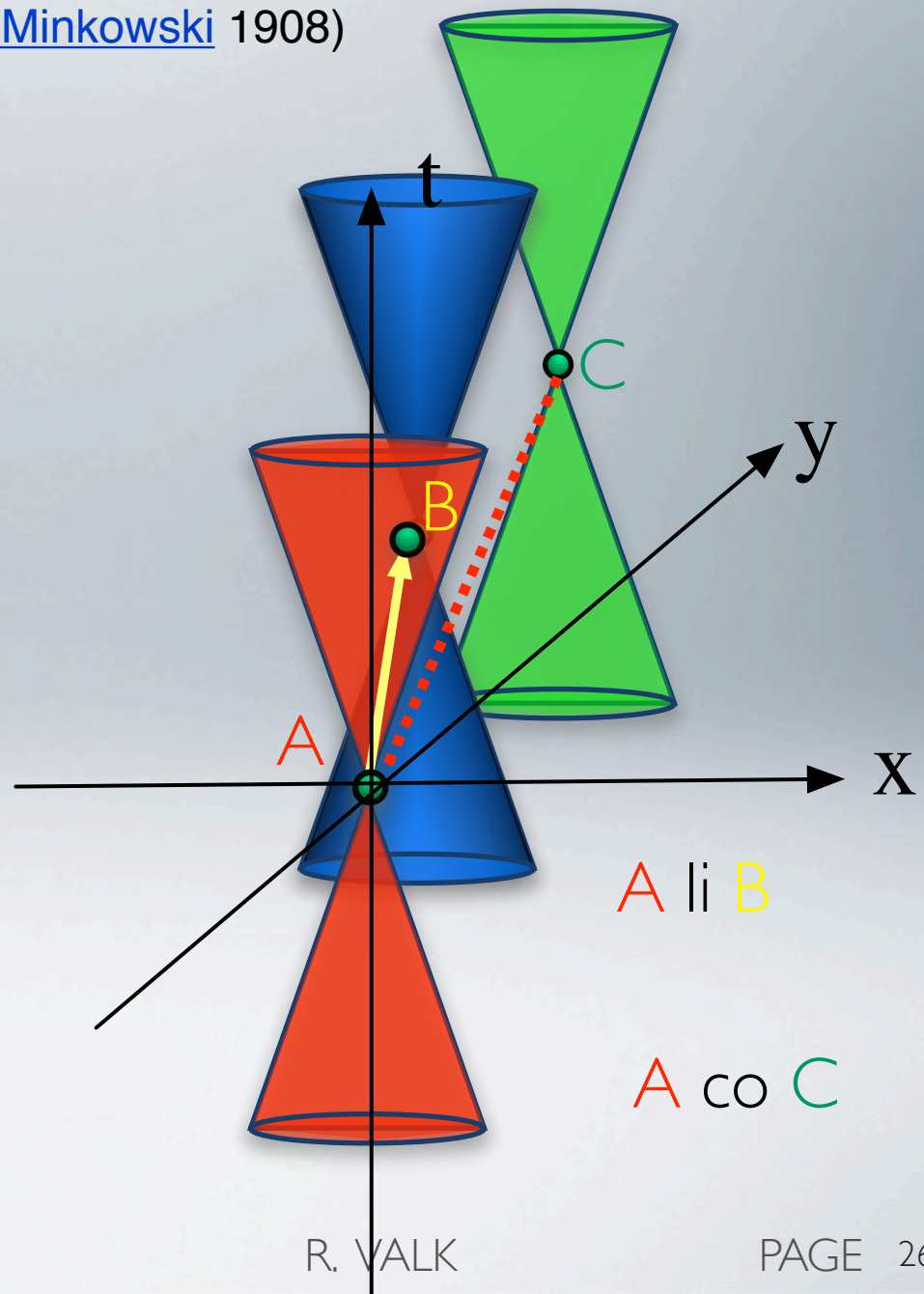
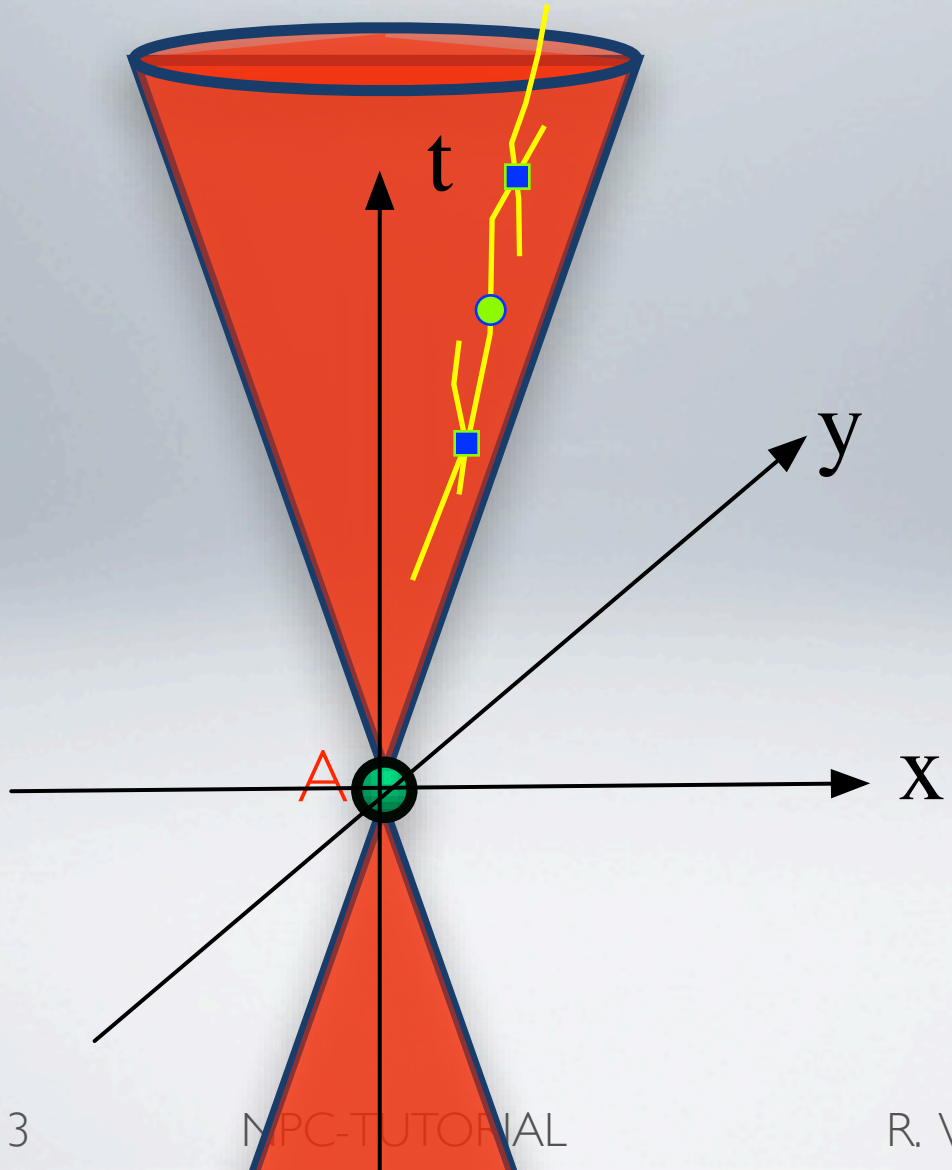
Minkowski-Diagrams (Hermann Minkowski 1908)



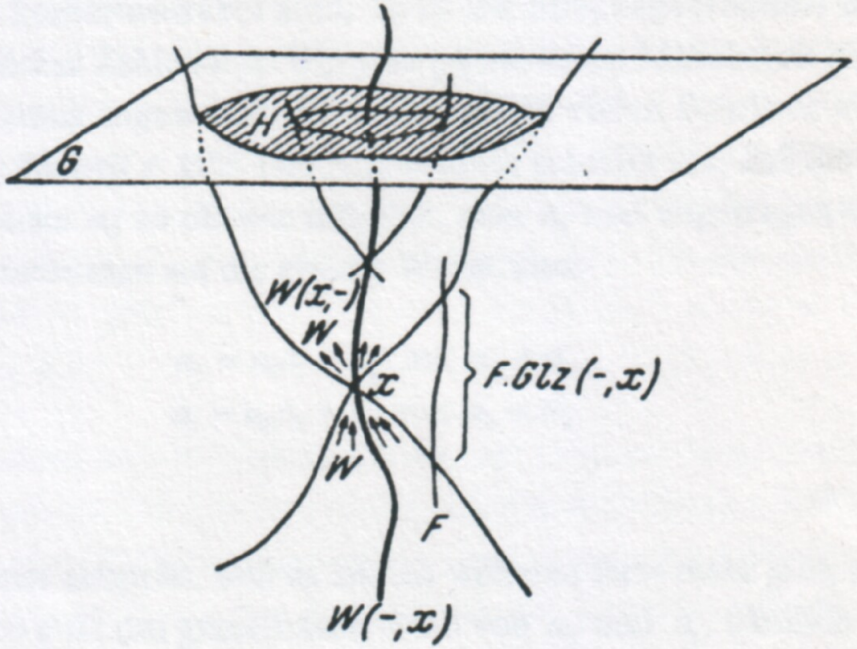


Minkowski-Diagrams (Hermann Minkowski 1908)



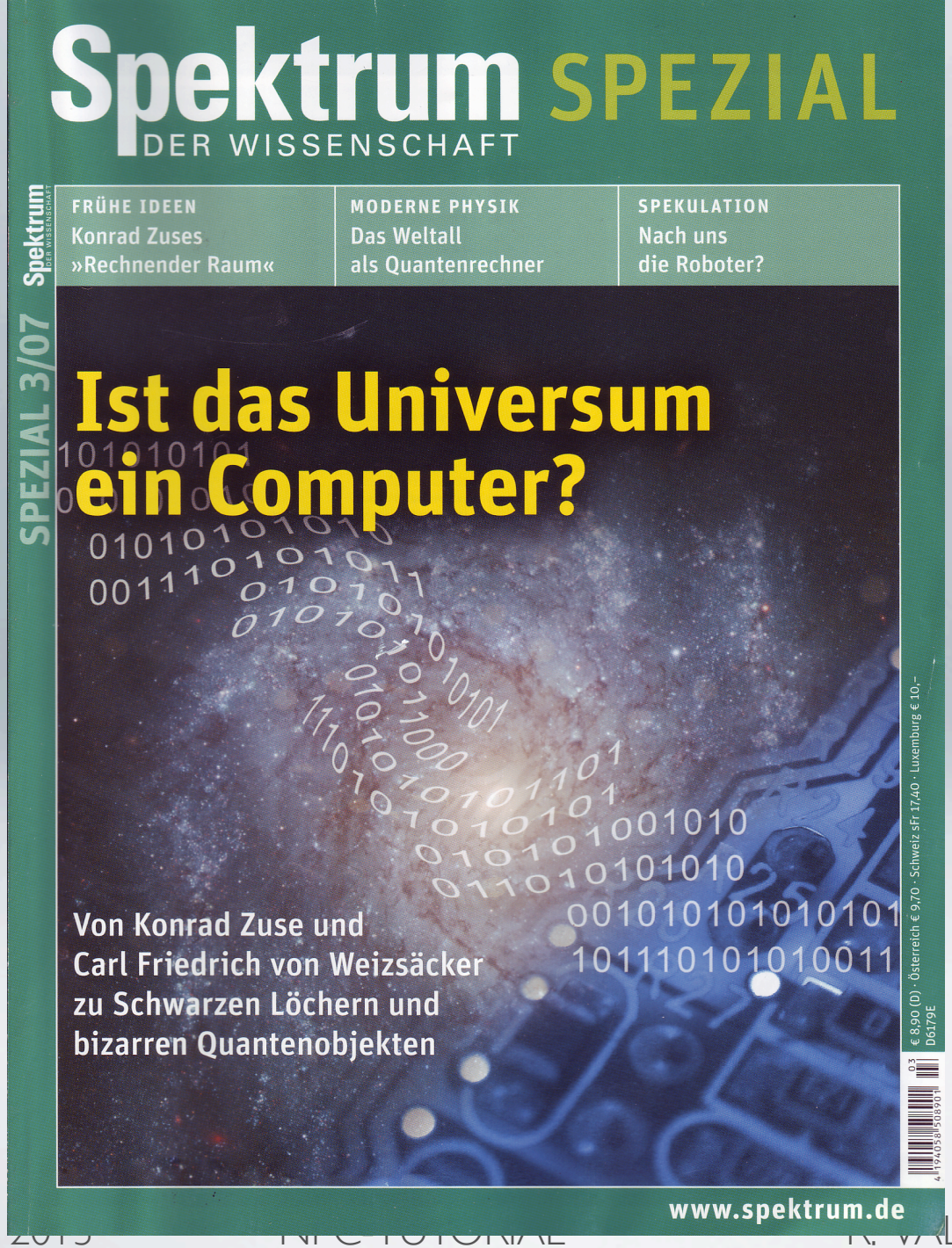
»Wir definieren: ein Raum ist eine Klasse von untereinander gleichzeitigen Weltpunkten, die mit jeder Weltlinie mindestens einen Weltpunkt gemein hat. …
Hieraus folgt, daß jeder Raum mit jeder Weltlinie genau einen Punkt gemein hat«
[Carnap-SL; 205]

from: R. Carnap: Symbolische Logik, Berlin 1973



Hartmut Müller: History and Development of Concurreny Theory
diploma thesis 1993

Is the Universe a Computer?



The Universe as a big Net

Das Universum als großes Netz

Die Arbeit von Konrad Zuse wurde in den 1960er Jahren mit der Netztheorie verteilter Systeme weiterentwickelt. Dabei gelang es, einige der Einwände gegen Zuses Konzept auszuräumen.

Von Carl Adam Petri

Wer einen neu konstruierten Computer zum Laufen bringen will, muss eine große Zahl von Einzelteilen zum präzise koordinierten Zusammenspiel bringen – nach dem Vorbild des strengen Wirkens der Naturgesetze. Das hat niemand intensiver erfahren als Konrad Zuse, der Erfinder und Erbauer des weltweit ersten programmgesteuerten Computers.

Der Gedanke liegt nahe, die Naturgesetze direkt für die dauerhafte Stabilität von Computern heranzuziehen. Dann würde der Computer funktionieren, weil er auf Grund der Naturgesetze nicht anders kann. Dazu wäre es freilich notwendig, diese Gesetze in die Sprache des Ingenieurs zu übersetzen. Ist das überhaupt möglich? Wäre es nicht einfacher, die physikalische Natur direkt in der Sprache des Computer-Ingenieurs neu zu beschreiben? Wenn das gelänge, erschiene die Welt als gigantischer Computer.

Seiner Zeit wieder einmal weit voraus, unternahm Zuse ernsthaft diesen Versuch, zum Kopfschütteln der meisten Zeitgenossen. Er begann mit dem damals bereits bekannten Konzept des »zellulären Automaten«: Hier wird der Raum in lauter gleichartige Zellen aufgeteilt, die wie kleine Maschinen nach einem vorgegebenen Verhaltensmuster (»Programm«) mit ihren Nachbarzellen Information austauschen. Es gelang ihm, wenigstens auf dem Papier, die Phänomene »Fortpflanzung« und »Bewegung« von Zustandsmustern zu beschreiben (siehe seinen Artikel auf S. 6). Für eine Nachprüfung durch Computer-Simulation waren die damaligen Maschinen nicht leistungsfähig genug.

Besser ausgerüstet gelang später anderen die Simulation einfacher zellulärer Automaten mit großem rechnerischem Aufwand. Der zelluläre Automat »Spiel des Lebens« von John Horton Conway ist weithin bekannt geworden. Vor allem Stephen Wolfram experimentierte mit vielen verschiedenen Zell-Programmen und stellte der Welt 2002 das Ergebnis seiner Versuche mit einem über 1000-seitigen Buch als »Eine Neue Art von Wissenschaft« vor – gewiss ein vollmundiger Titel.

Eines Tages besuchte mich Konrad Zuse in meinem Arbeitszimmer. Ich war ihm empfohlen worden als erfinderischer Theoretiker mit langjähriger Erfahrung als Leiter eines großen Rechenzentrums. Er bat mich, ihn bei seiner Arbeit zum Rechnenden Raum zu beraten. Ich fühlte mich hoch geehrt und stimmte sofort zu. Es entstand eine äußerst fruchtbare Zusammenarbeit, die sich wider Erwarten über mehr als drei Jahre erstreckte. In ungezählten (Streit-)Gesprächen kamen wir uns wissenschaftlich und persönlich immer näher.

Konrad Zuse vertrat zunächst seinen Ansatz der zellulären Automaten und verteidigte ihn vehement gegen meine Bedenken. Diese bestanden hauptsächlich in Folgendem: Erstens ergibt sich aus der räumlichen Anordnung der Zellen eine Auszeichnung von drei bestimmten Richtungen im Raum, die sich nur durch Einführung zufälliger Prozesse aufheben lässt. (Zuse hielt die Annahme von Zufälligkeit nicht für zielführend.)

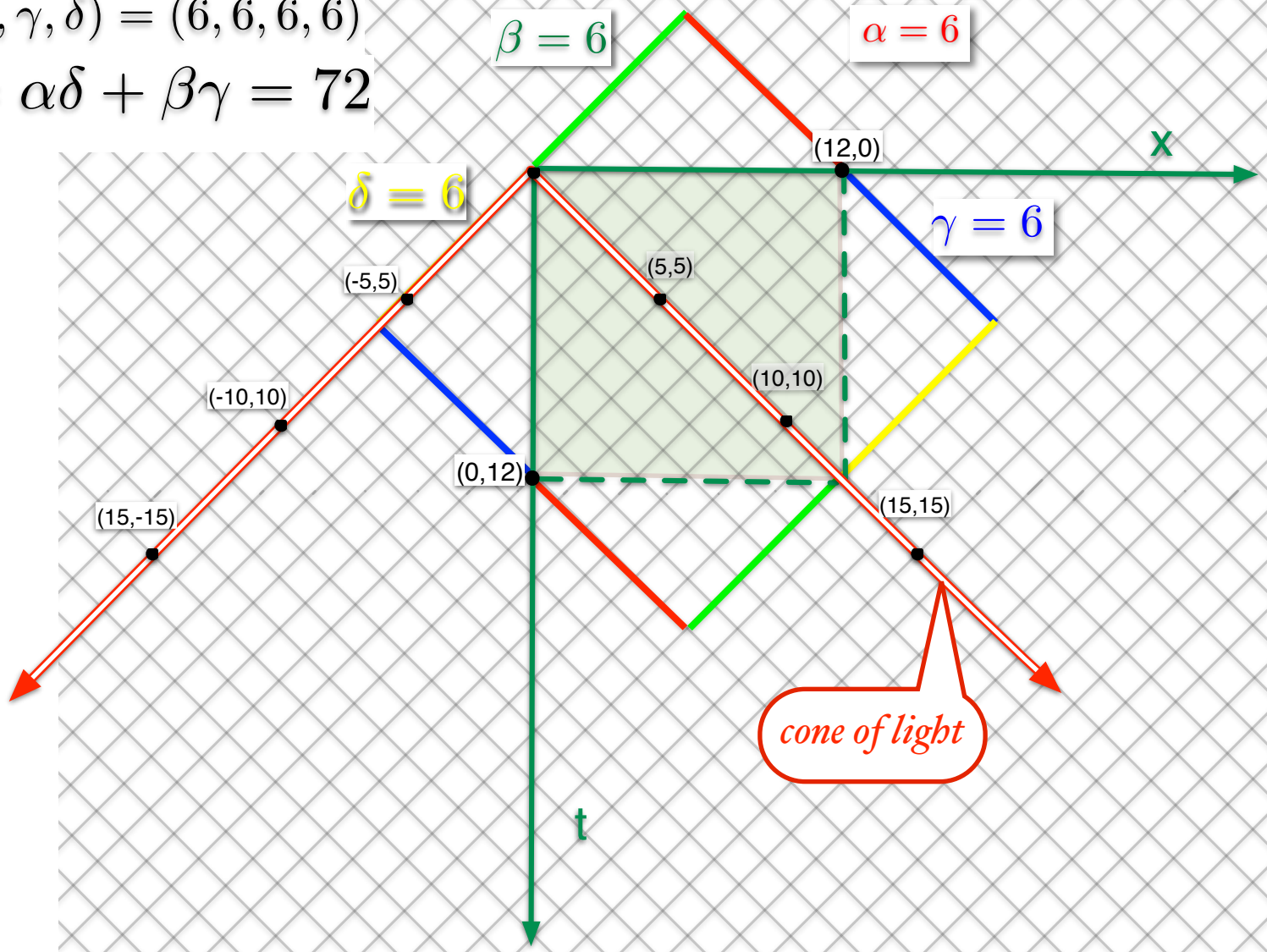
Zweitens wandte ich ein, dass sich in diesem Modell die gesamte Physik in den Programmen jeder Zelle verstecke, sozusagen als nicht analysierbare DNA und ohne direkten Bezug zu physikalischen Größen. Ich schlug ihm deshalb eine andere Modellierungstechnik vor: die Netztheorie verteilter Systeme. Heute unter dem Namen »Petri-Netze« bekannt, hatte sie bereits viele erfolgreiche Anwendungen gefunden, so in Bankwesen, Ökonomie, Telekommunikation, Workflow Management, Konfliktlösung, Prozesssteuerung und Biochemie, nur leider noch nicht an ihrem Geburtsort,

Carl Adam Petri (links) im Gespräch mit Konrad Zuse, um 1975

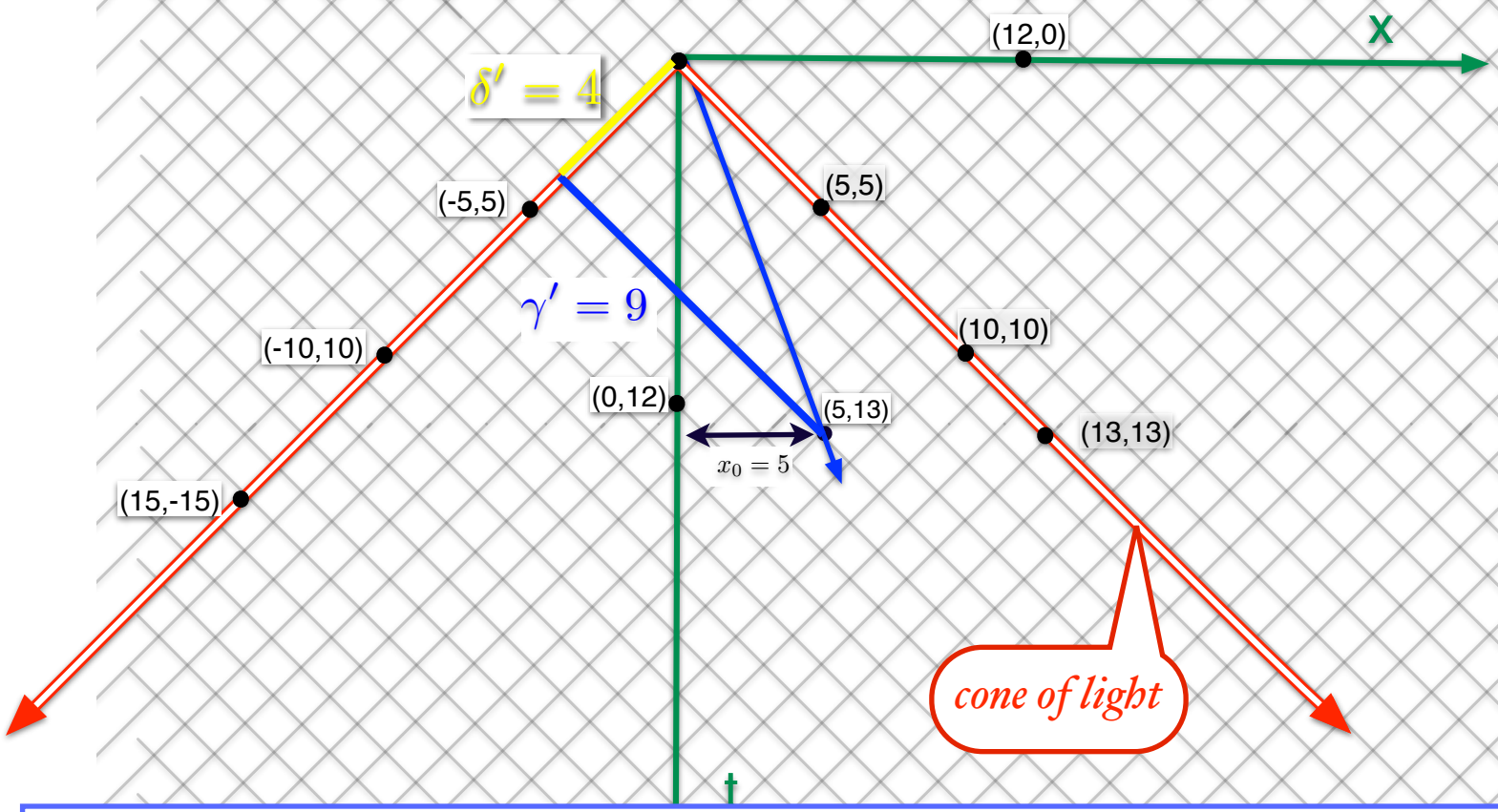
FOTO: MIT FRDL. GEN. DER HANNOVER GESELLSCHAFT, ALLE GRAFIKEN: SPEKTRUM DER WISSENSCHAFT, NACH: CARL A. PETRI

$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$

$$A = \alpha\delta + \beta\gamma = 72$$



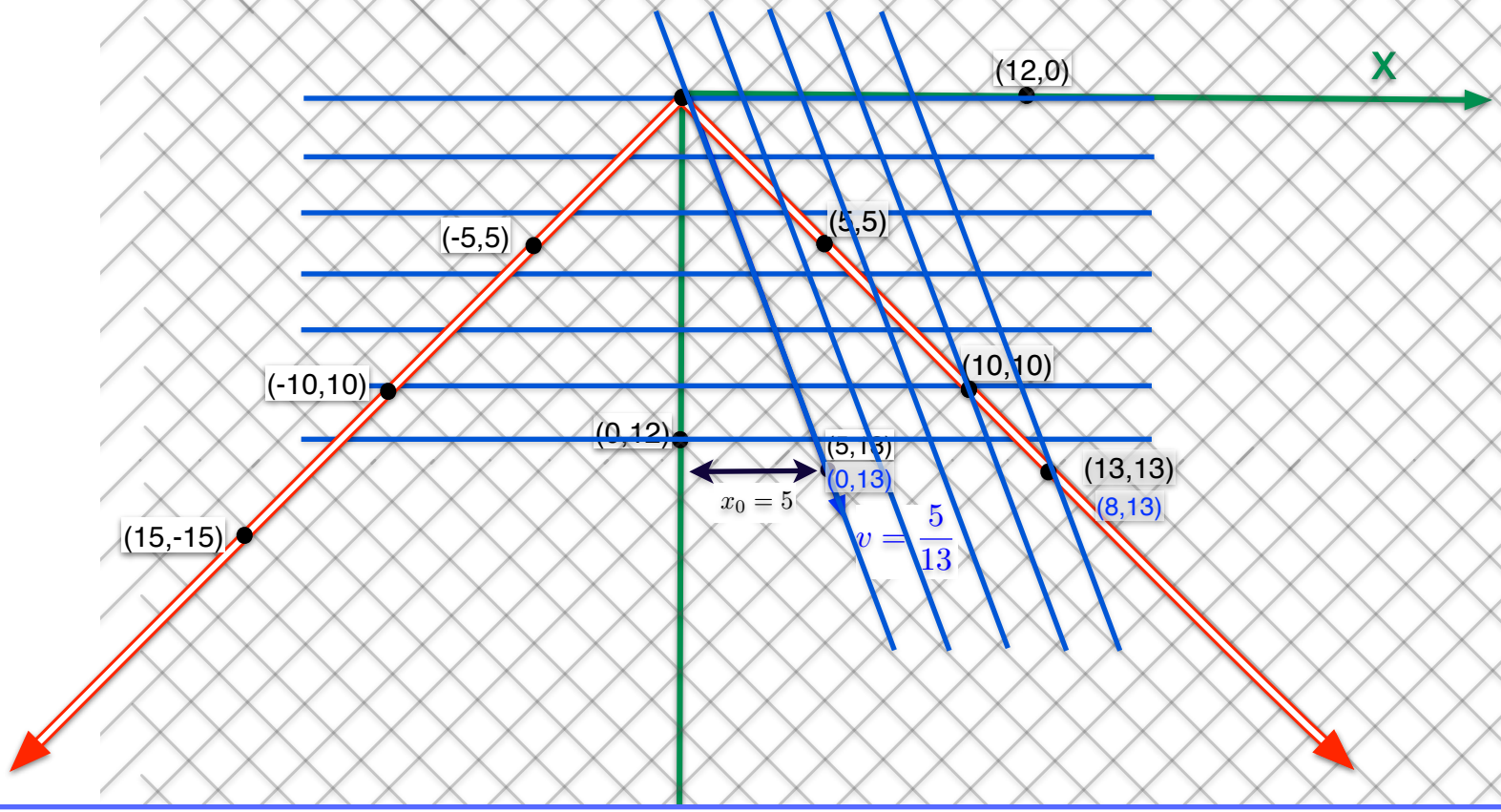
Galilei-Transformation (GT)



$$\text{velocity} = \frac{displacement}{time\ \ taken} = v = \frac{\gamma' - \delta'}{\gamma' + \delta'} = \frac{9-4}{9+4} = \frac{5}{13} \qquad \cdot c \quad [\frac{m}{sec}]$$

v is measured in meters (gained) per second

Galilei-Transformation (GT)



$$x' = x - v \cdot t \qquad t' = t$$

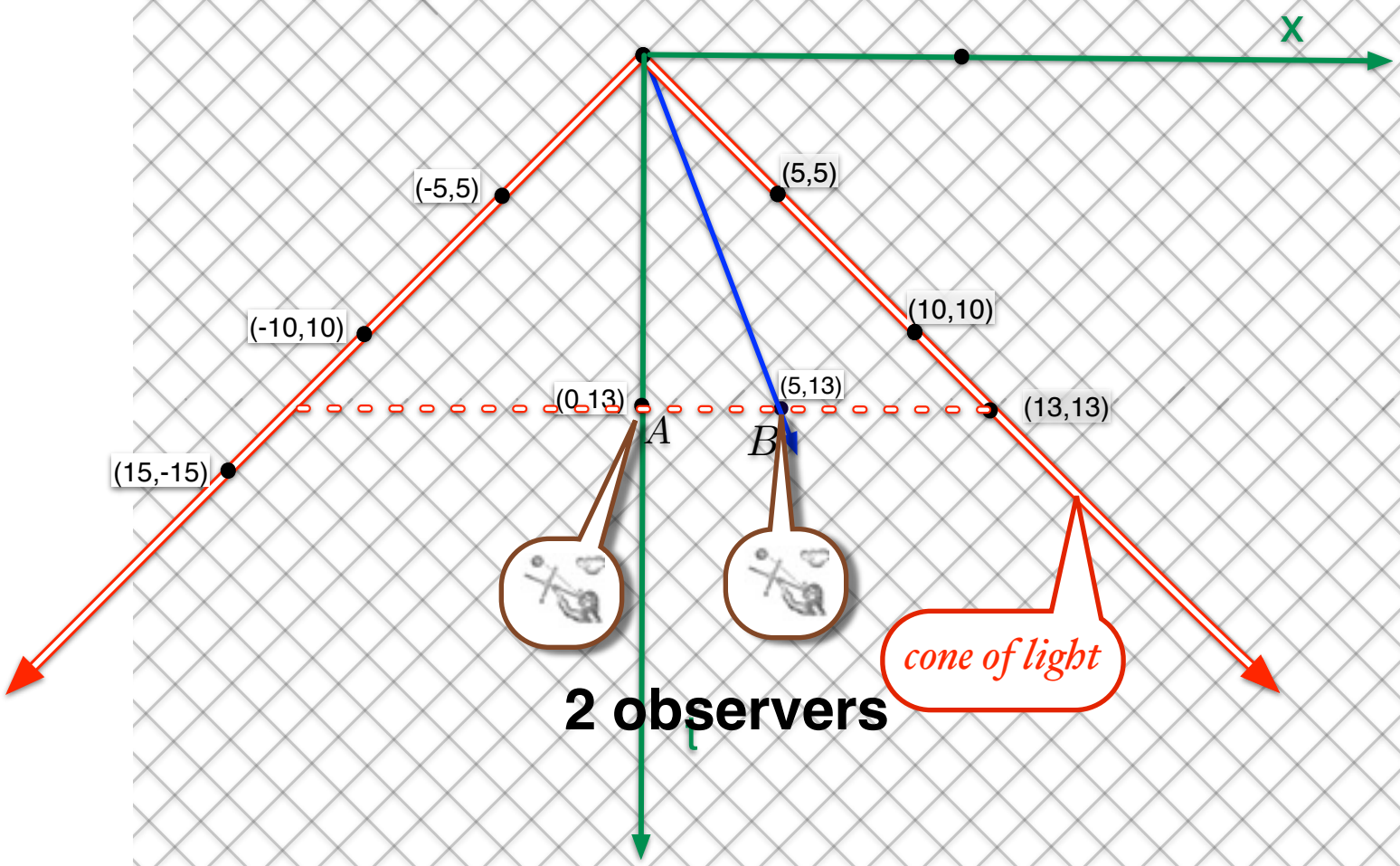
$$x' = x - \frac{5}{13} \cdot t$$

$$(5, 13) \mapsto (0, 13)$$

$$(13, 13) \mapsto (8, 13)$$

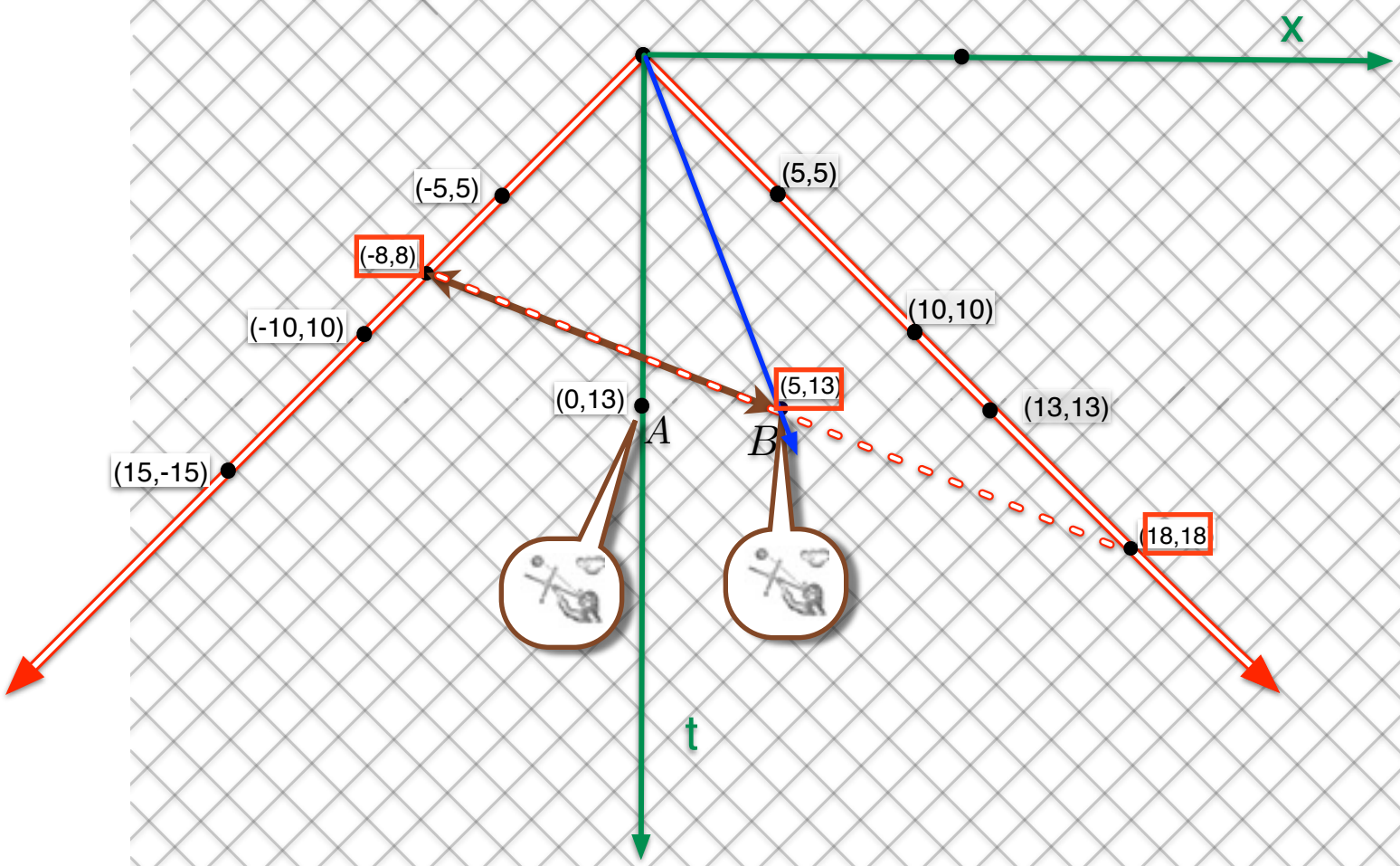
Michelson–Morley experiment

Lorentz-Transformation (LT)



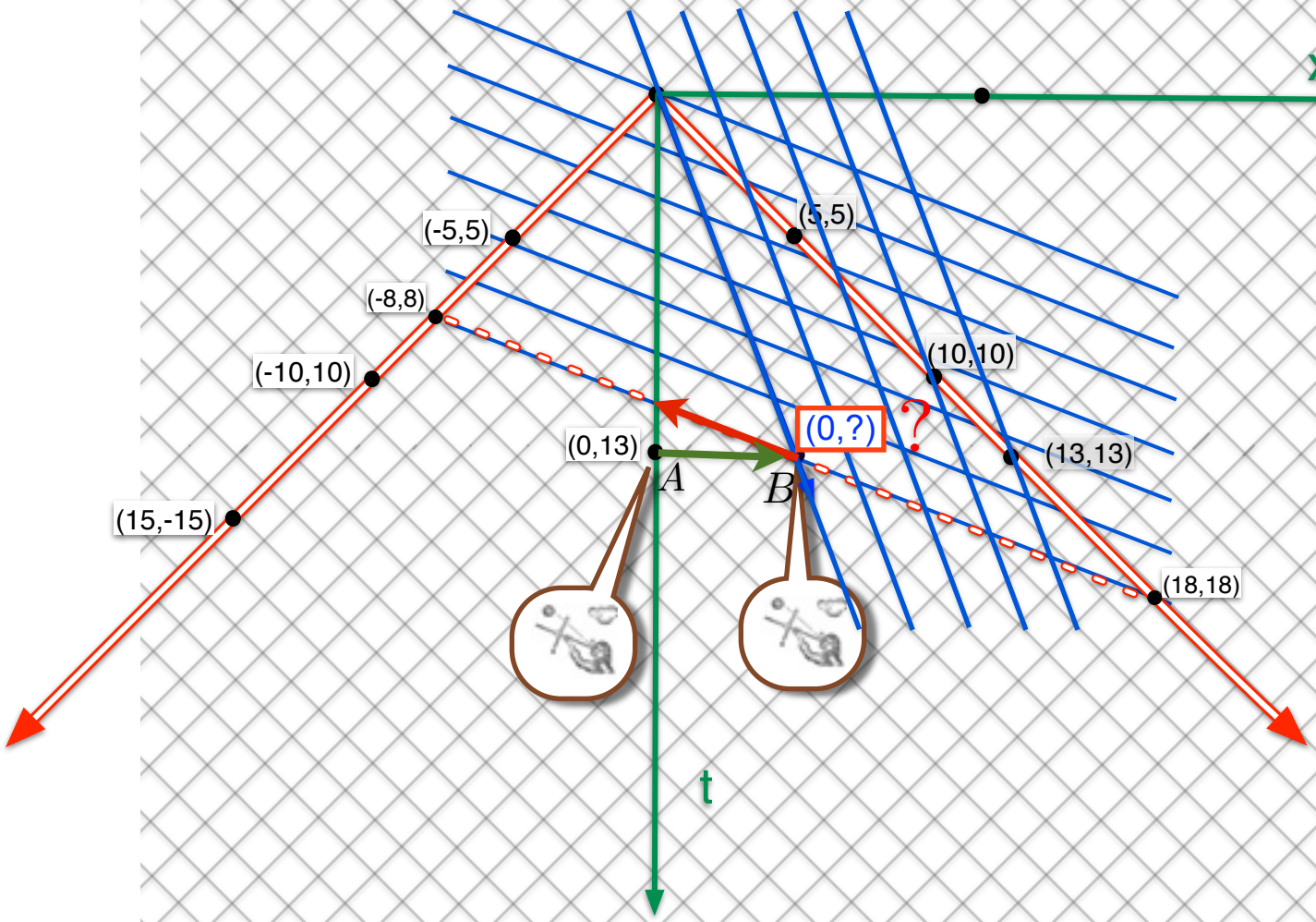
Michelson–Morley experiment

Lorentz-Transformation (LT)

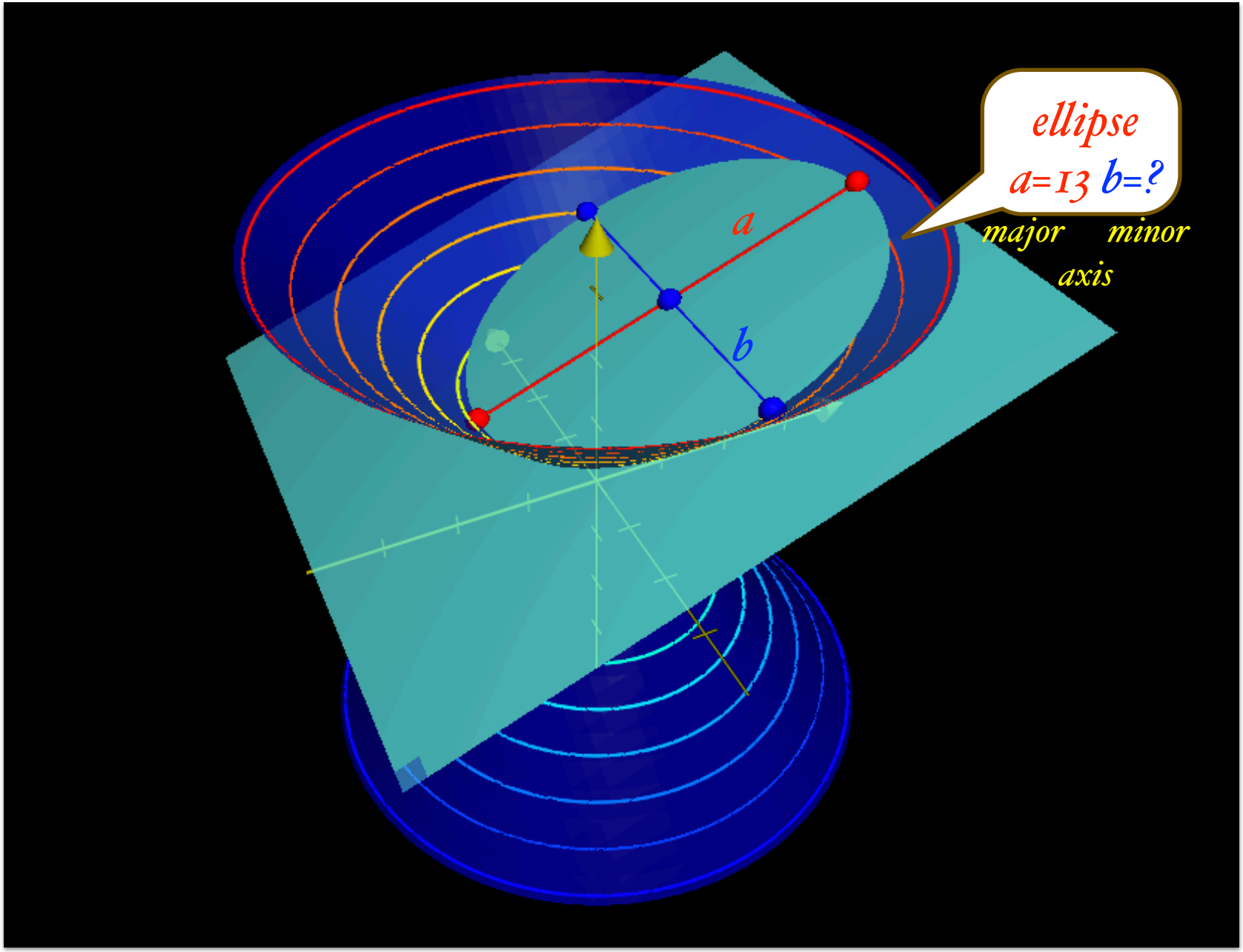


observer B observes the same speed of light

Lorentz-Transformation (LT)



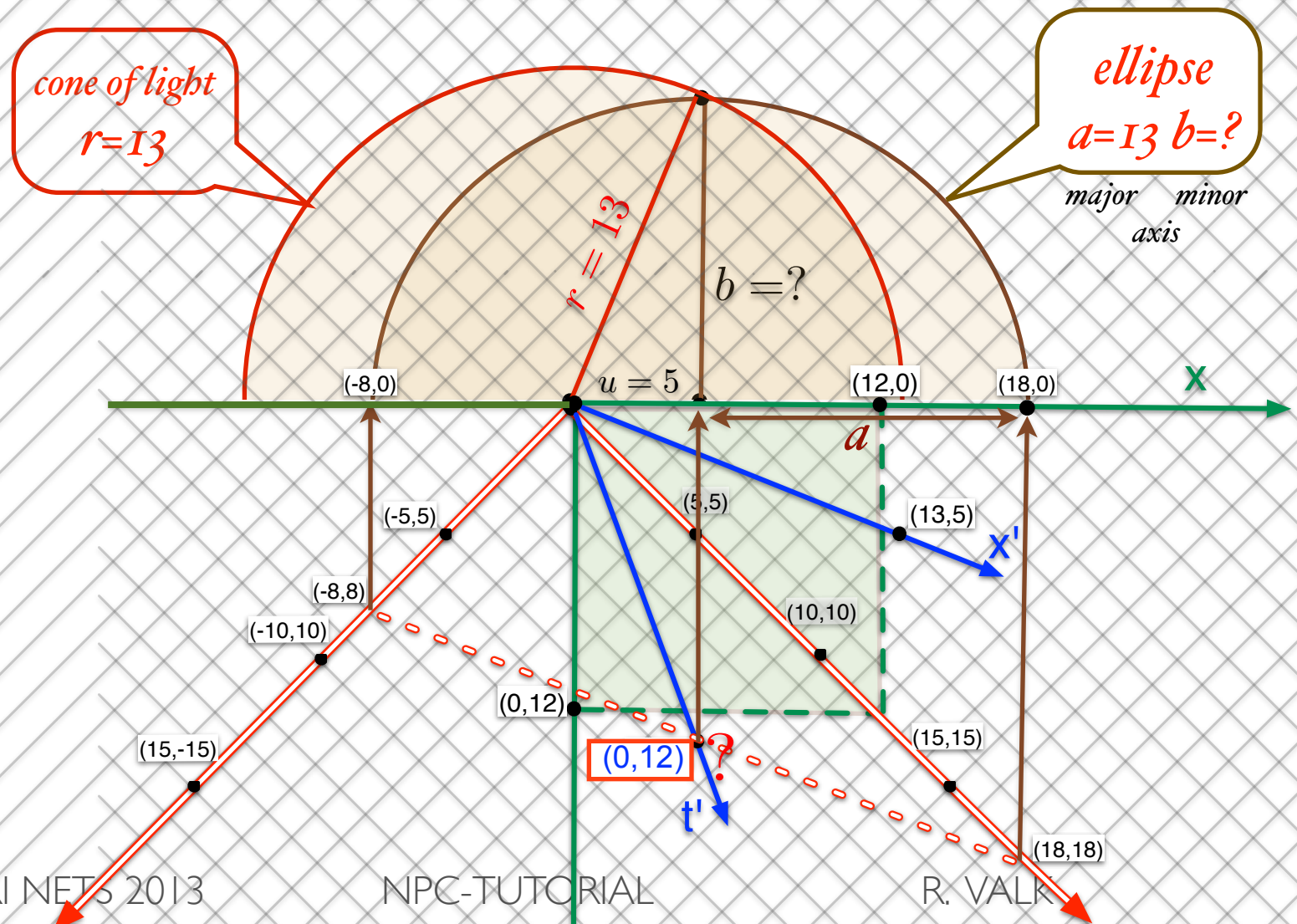
observer B observes the same speed of light


ellipse
a=13 b=?
major
minor
axis
a
b

$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$

$$(\alpha', \beta', \gamma', \delta') = (9, 4, 9, 4)$$

$$\boxed{b = \sqrt{r^2 - u^2} = \sqrt{13^2 - 5^2} = 12}$$



LT by tile-parameters

$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$

$$(\alpha', \beta', \gamma', \delta') = (9, 4, 9, 4)$$

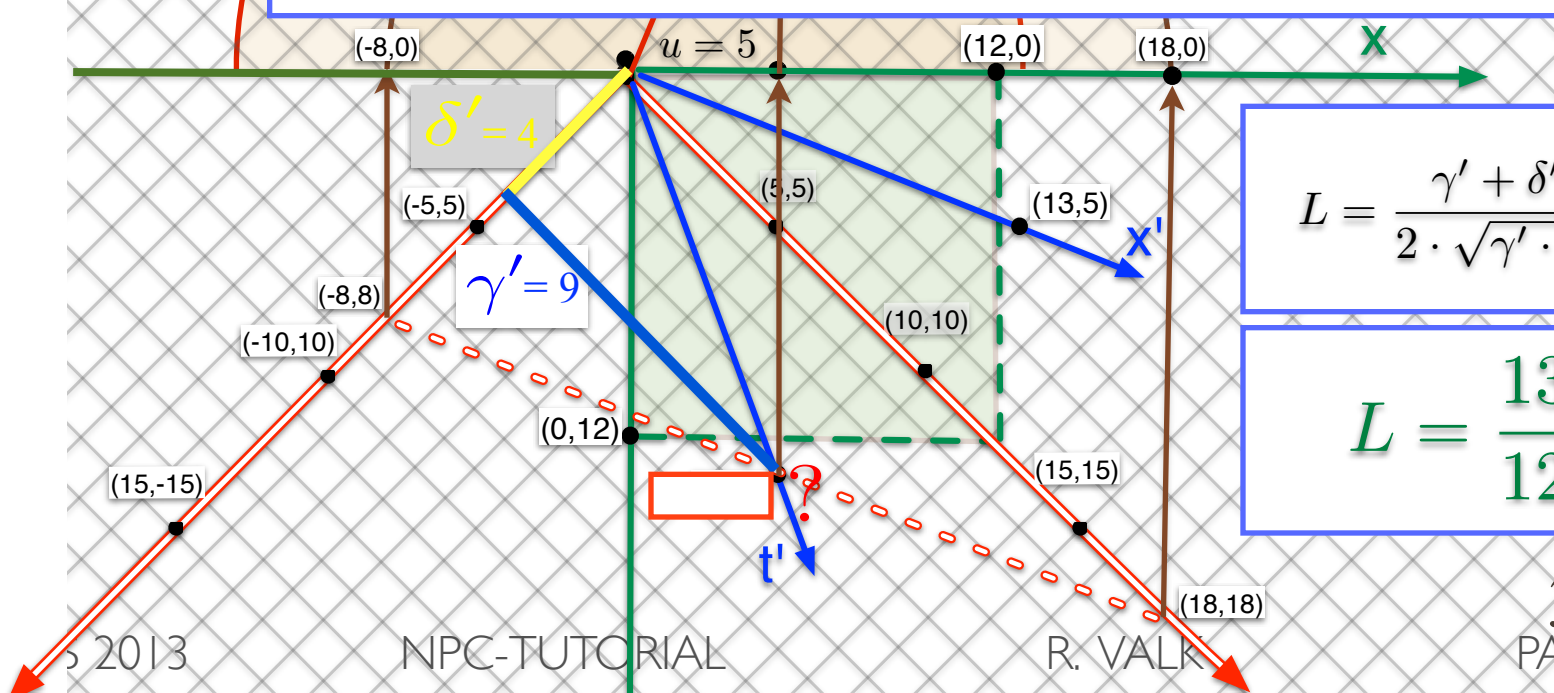
$$b = \sqrt{r^2 - u^2} = \sqrt{13^2 - 5^2} = 12$$

$$r = \gamma' + \delta' \qquad u = \gamma' - \delta'$$

$$b = \sqrt{(\gamma' + \delta')^2 - (\gamma' - \delta')^2}$$

$$= \sqrt{[(\gamma' + \delta') + (\gamma' - \delta')] \cdot [(\gamma' + \delta') - (\gamma' - \delta')]}$$

$$= \sqrt{2 \cdot \gamma' \cdot 2 \cdot \delta'} = 2 \cdot \sqrt{\gamma' \cdot \delta'} = 2 \cdot \sqrt{9 \cdot 4} = 12$$



$$L = \frac{\gamma' + \delta'}{2 \cdot \sqrt{\gamma' \cdot \delta'}}$$

$$L = \frac{13}{12}$$

»Wir zeichnen den Durchschnitt jener Schale mit der Ebene der x- und der t-Achse, den oberen Ast der Hyberbel $c^2t^2 - x^2 = 1$, mit seinen Asymptoten. Ferner werde ein beliebiger Radiusvektor OA' dieses Hyperbelastes vom Nullpunkt O aus eingetragen, die Tangente in A' an die Hyperbel bis zum Schnitte B' mit der Asymptote rechts gelegt, $OA'B'$ zum Parallelogramm $OA'B'C'$ vervollständigt, endlich für das spätere noch $B'C'$ bis zum Schnitt D' mit der x-Achse durchgeführt. Nehmen wir nun OC' und OA' als Achsen für die Parallelkoordinaten x', t' mit den Maßstäben $OC' = 1$, $OA' = 1/c$, so erlangt jener Hyperbelast wieder den Ausdruck $c^2t'^2 - x'^2 = 1$, $t' > 0$, und der Übergang von x, y, z, t zu x', y, z, t' ist eine der fraglichen Transformationen.«

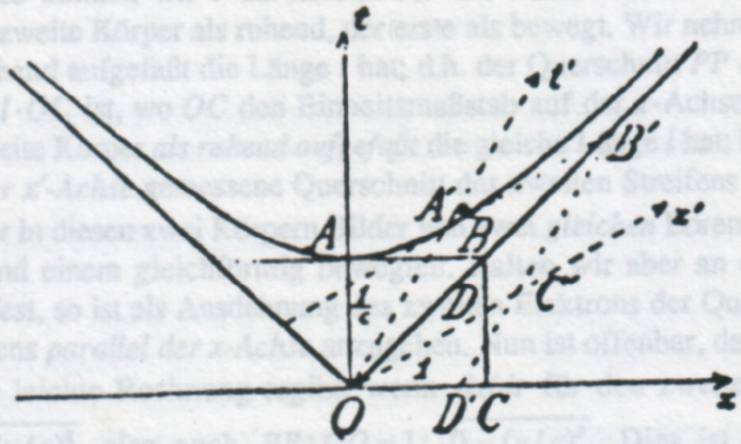
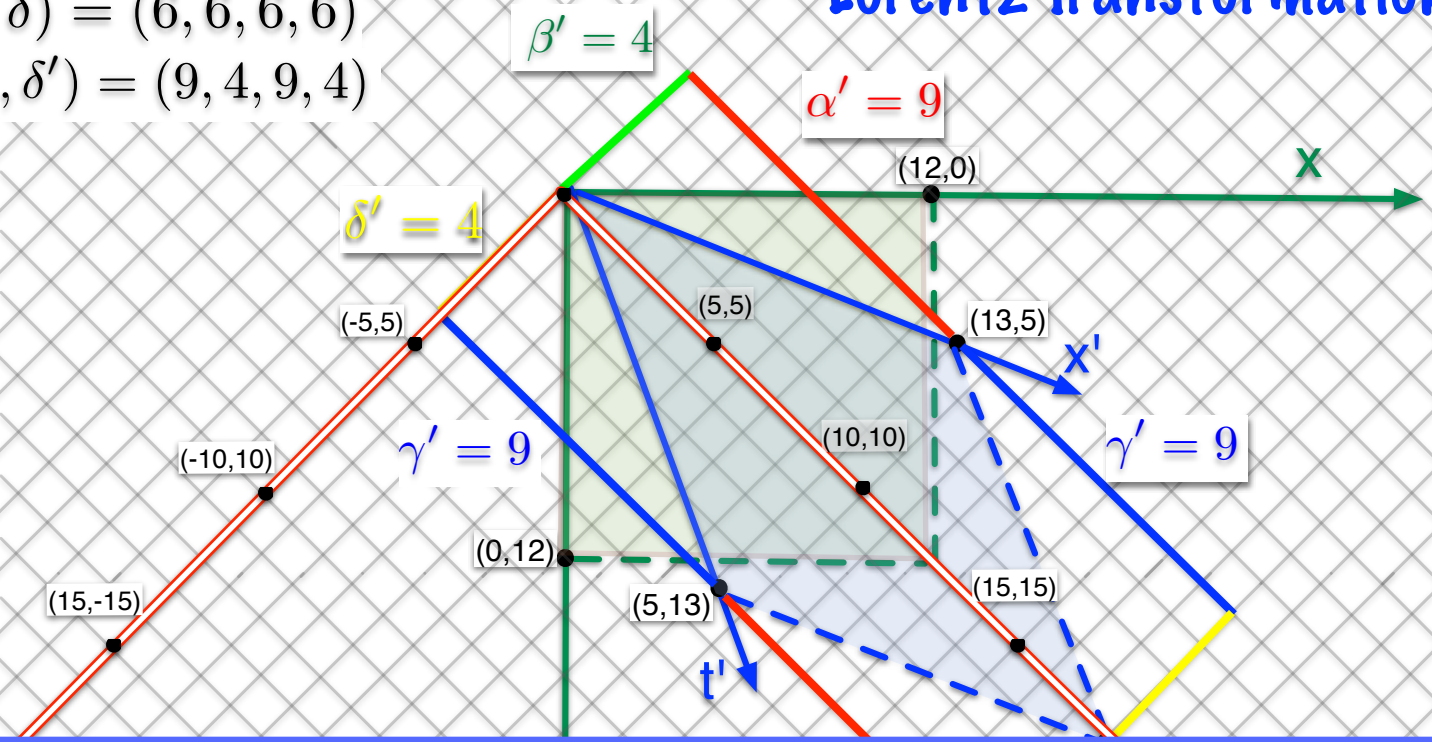


Abb. 1.2.1 Minkowskis Orginaldarstellung der Konstruktion zur Lorentz-Transformation

Hartmut Müller: Geschichte und Entwicklung der Concurrency-Theorie
Diplomarbeit 1993

Lorentz-Transformation (LT)

$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$
$$(\alpha', \beta', \gamma', \delta') = (9, 4, 9, 4)$$



$$x' = \frac{(x - vt)}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z \qquad t' = \frac{(t - \frac{vx}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz-Transformation

$$L = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = L(x - vt)$$

$$t' = L(t - \frac{v}{c^2} \cdot x) = L(t - w \cdot x)$$

slowness $w = \frac{v}{c^2}$

Lorentz-Transformation (LT)

$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$
$$(\alpha', \beta', \gamma', \delta') = (9, 4, 9, 4)$$

These results pertain also to macroscopic levels!

$$x' = \frac{(x - vt)}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z \qquad t' = \frac{(t - \frac{vx}{c^2})}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz-Transformation

$$L = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

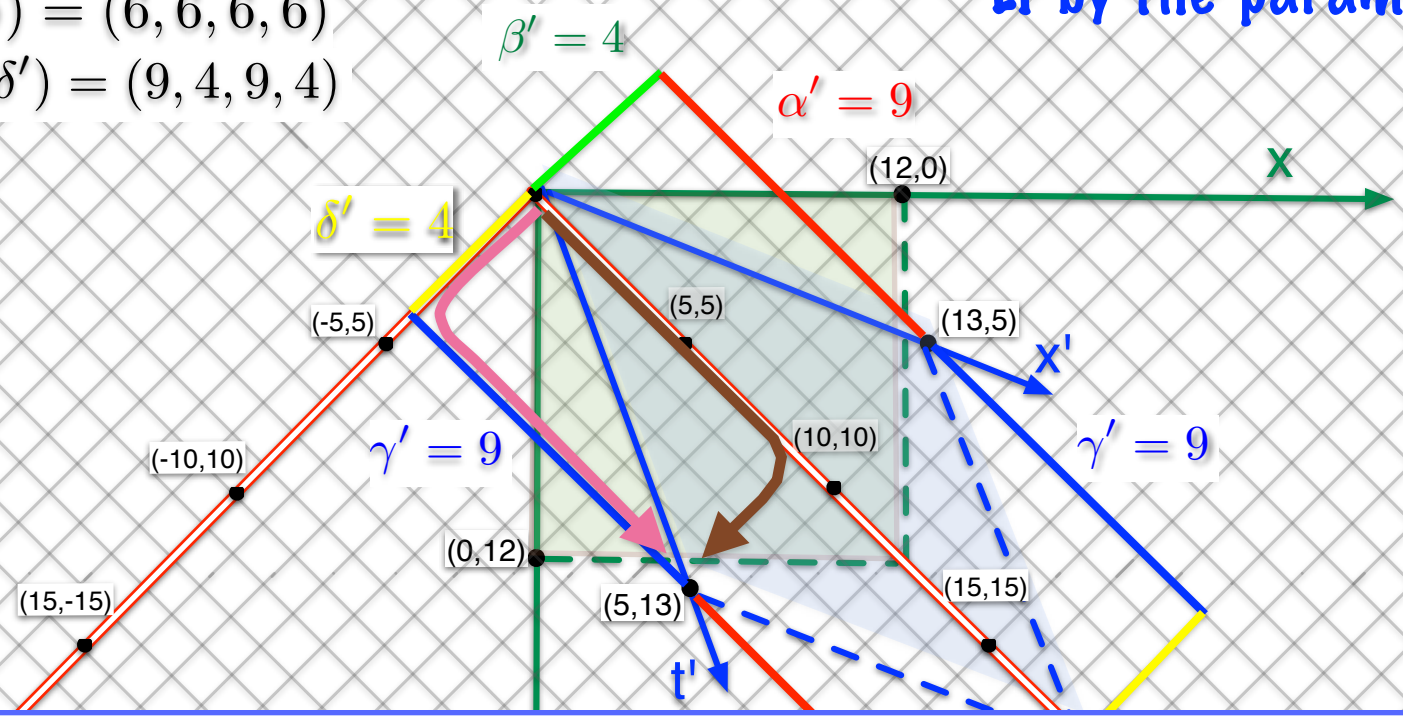
$$x' = L(x - vt)$$
$$t' = L(t - \frac{v}{c^2} \cdot x) = L(t - w \cdot x)$$

slowness $w = \frac{v}{c^2}$

no mention of c !

LT by tile-parameters

$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$
$$(\alpha', \beta', \gamma', \delta') = (9, 4, 9, 4)$$



$$\text{velocity} = \frac{displacement}{time\ \ taken} = v = \frac{\gamma' - \delta'}{\gamma' + \delta'} = \frac{9-4}{9+4} = \frac{5}{13} \quad \cdot c \quad [\frac{m}{sec}]$$

v is measured in meters (gained) per second

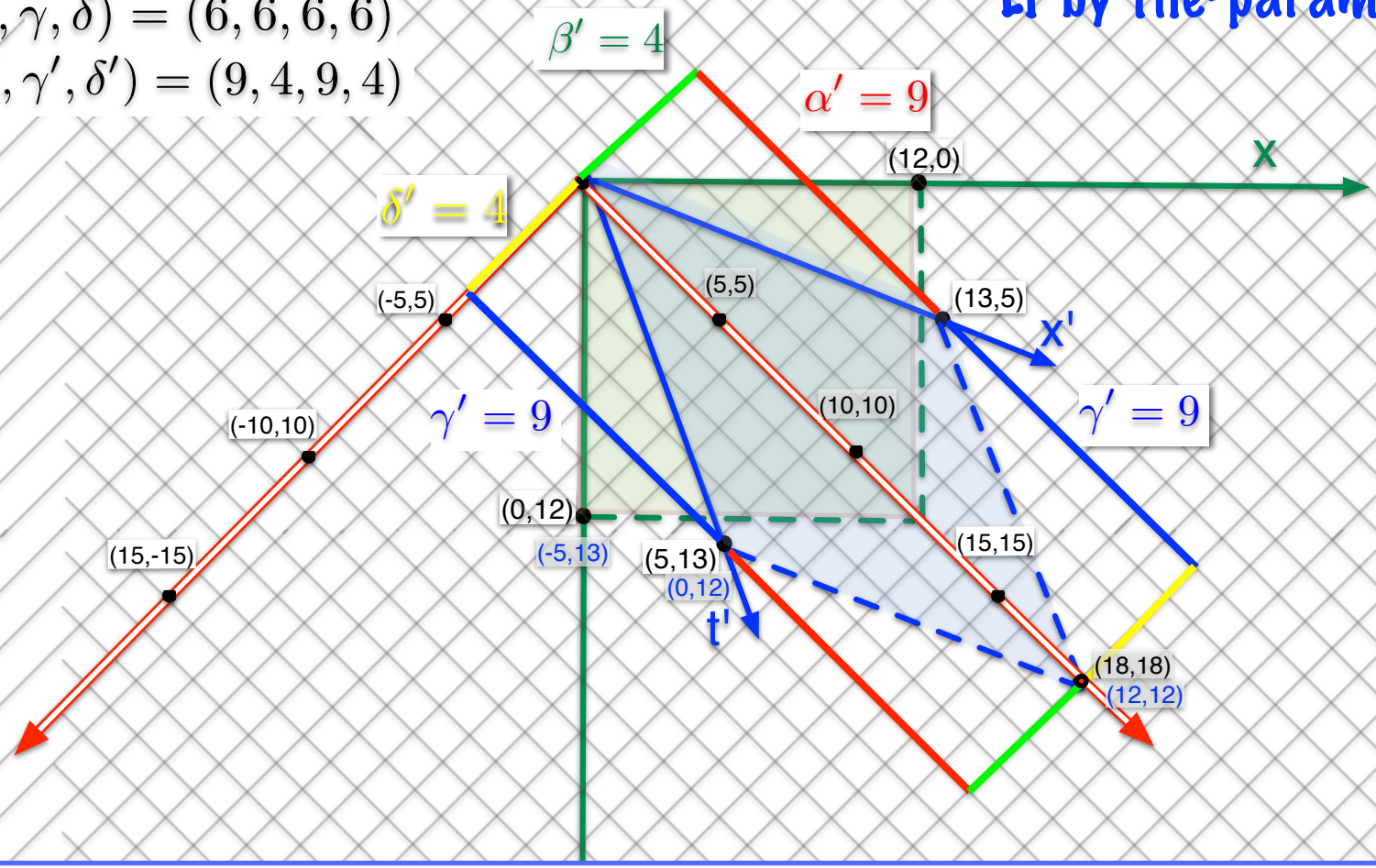
$$\text{slowness} = \frac{time\ \ lost}{displacement} = w = \frac{\alpha' - \beta'}{\alpha' + \beta'} = \frac{9-4}{9+4} = \frac{5}{13} \quad \cdot \frac{1}{c} \quad [\frac{sec}{m}]$$

w is measured in seconds (lost) per meter

$$L = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(\gamma' - \delta')^2}{(\gamma' + \delta')^2}}} = \frac{\gamma' + \delta'}{2} \cdot \frac{1}{\sqrt{\gamma' \cdot \delta'}} = \frac{9+4}{2} \cdot \frac{1}{\sqrt{9 \cdot 4}} = \frac{13}{2} \cdot \frac{1}{6} = \frac{13}{12}$$

LT by tile-parameters

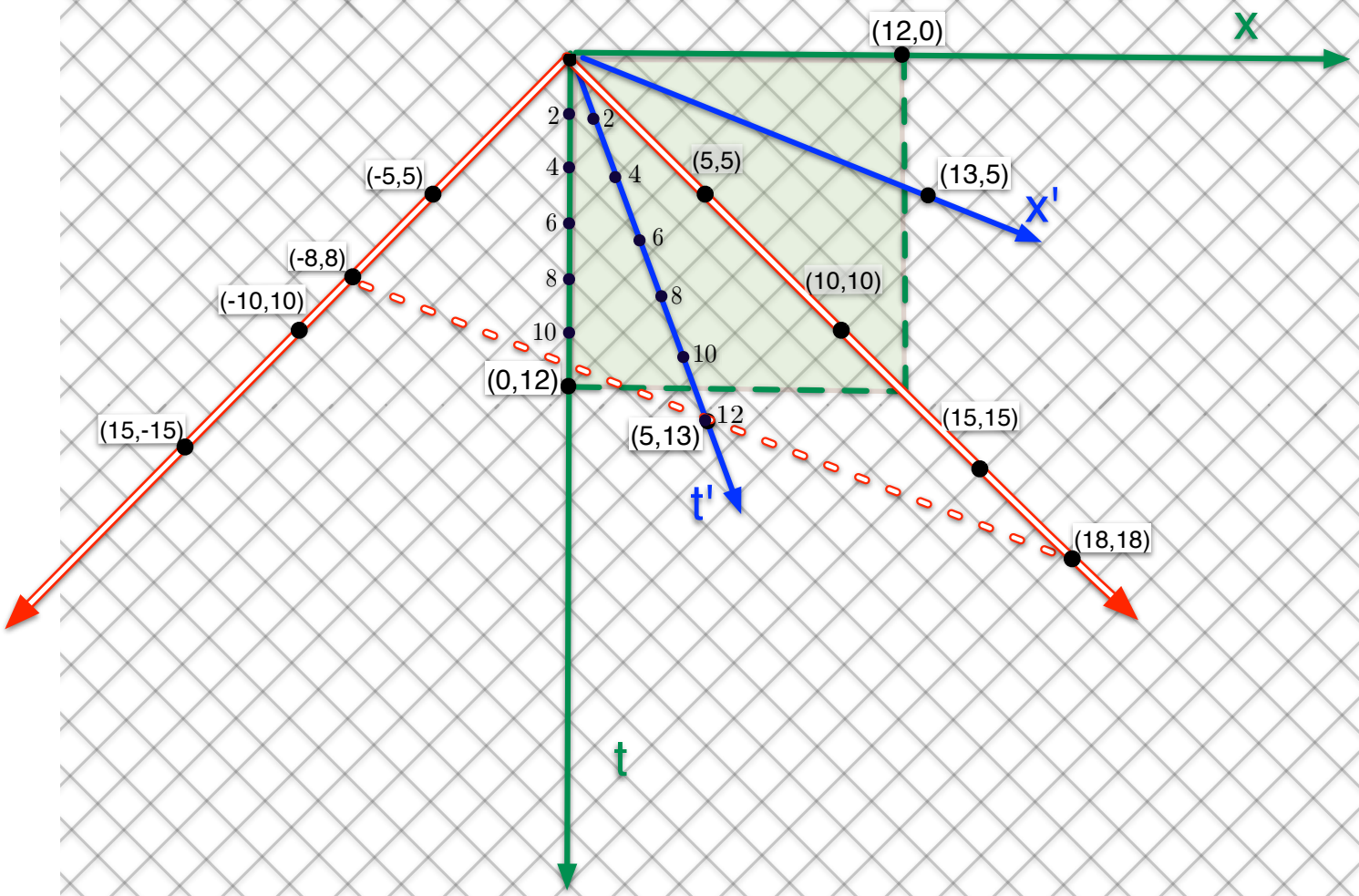
$$(\alpha, \beta, \gamma, \delta) = (6, 6, 6, 6)$$
$$(\alpha', \beta', \gamma', \delta') = (9, 4, 9, 4)$$



$$x' = L(x - vt)$$
$$t' = L(t - wx)$$

$$x' = \frac{13}{12}(x - \frac{5}{13}t) \qquad t' = \frac{13}{12}(t - \frac{5}{13}x)$$

$$(5, 13) \mapsto (0, 12) \qquad (0, 12) \mapsto (-5, 13) \qquad (18, 18) \mapsto (12, 12)$$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x'
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(18,18)
(-5,5)
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(15,-15)
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(5,13)
t'
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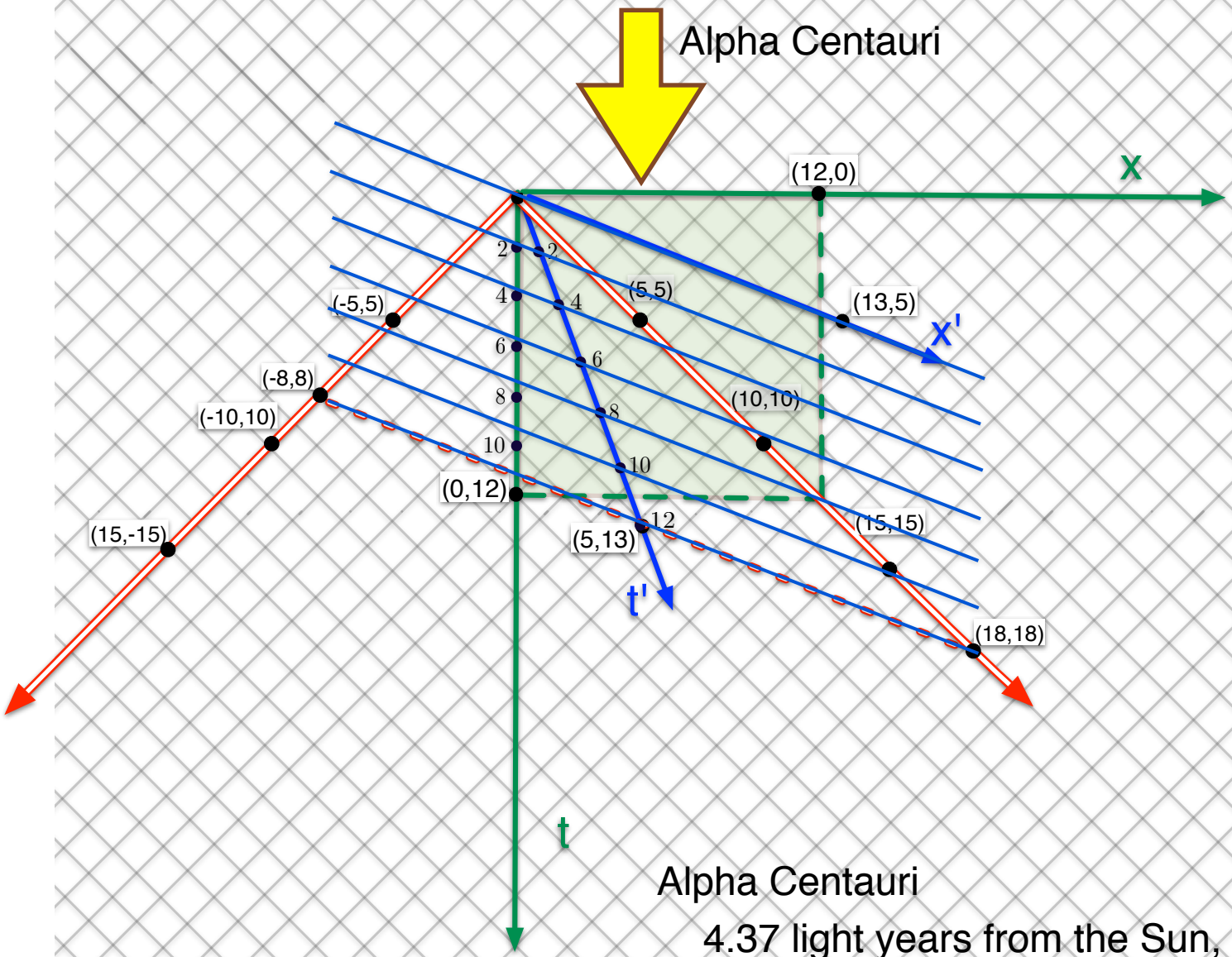
12

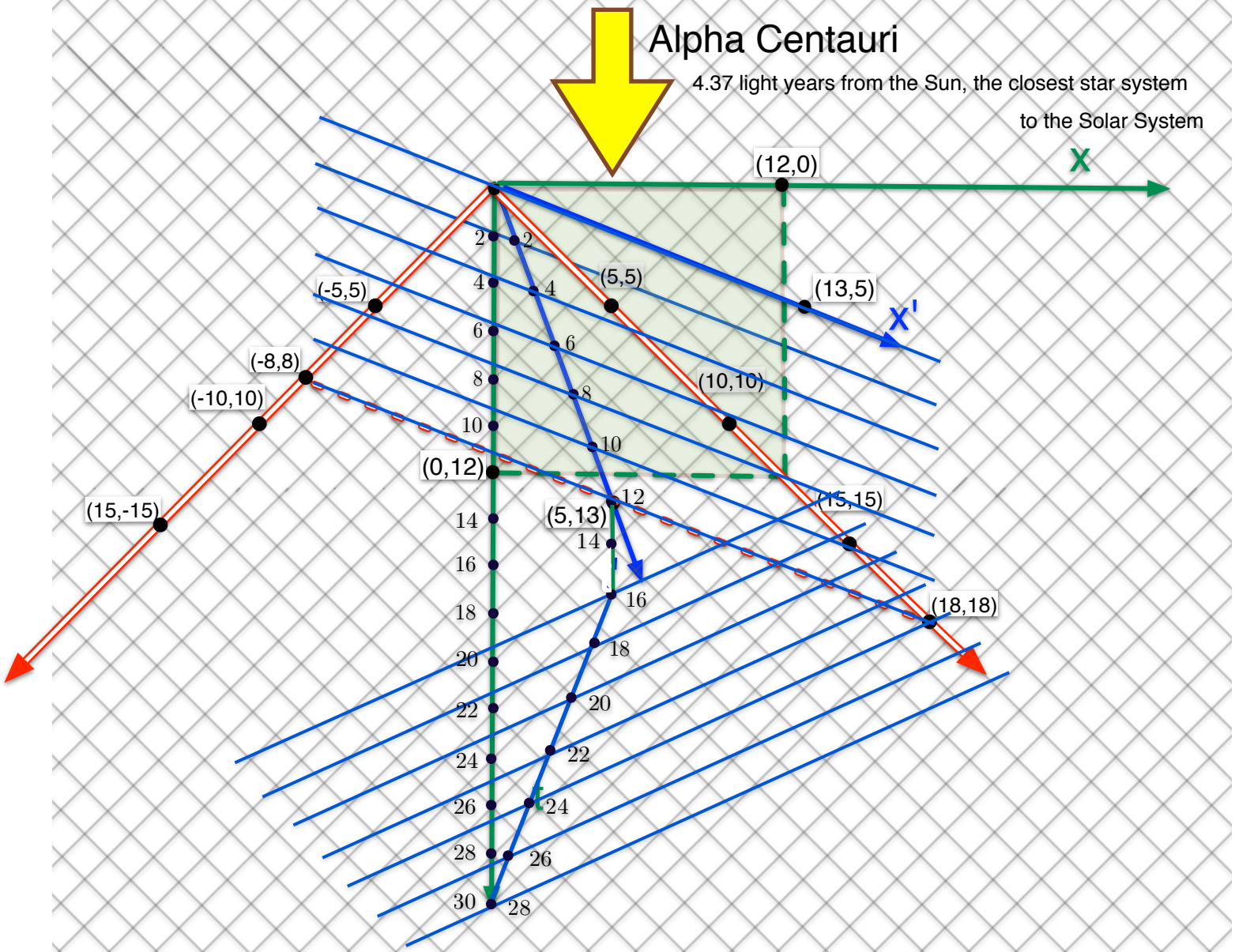


Alpha Centauri

4.37 light years from the Sun,

the closest star system

to the Solar System


Alpha Centauri
4.37 light years from the Sun, the closest star system
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X
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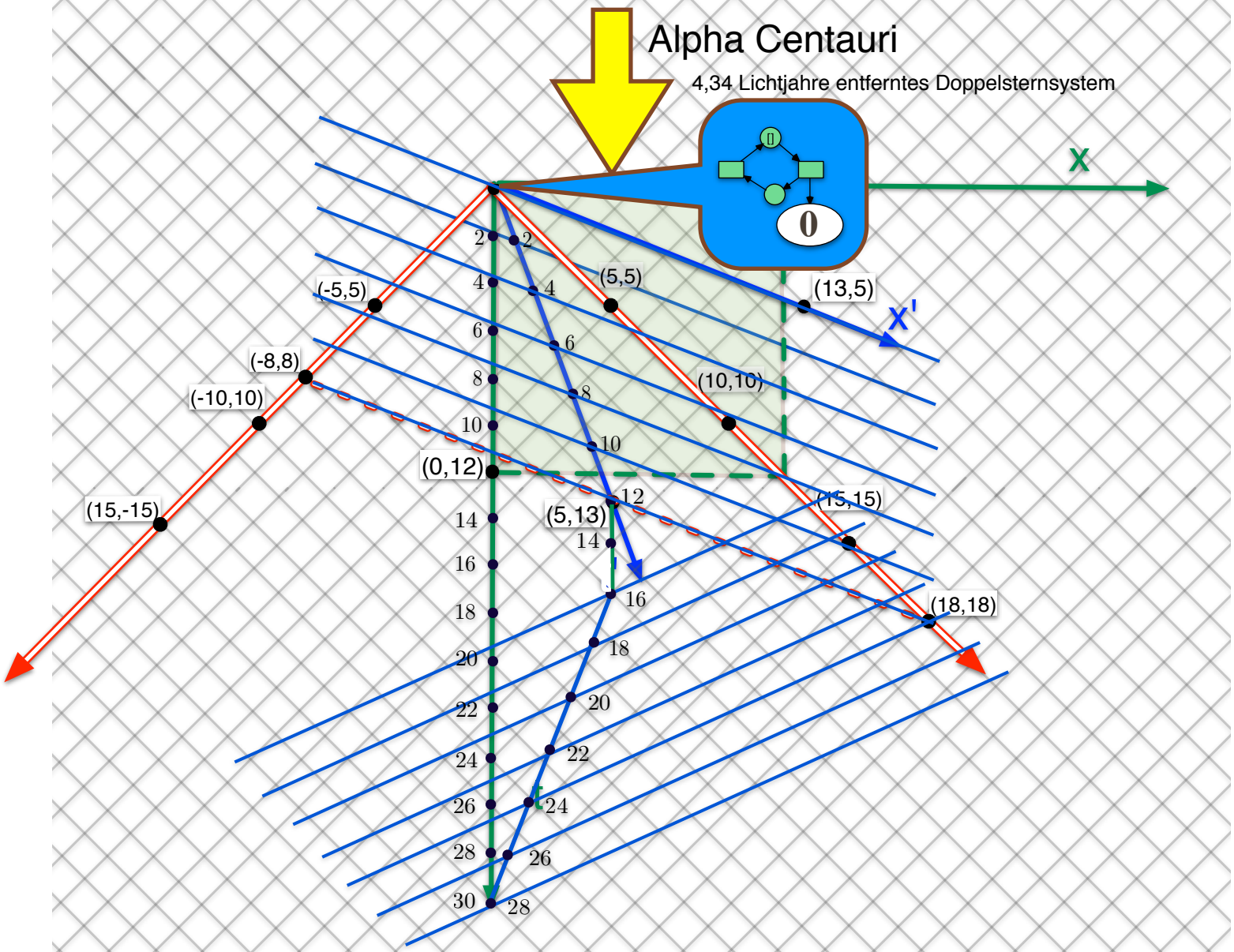
Alpha Centauri
4,34 Lichtjahre entferntes Doppelsternsystem
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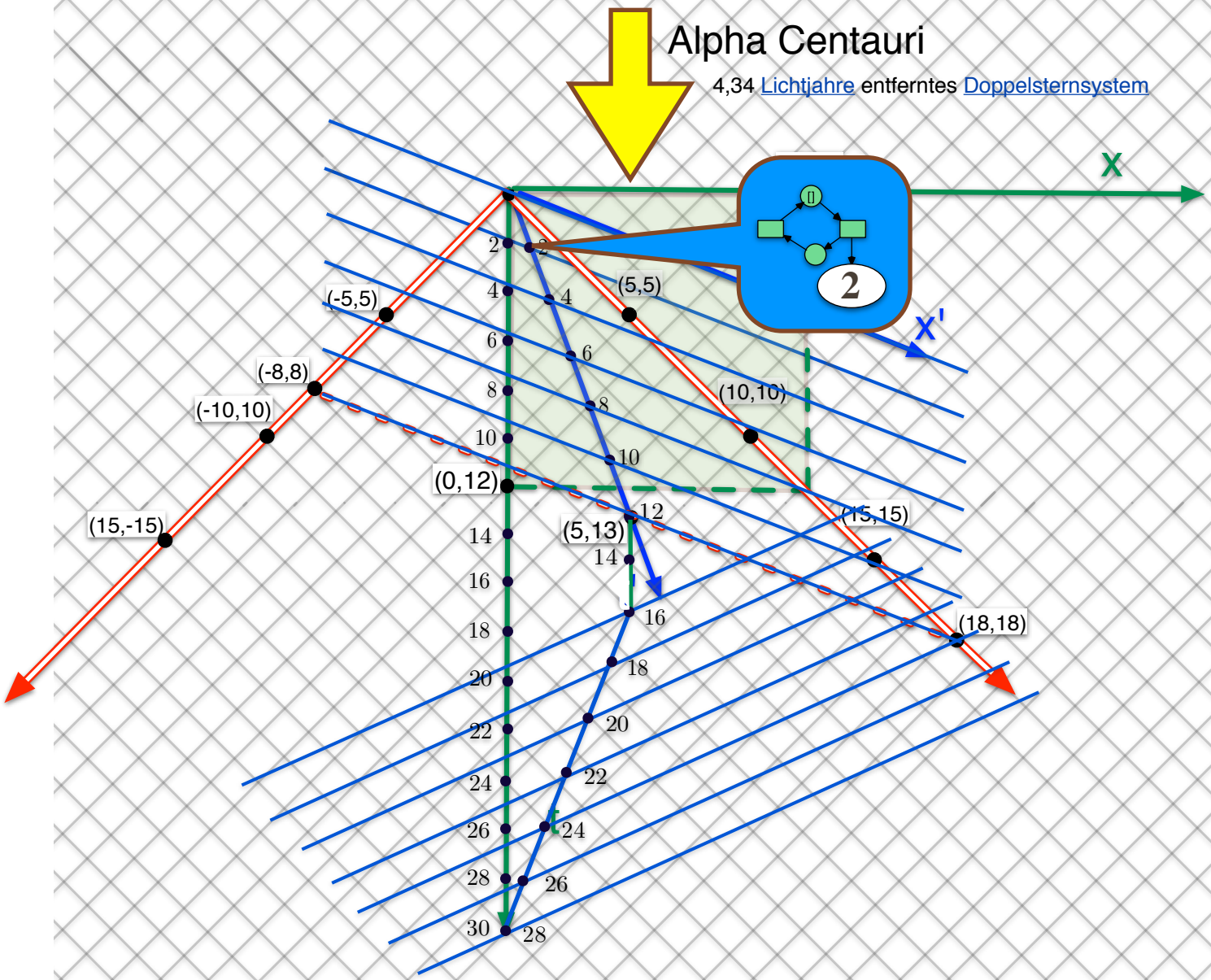
Alpha Centauri
4,34 Lichtjahre entferntes Doppelsternsystem
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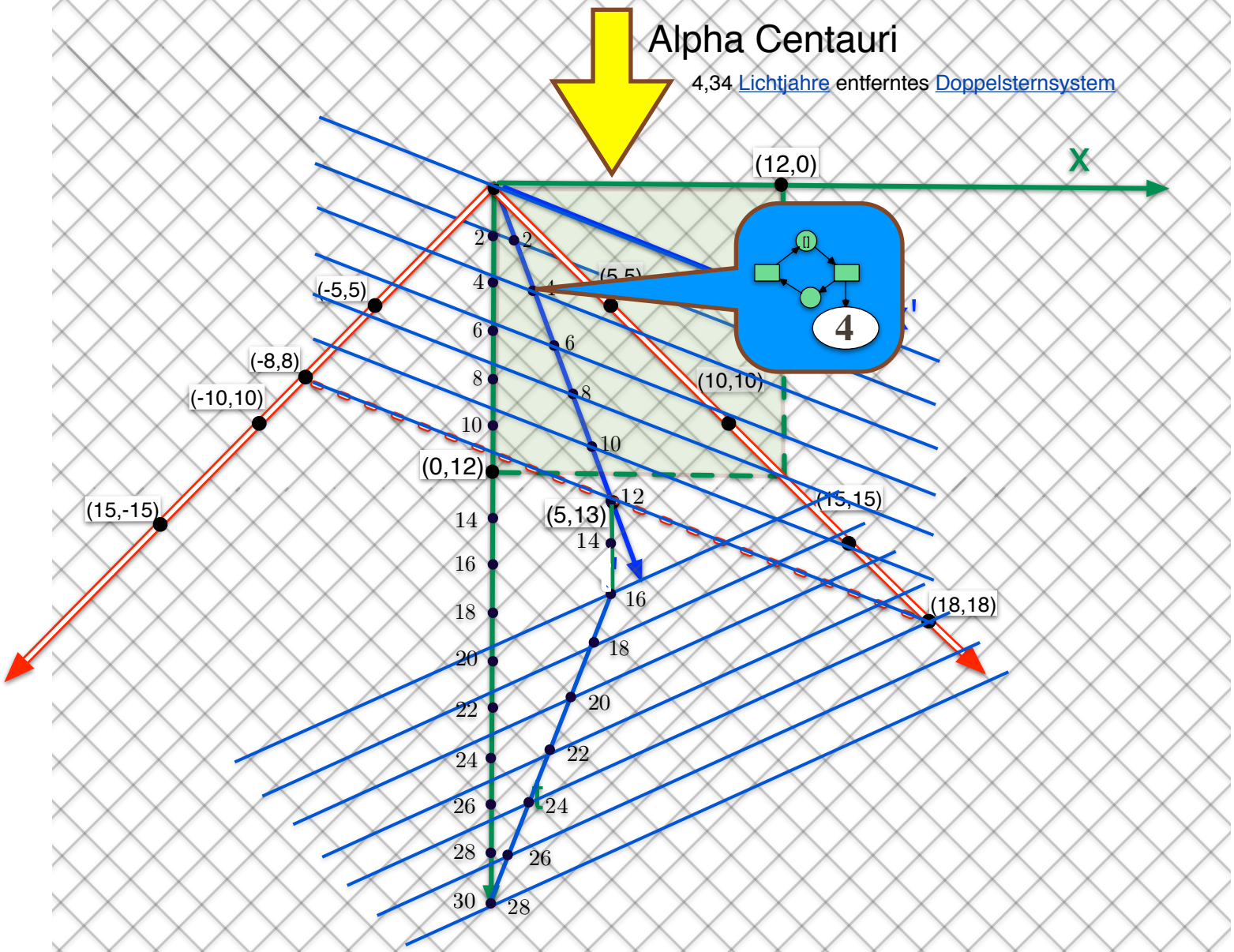
Alpha Centauri
4,34 Lichtjahre entferntes Doppelsternsystem
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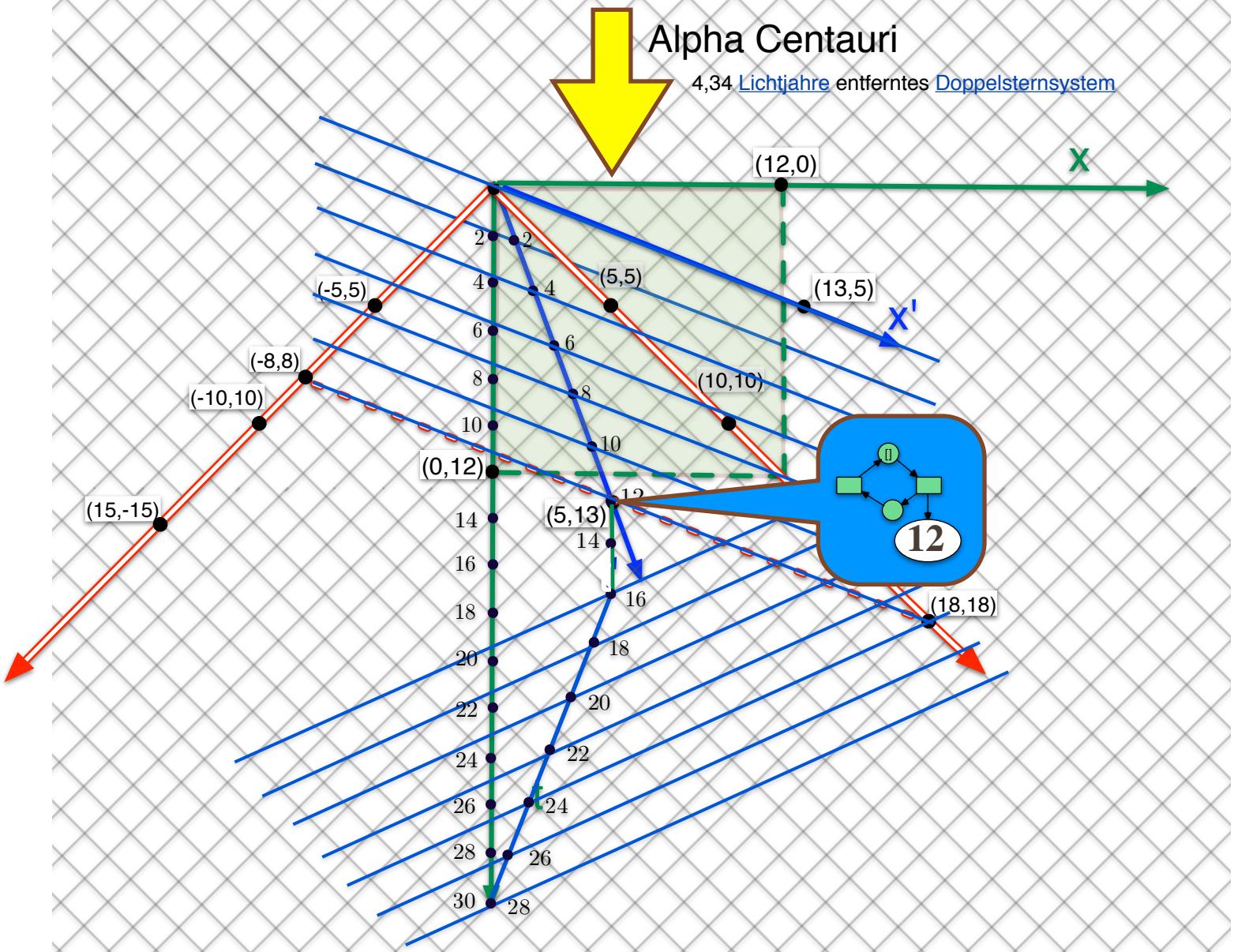
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4,34 Lichtjahre entferntes Doppelsternsystem
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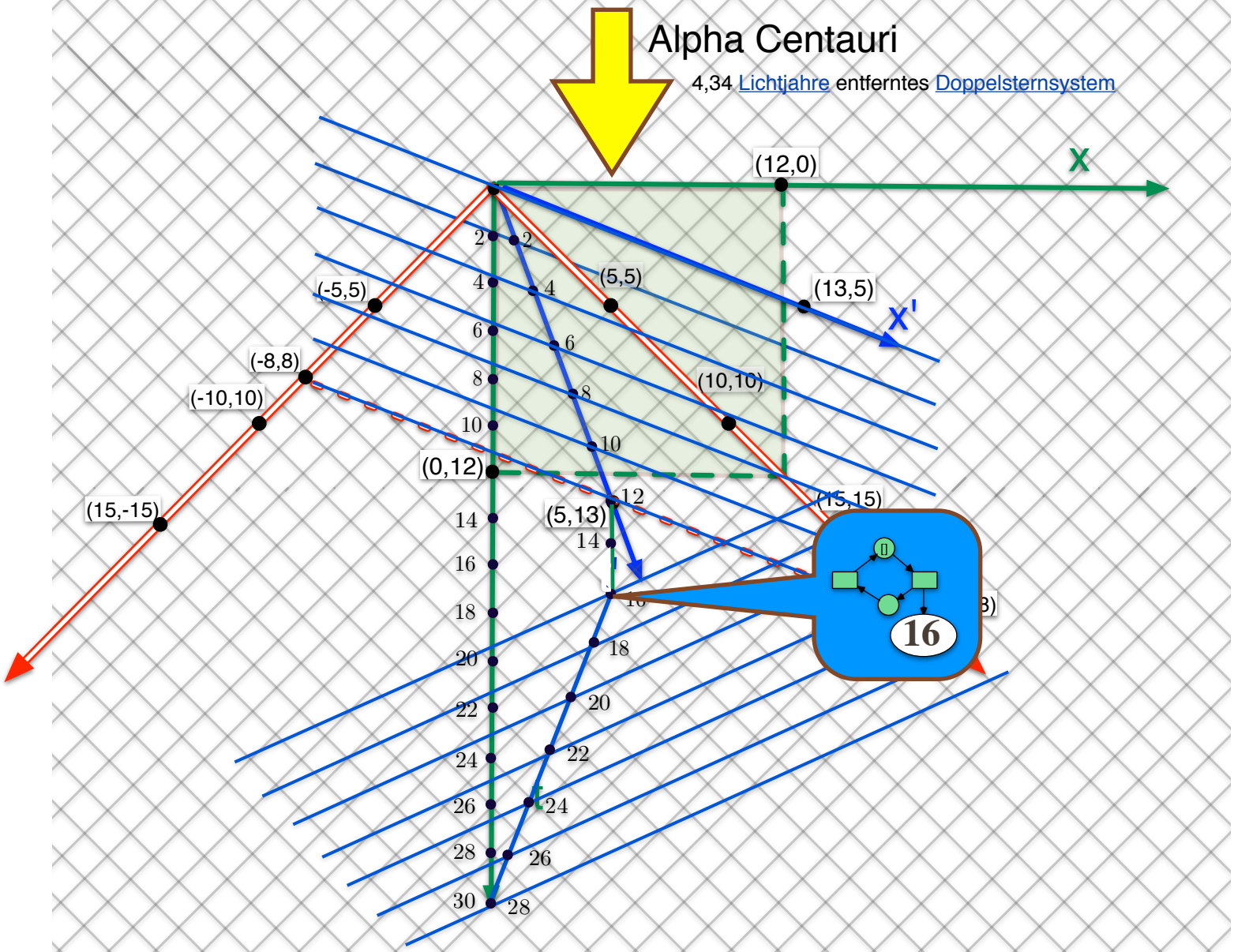
Alpha Centauri
4,34 Lichtjahre entferntes Doppelsternsystem
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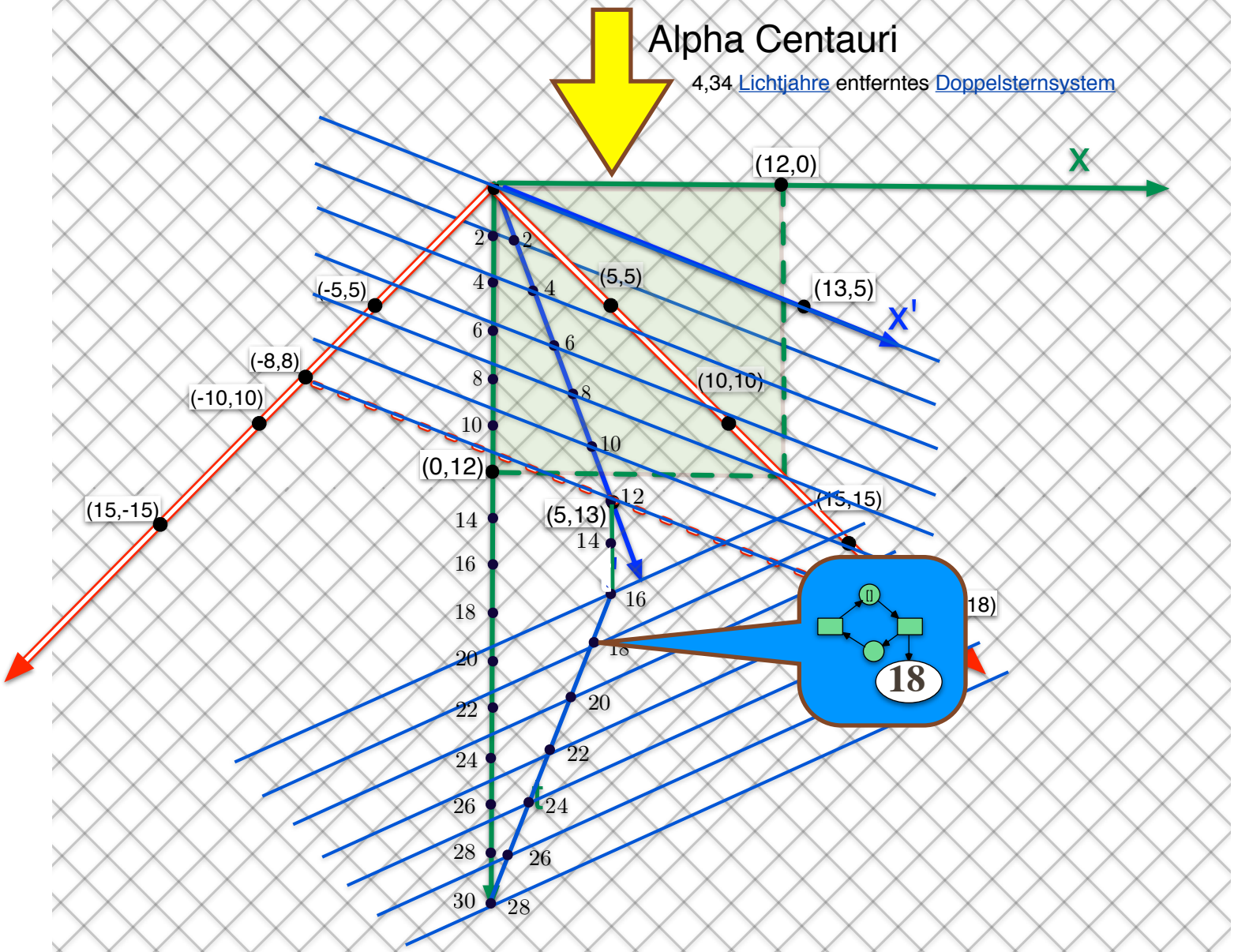
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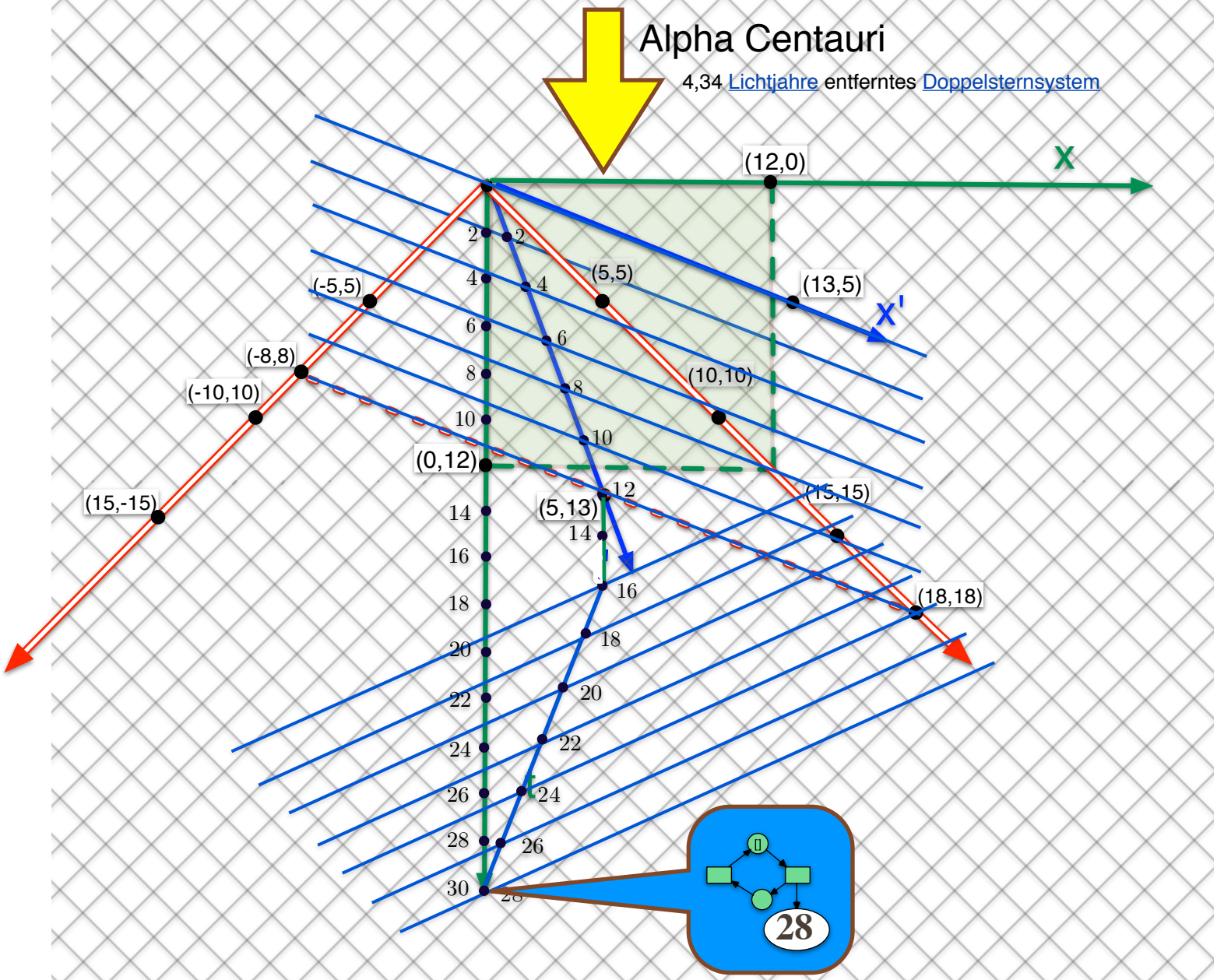
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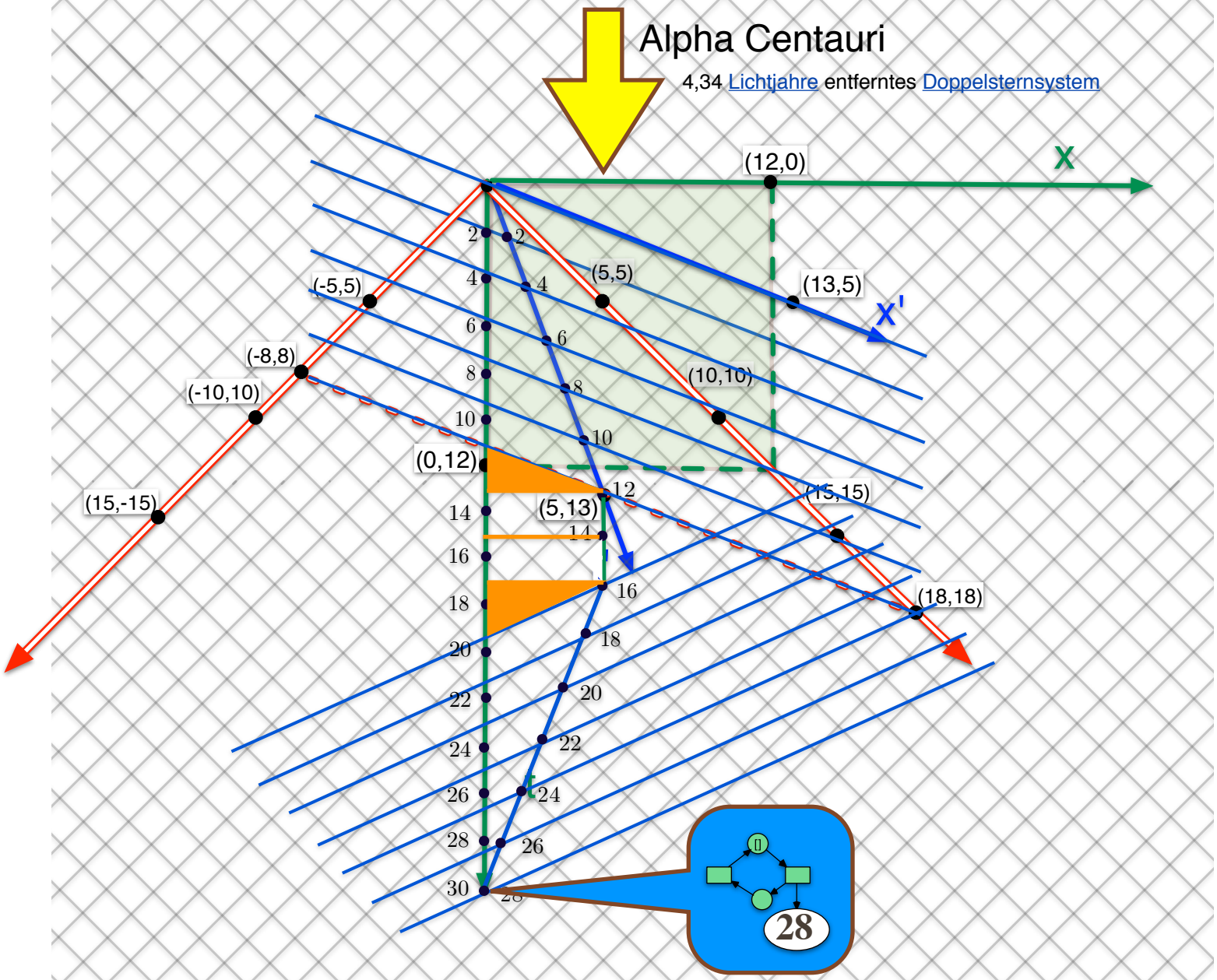
Alpha Centauri
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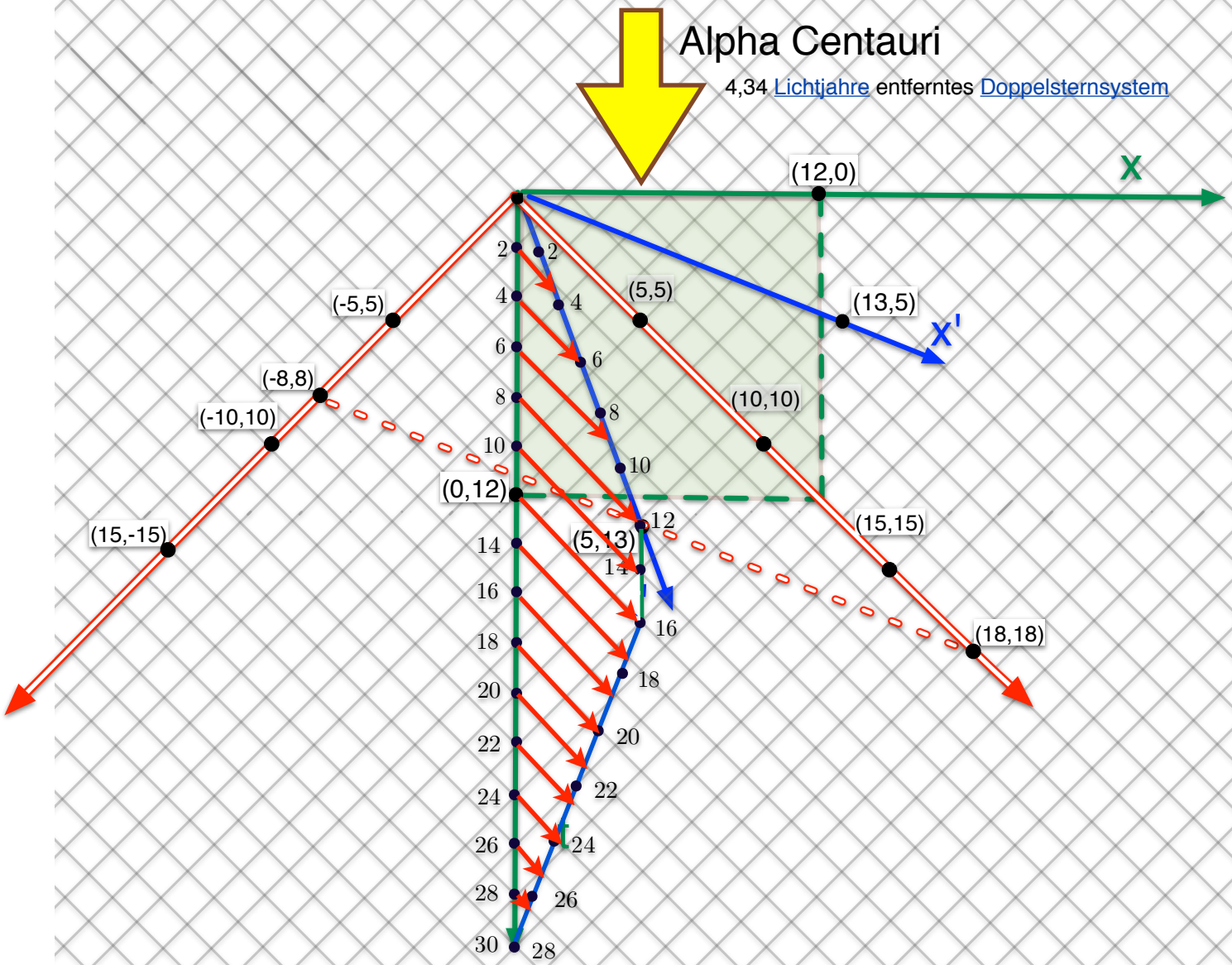
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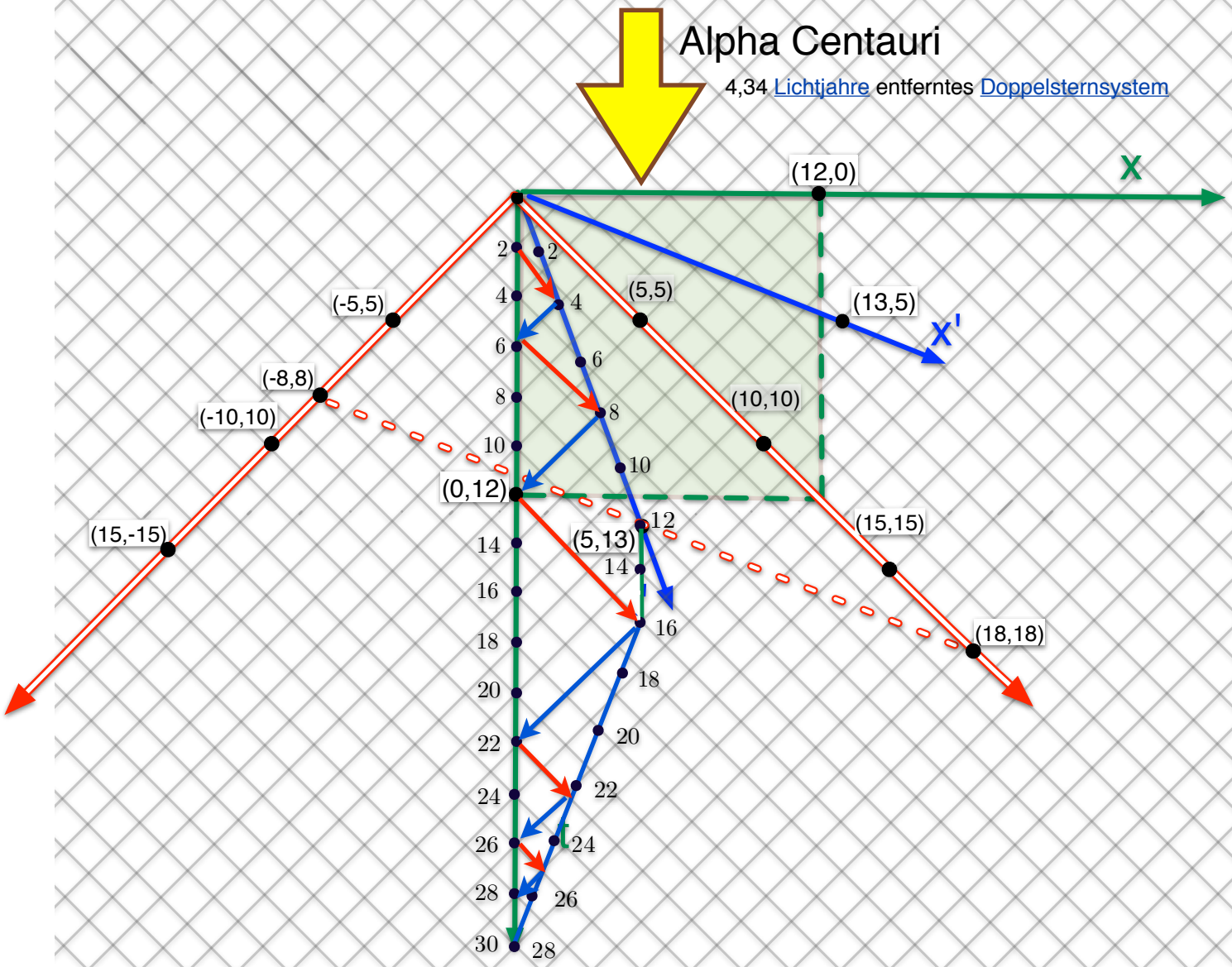
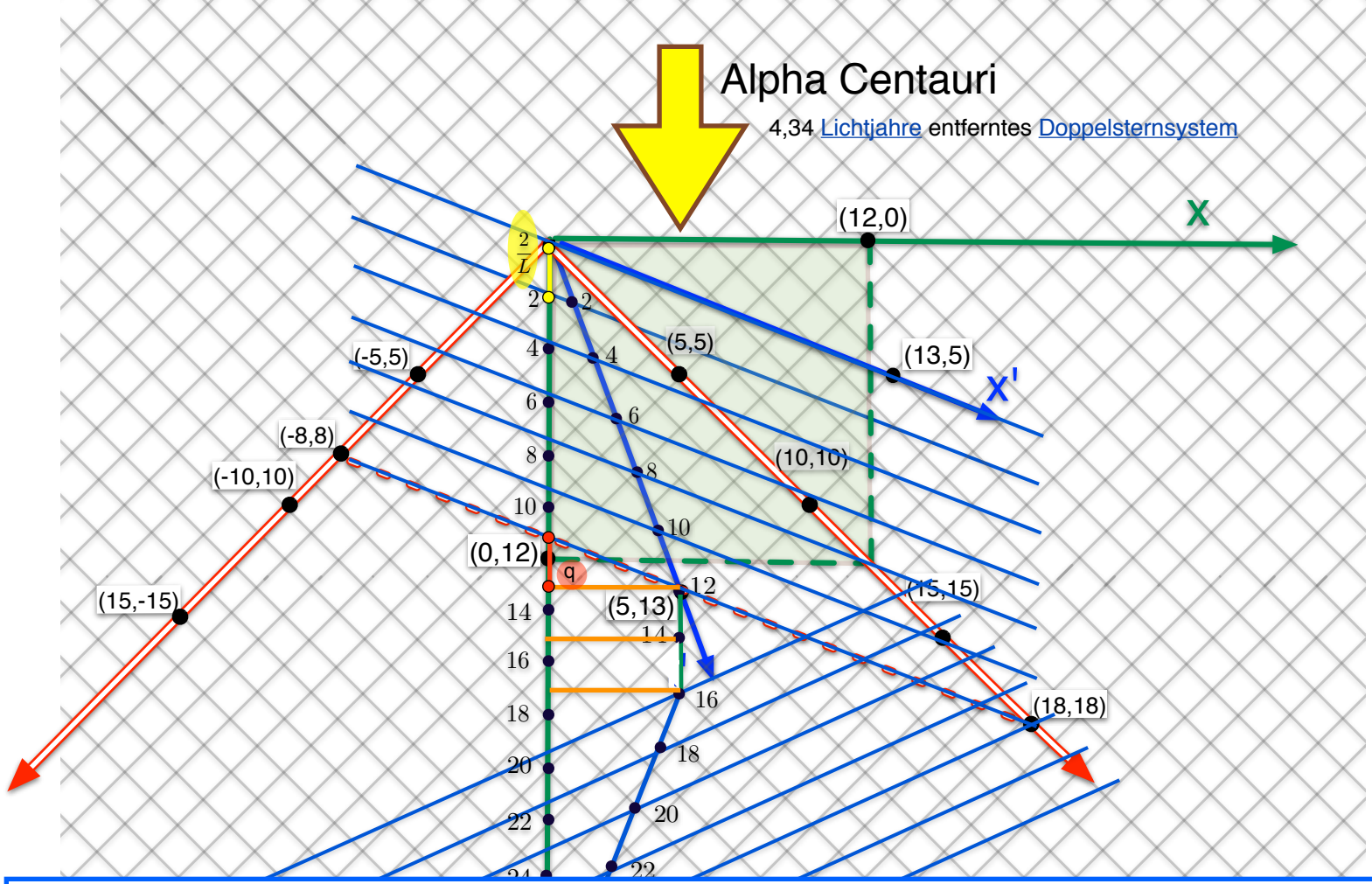
Alpha Centauri

4,34 Lichtjahre entferntes Doppelsternsystem

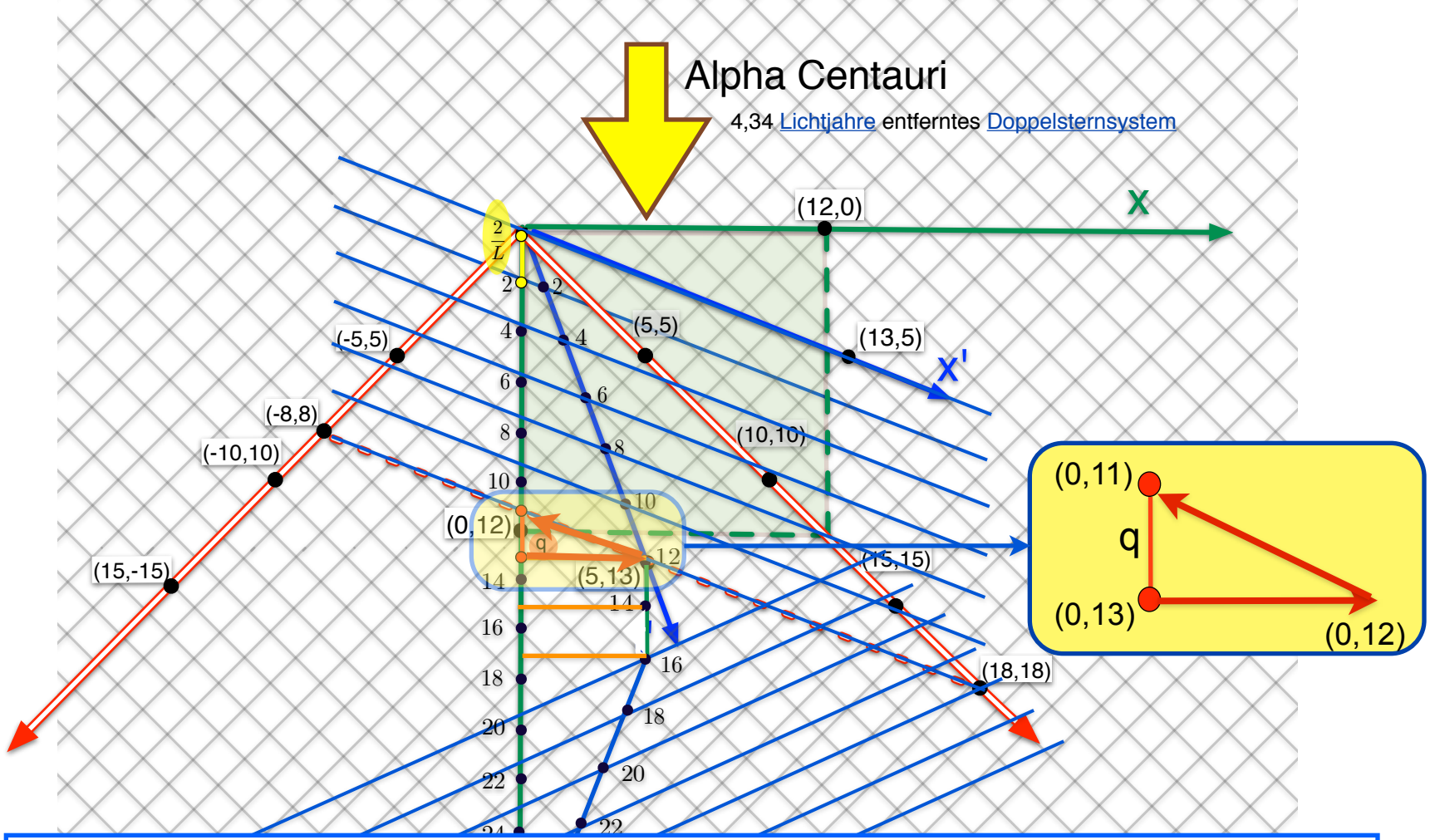




$$q = (\alpha+\beta) - \frac{1}{L}\cdot 2\sqrt{\alpha\cdot\beta} \quad = (\alpha+\beta) - \frac{2\sqrt{\alpha\cdot\beta}}{\alpha+\beta}\cdot 2\sqrt{\alpha\cdot\beta} \quad = (\alpha+\beta) - \frac{4\cdot\alpha\cdot\beta}{\alpha+\beta}$$

$$= \frac{(\alpha+\beta)^2 - 4\cdot\alpha\cdot\beta}{\alpha+\beta} \quad = \frac{(\alpha-\beta)^2}{\alpha+\beta} \quad = \frac{5^2}{13} = 1,9230\ldots$$

$$\text{slowdown ratio} = \quad \frac{q}{\alpha+\beta} = \frac{(\alpha-\beta)^2}{(\alpha+\beta)^2} \quad = w^2$$



$$q = (\alpha+\beta) - \frac{1}{L}\cdot 2\sqrt{\alpha\cdot\beta} \quad = (\alpha+\beta) - \frac{2\sqrt{\alpha\cdot\beta}}{\alpha+\beta}\cdot 2\sqrt{\alpha\cdot\beta} \quad = (\alpha+\beta) - \frac{4\cdot\alpha\cdot\beta}{\alpha+\beta}$$

$$= \frac{(\alpha+\beta)^2 - 4\cdot\alpha\cdot\beta}{\alpha+\beta} \quad = \frac{(\alpha-\beta)^2}{\alpha+\beta} \quad = \frac{5^2}{13} = 1,9230\ldots$$

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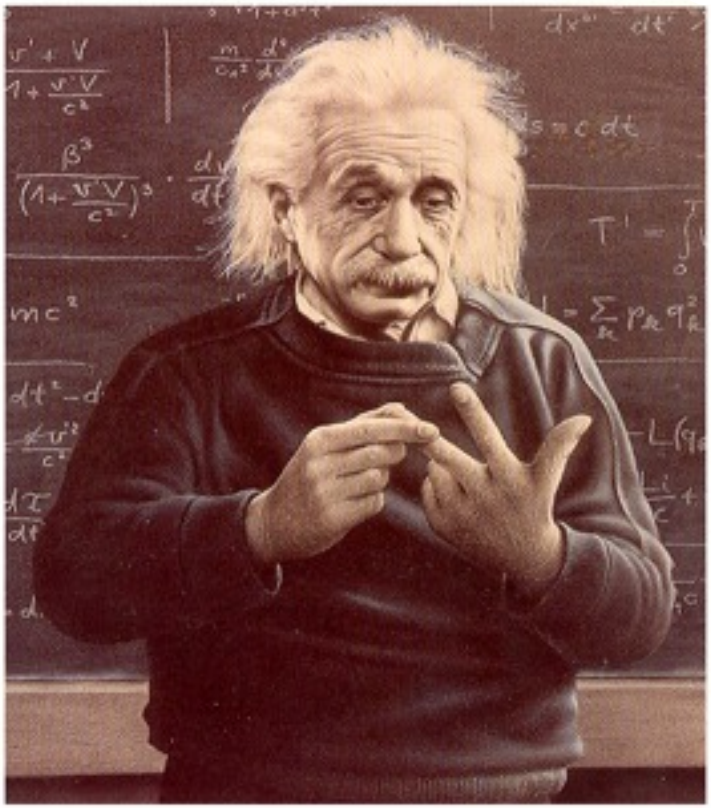
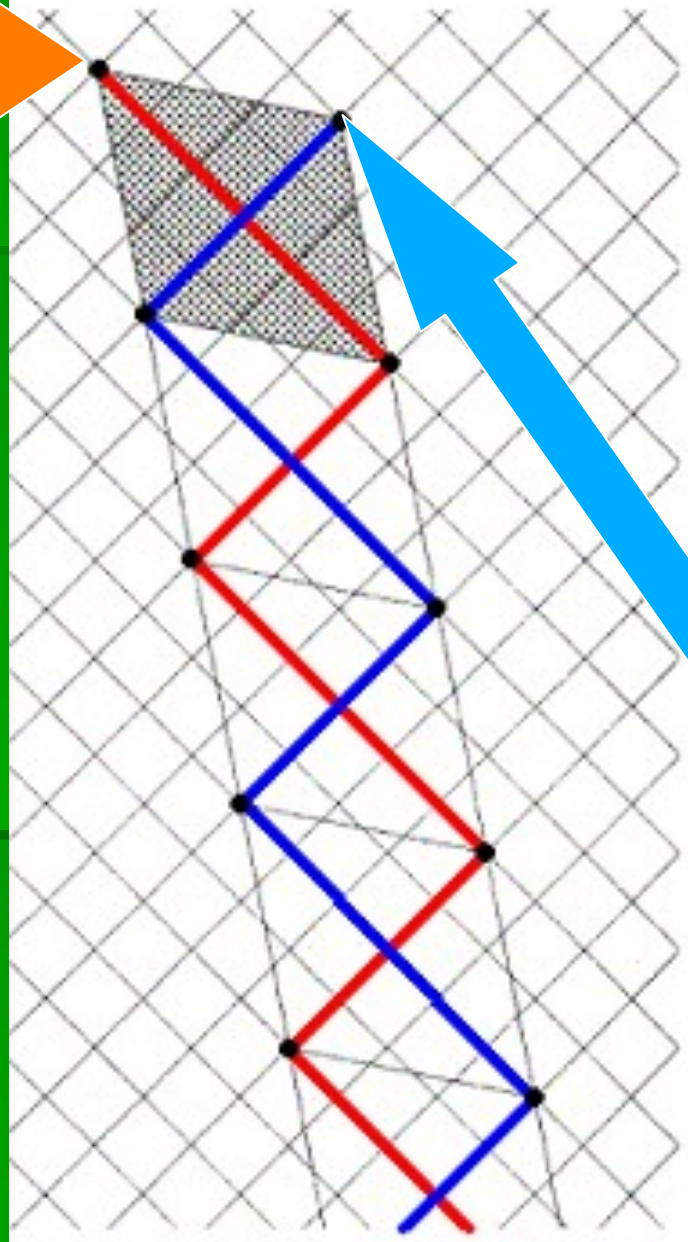
Einstein's Regimen



"Time is Number; and Number is Time"
Aristoteles



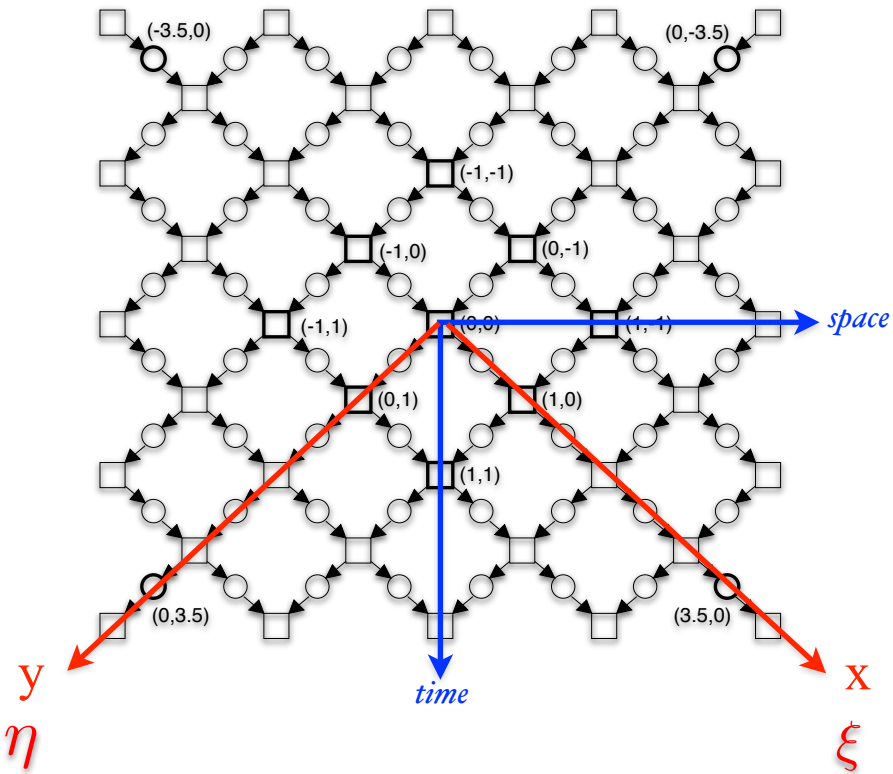
Two Clocks, travelling at a fixed distance, sending signals to each other in every time unit

Einstein's Regimen

We see the sun's light, and measure its speed c carefully. Now we fly towards the sun, and measure the new speed c'. Against our expectations, we find c = c' ! What happened?

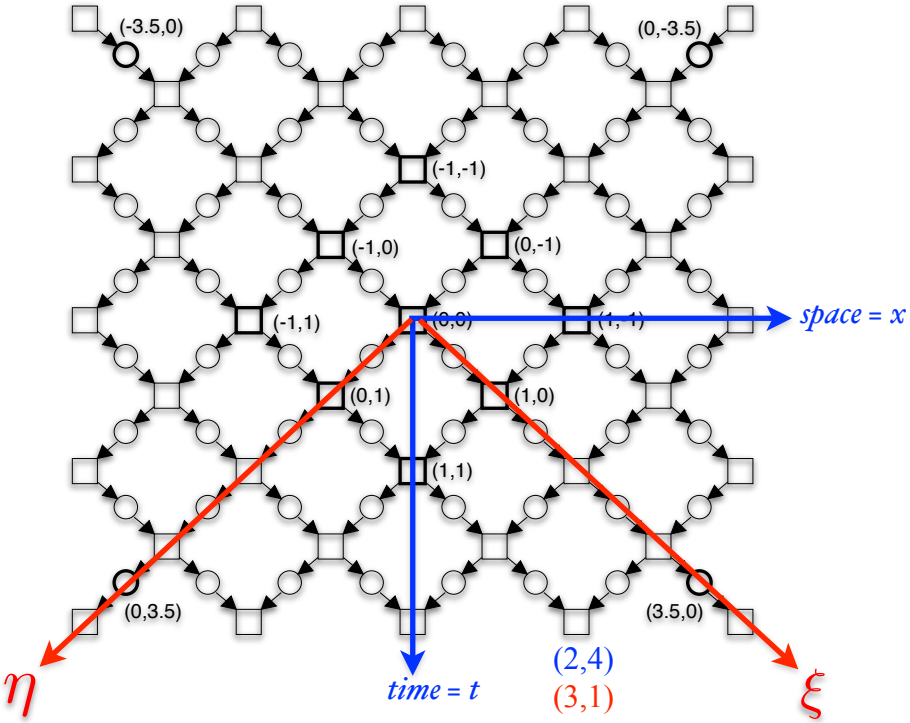
The invariance of c shakes the belief in our intuitive use of clocks. Are we sure that the two clocks at the front and back of the measurement device are still synchronous in flight? How can we check? Einstein proposed a strict regimen for that fundamental test; it is COMBINATORIAL rather than metrical: we have to send Light Signals forth and back, to establish Synchrony :

Formal Definition of Petri's Standard Grid



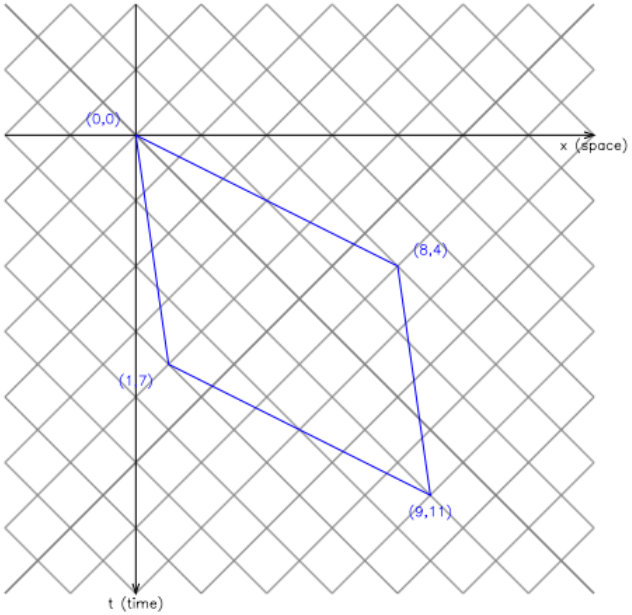
$$
\begin{aligned}
N_{SG} &:= (S_{SG}, T_{SG}, F_{SG}) \\
S_{SG} &:= \{(x, y+\tfrac{1}{2}) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \cup \\
&\quad\ \{(x+\tfrac{1}{2}, y) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \\
T_{SG} &:= \{(x, y) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \\
F_{SG} &:= \{((x,y),(x,y+\tfrac{1}{2})) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \cup \\
&\quad\ \{((x,y),(x+\tfrac{1}{2},y)) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \cup \\
&\quad\ \{((x,y-\tfrac{1}{2}),(x,y)) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \cup \\
&\quad\ \{((x-\tfrac{1}{2},y),(x,y)) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z}\} \\
X_{SG} &:= S_{SG} \cup T_{SG} \\
li_{SG} &:= {F_{SG}}^{+} \cup (F_{SG}^{-1})^{+} \\
co_{SG} &:= X_{SG} \times X_{SG} - li_{SG} - \mathrm{id}_{X_{SG}}
\end{aligned}
$$

Olaf Kummer: Axiomatic Systems in Concurrency Theory,
Logos Verlag Berlin, 2001



$$x = \xi - \eta$$

$$t = \xi + \eta$$



app by Peter Langner:

http://cycloids.adventas.de

Choose predefined cycloids: Bit-pair-Equality

Cycloid = α: 6 β: 2 γ: 4 δ: 3 Show Cycloid

- [x] Show Standard Grid
- [x] Show (x,t) coordinates of Cycloid
- [x] Show (x,t) axis, with $x = \xi - \eta$ and $t = \xi + \eta$
- [ ] Show *Minkowski* coordinates (ξ,η) of Cycloid
- [ ] Show *Minkowski* (ξ,η) axis
- [ ] Show $\alpha, \beta, \gamma, \delta$, with $\alpha = +\xi_1$, $\beta = -\eta_1$, $\gamma = +\xi_2$ and $\delta = +\eta_2$

- [x] Use MathML to show formulae

It's a cycloid ($\alpha \geq 2 \wedge \beta \geq 2 \wedge \gamma \geq 2 \wedge \delta \geq 2$). They are live and secure nets, which are made from 4-meshes.

Lorenz Factor $L=\frac{(\gamma+\delta)/2}{\sqrt{\gamma\cdot\delta}}=\frac{(4+3)/2}{\sqrt{4\cdot 3}}=1.0103629710818451$

Velocity $v=\frac{(\gamma - \delta)}{(\gamma + \delta)}=\frac{(4 - 3)}{(4 + 3)}=\frac{1}{7}=0.14285714285714285c$, and

Slowness $w=\frac{(\alpha - \beta)}{(\alpha + \beta)}=\frac{(6 - 2)}{(6 + 2)}=\frac{4}{8}=0.5\frac{1}{c}$

Area $A=\alpha\delta+\beta\gamma=6\cdot 3+2\cdot 4=26$

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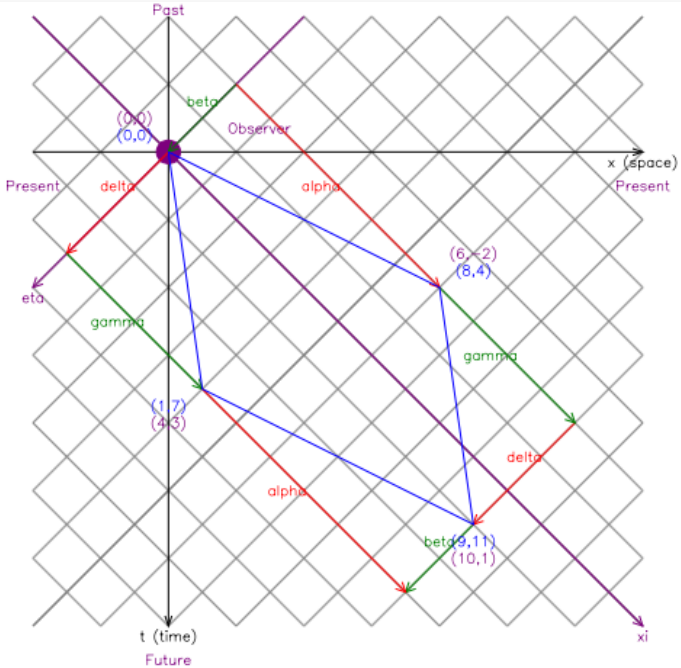

http://cycloids.adventas.de

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Contents:

1. Slowness: introduction and examples
2. From Physics to cycloids
3. Lorentz-transformation and slowness
4. **Characterisation of cycloids**
5. Slowness: looking back

Definition 3.46 [*Cycloid*]

For any $\alpha, \beta, \gamma, \delta \geq 1$ the structure

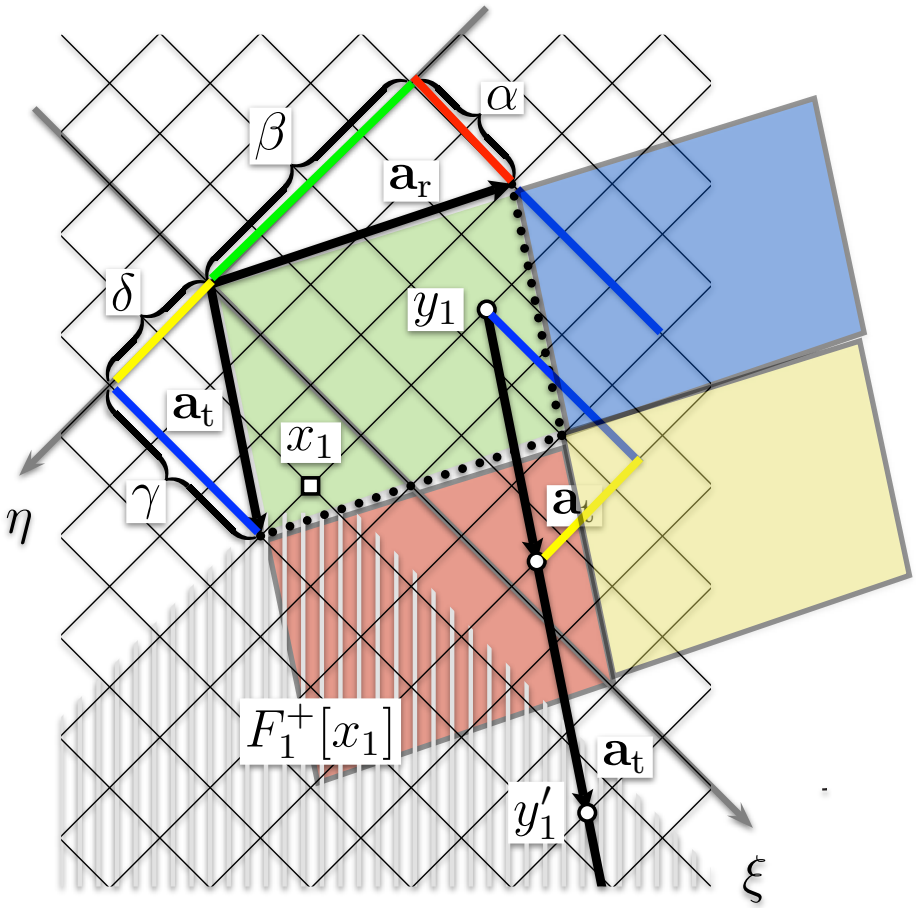
$$
\begin{aligned}
N_{\alpha\beta\gamma\delta} &:= (S_{\alpha\beta\gamma\delta}, T_{\alpha\beta\gamma\delta}, F_{\alpha\beta\gamma\delta}; M_{\alpha\beta\gamma\delta}) \\
(x, y) \equiv (u, v) &:\Leftrightarrow \exists m \in \mathbb{Z} : \exists n \in \mathbb{Z} : x = u + m\alpha + n\gamma \wedge y = v - m\beta + n\delta \\
S_{\alpha\beta\gamma\delta} &:= S_{SG}/\equiv \\
T_{\alpha\beta\gamma\delta} &:= T_{SG}/\equiv \\
F_{\alpha\beta\gamma\delta} &:= F_{SG}/\equiv \\
M_{\alpha\beta\gamma\delta} &:= \{(x, y + \tfrac{1}{2}) \mid x, y \in \mathbb{Z} \wedge x\beta + y\alpha < 0 \wedge x\beta + (y+1)\alpha \geq 0\}/\equiv \cup \\
&\quad\ \{(x + \tfrac{1}{2}, y) \mid x, y \in \mathbb{Z} \wedge x\beta + y\alpha < 0 \wedge (x+1)\beta + y\alpha \geq 0\}/\equiv
\end{aligned}
$$

is called the cycloid $(\alpha, \beta, \gamma, \delta)$.

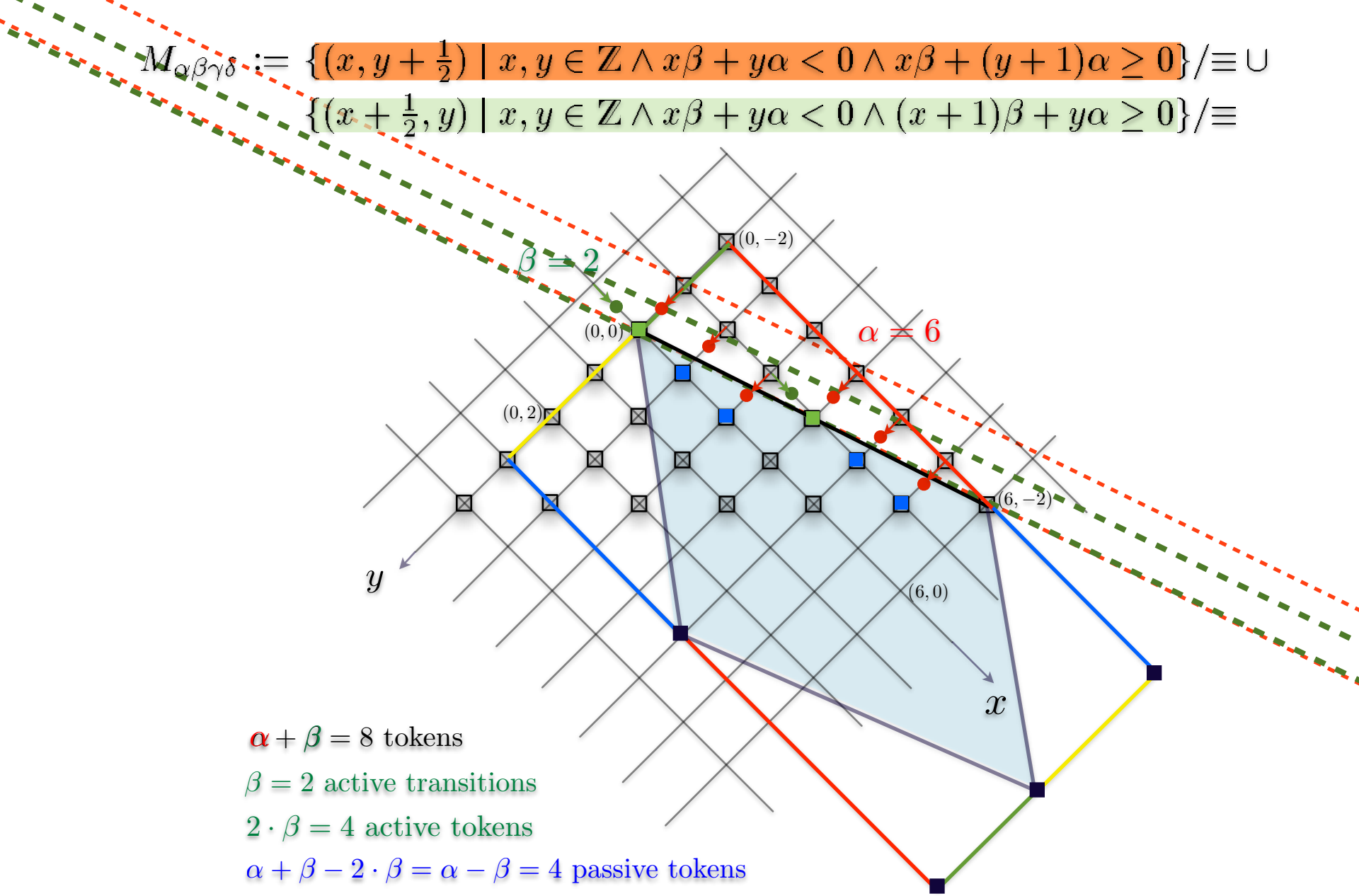
The folding is to be understood as the elements of standard grid onto the equivalence classes of $\equiv$

Olaf Kummer: Axiomatic Systems in Concurrency Theory,
Logos Verlag Berlin, 2001

$$(x,y) \equiv (u,v) :\Leftrightarrow \exists m \in \mathbb{Z} : \exists n \in \mathbb{Z} : x = u + m\alpha + n\gamma \wedge y = v - m\beta + n\delta$$



$$M_{\alpha\beta\gamma\delta} := \{(x, y+\tfrac{1}{2}) \mid x, y \in \mathbb{Z} \wedge x\beta + y\alpha < 0 \wedge x\beta + (y+1)\alpha \geq 0\}/\equiv \cup$$
$$\{(x+\tfrac{1}{2}, y) \mid x, y \in \mathbb{Z} \wedge x\beta + y\alpha < 0 \wedge (x+1)\beta + y\alpha \geq 0\}/\equiv$$



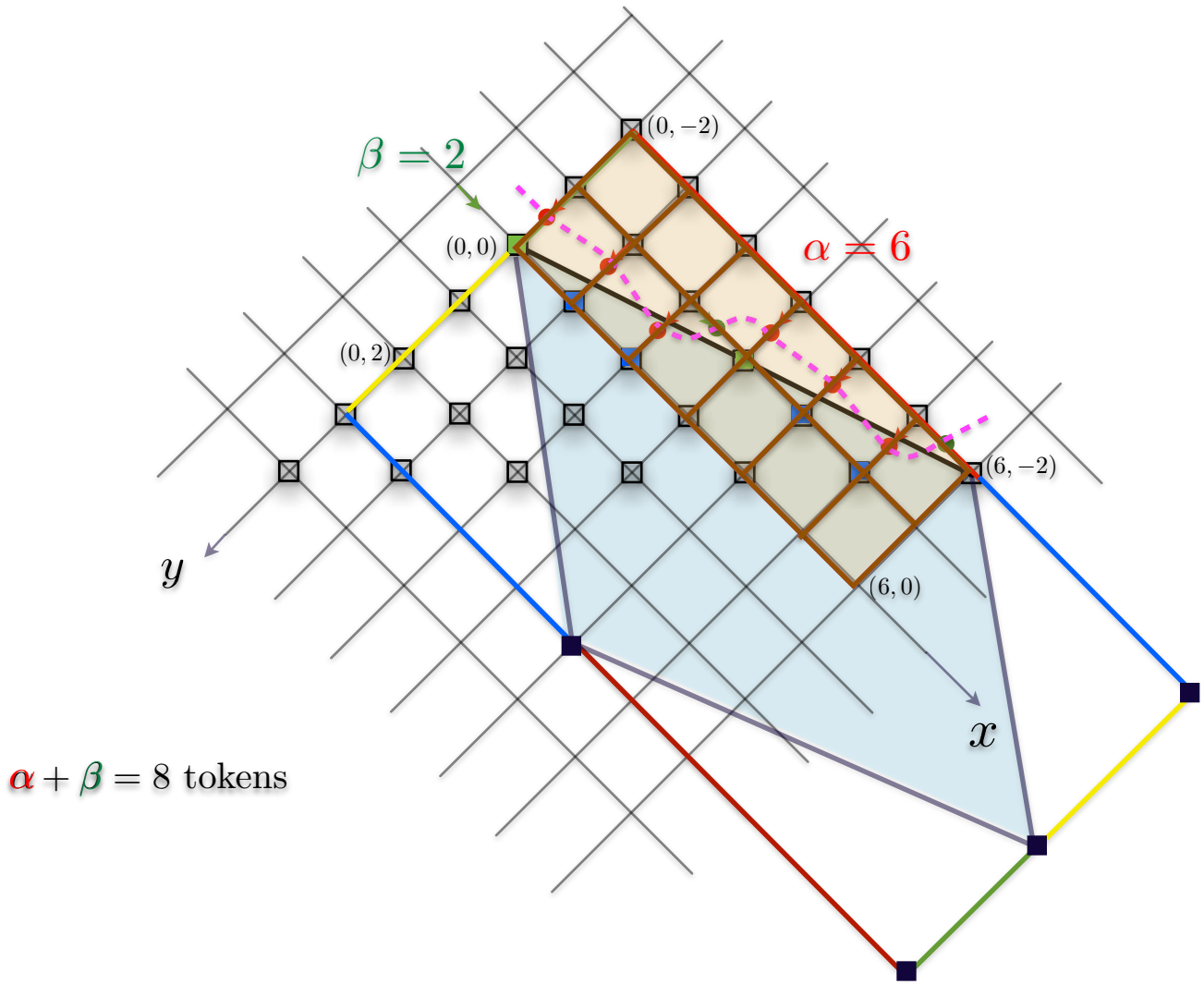
$\alpha + \beta = 8$ tokens

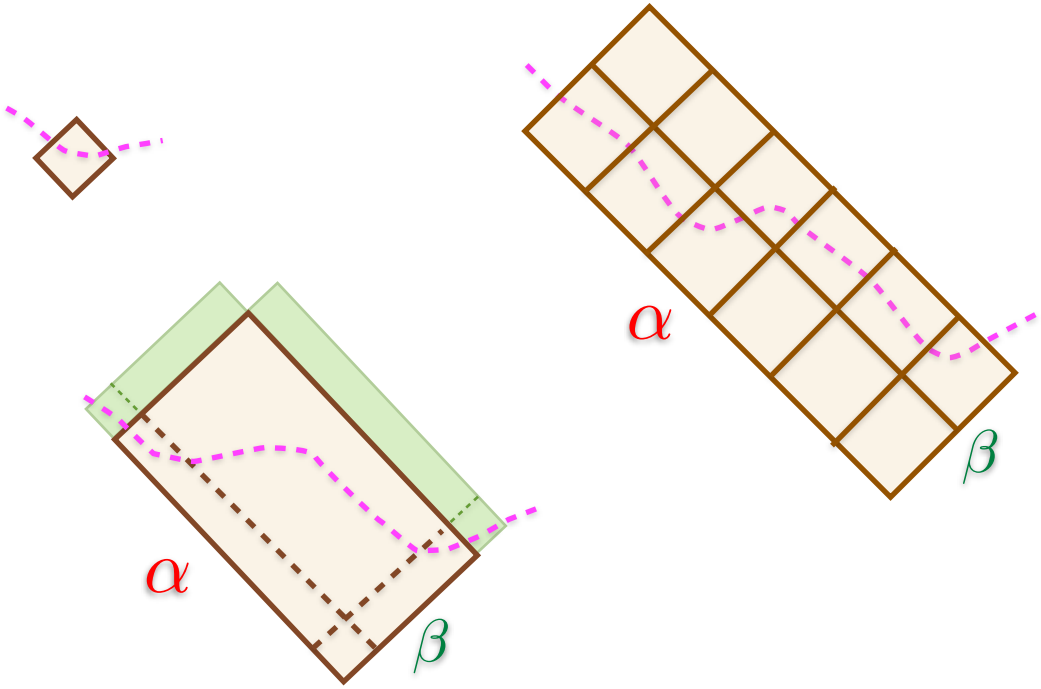
$\beta = 2$ active transitions

$2 \cdot \beta = 4$ active tokens

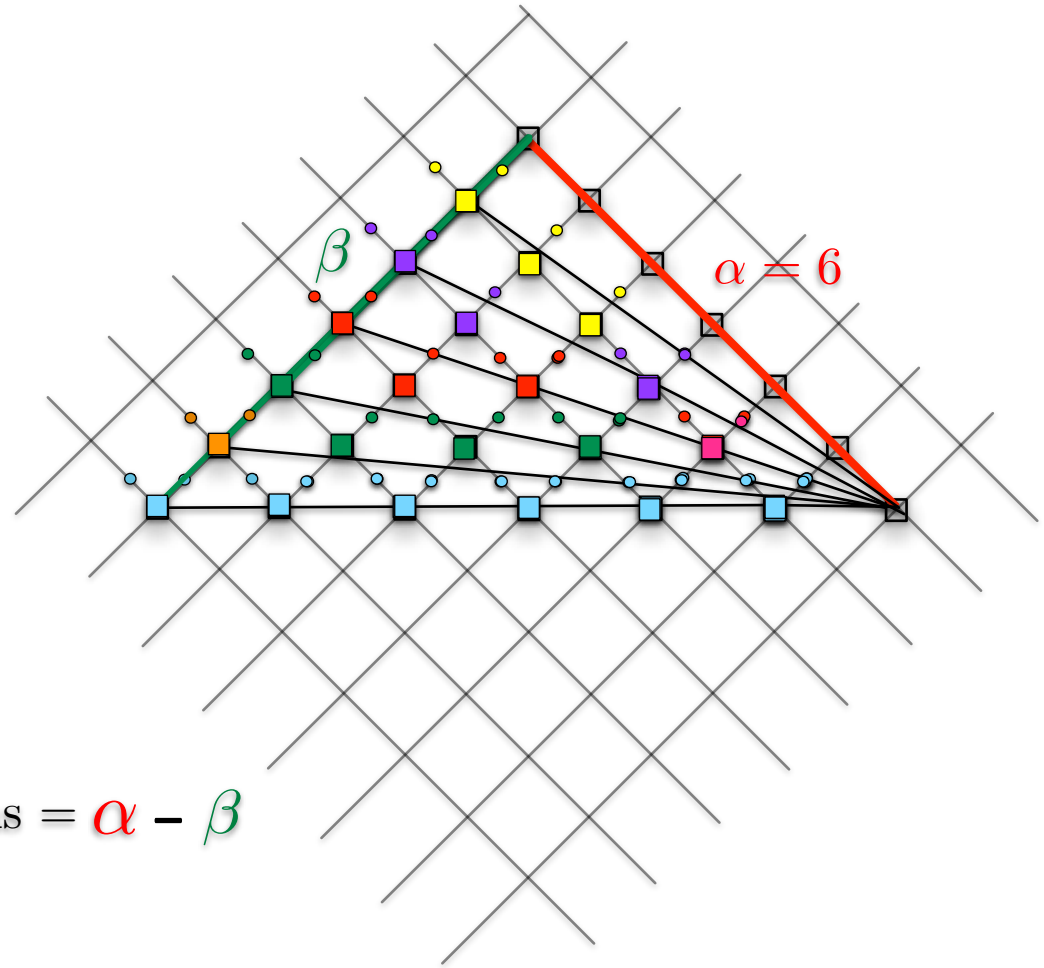
$\alpha + \beta - 2 \cdot \beta = \alpha - \beta = 4$ passive tokens

$\alpha - \beta = 4$ passive transitions


$\beta = 2$
$(0, -2)$
$(0, 0)$
$\alpha = 6$
$(0, 2)$
$(6, -2)$
y
$(6, 0)$
x
$\alpha + \beta = 8$ tokens

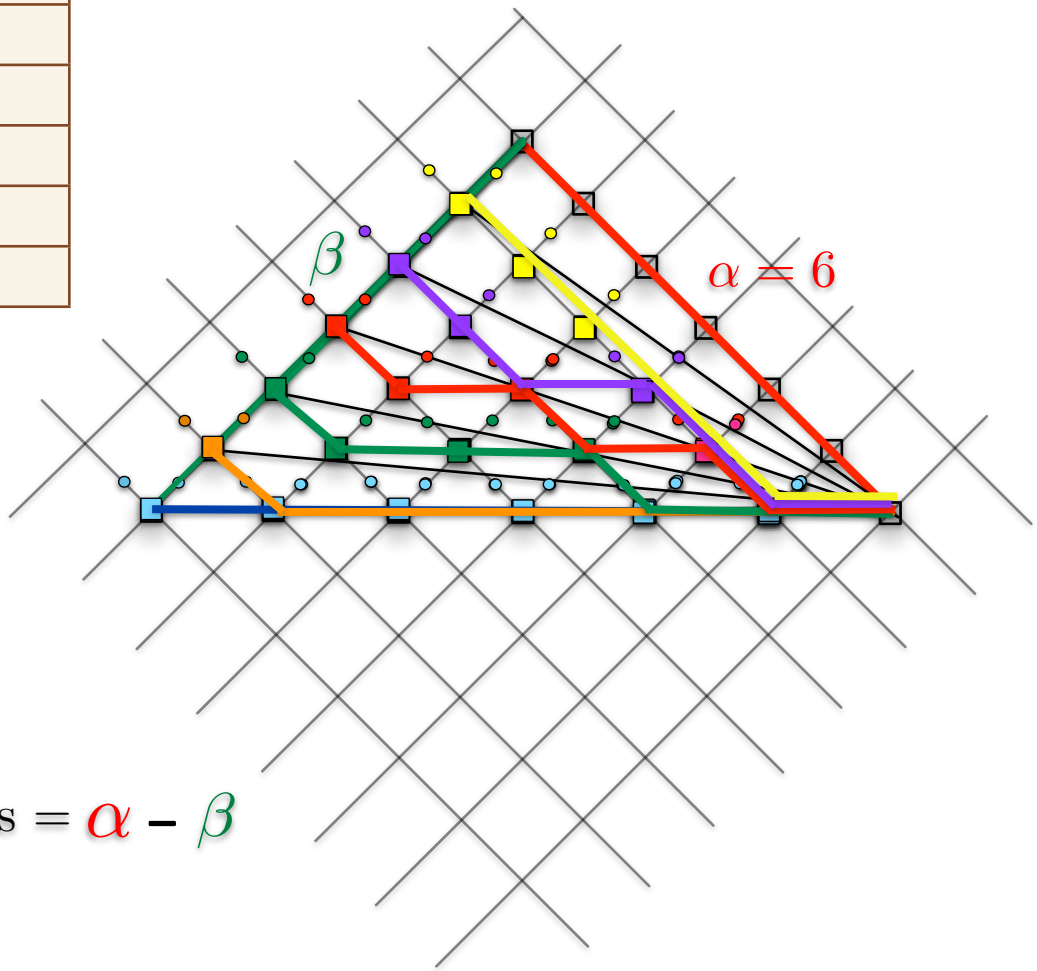

α
β
α
β

β	active transitions	passive tokens
1	1	5
2	2	4
3	3	3
4	4	2
5	5	1
6	6	0



number of passive tokens $= \alpha - \beta$

β	active transitions	passive tokens	number of edges
1	1	5	5
2	2	4	4
3	3	3	3
4	4	2	2
5	5	1	1
6	6	0	0



number of passive tokens $= \alpha - \beta$

Let be given a cycloid $(\alpha, \beta, \gamma, \delta)$ with $min(\alpha, \beta, \gamma, \delta) > 0$ and:

1. $c_0 := |M_0|$ the number of tokens in the initial marking, as defined by C.A. Petri and formalized by O. Kummer $\quad c_0 = \alpha + \beta$

2. n the number tokens not involved in a maximal (initial) step $\quad n = |\alpha - \beta|$

Then we claim

$\alpha = \frac{1}{2}(c_0 + n)$ and $\beta = \frac{1}{2}(c_0 - n)$ if $\alpha \geq \beta$ *or*

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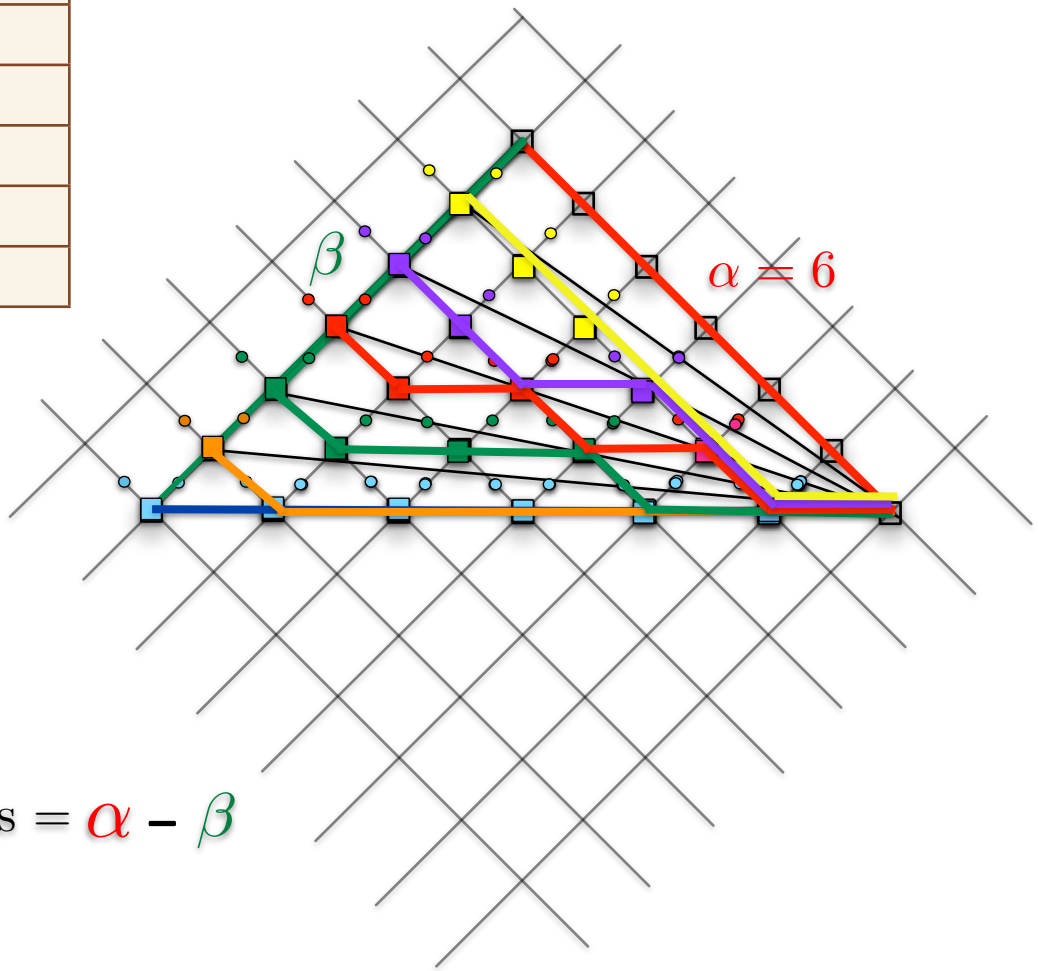
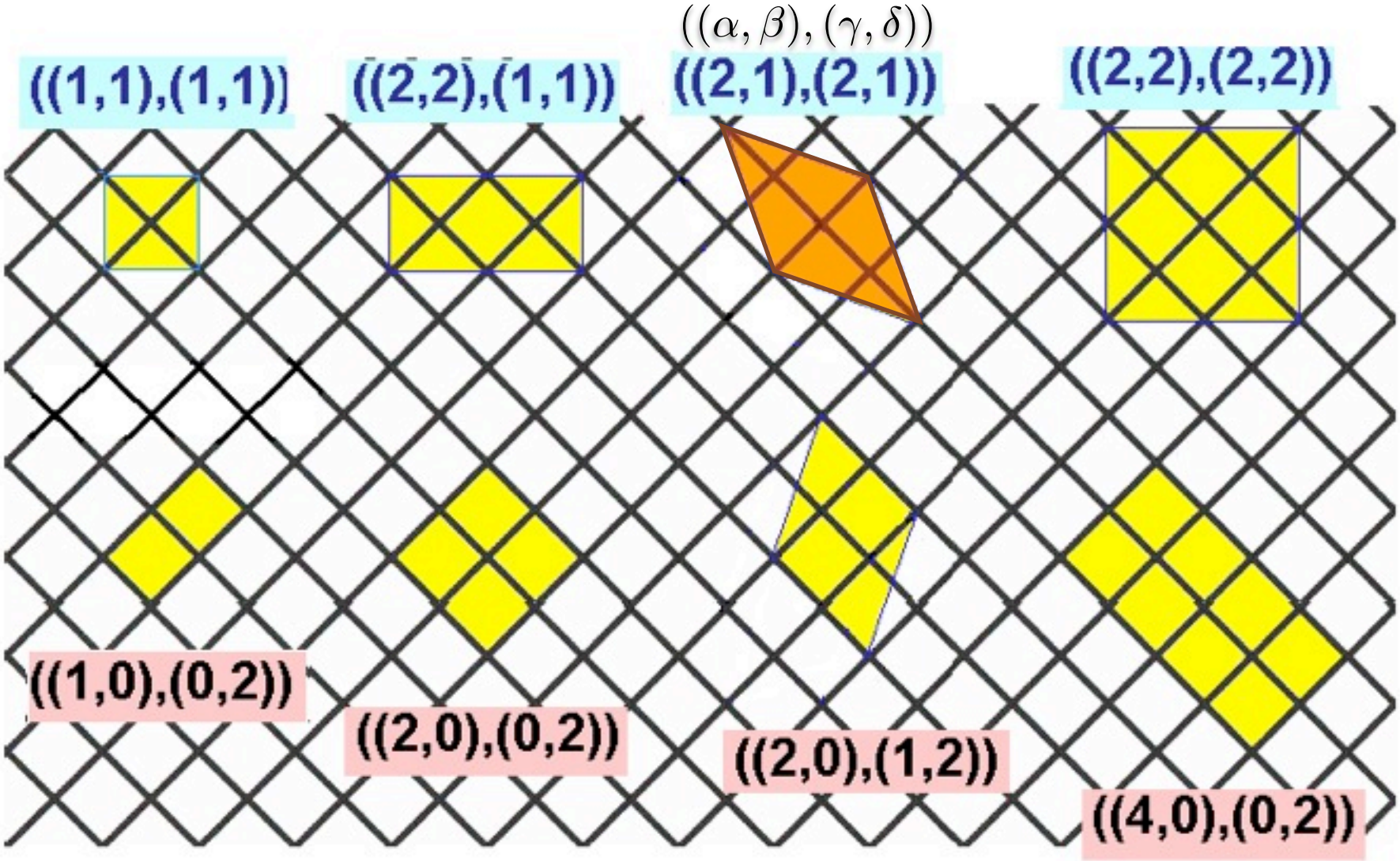


number of passive tokens $= \alpha - \beta$

$$\beta = 0\ ?$$

The smallest regular patterns of behavior



LT-compatible and degenerate Cycloids

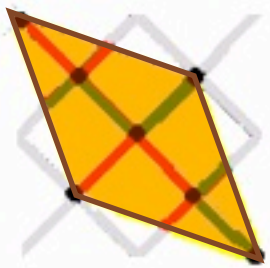
The smallest lossfree Boolean Transfe

$((\alpha, \beta), (\gamma, \delta))$

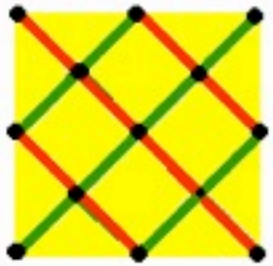
Bit-pair-Equality

Synchronizer, Bit Exchange

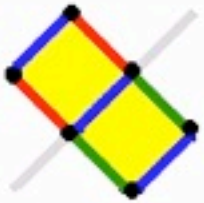
XOR-Transfer



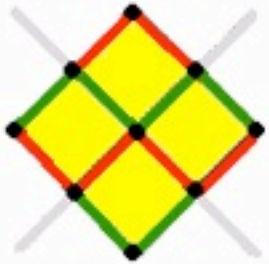
Majority Transfer



$((4, 0), (0, 2))$

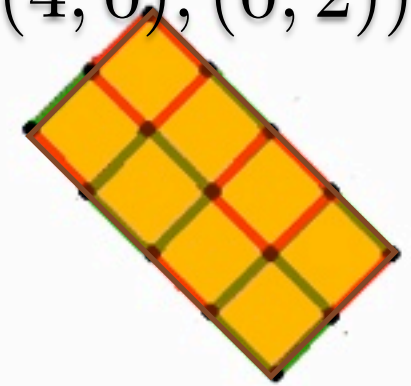


Dual of the two-state Automaton (not a net!)



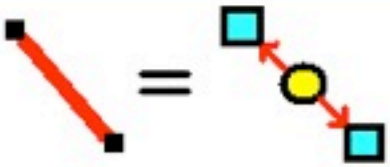
Synchronizer, Bit Exchange

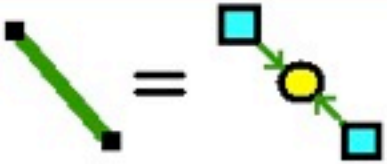
XOR-Transfer



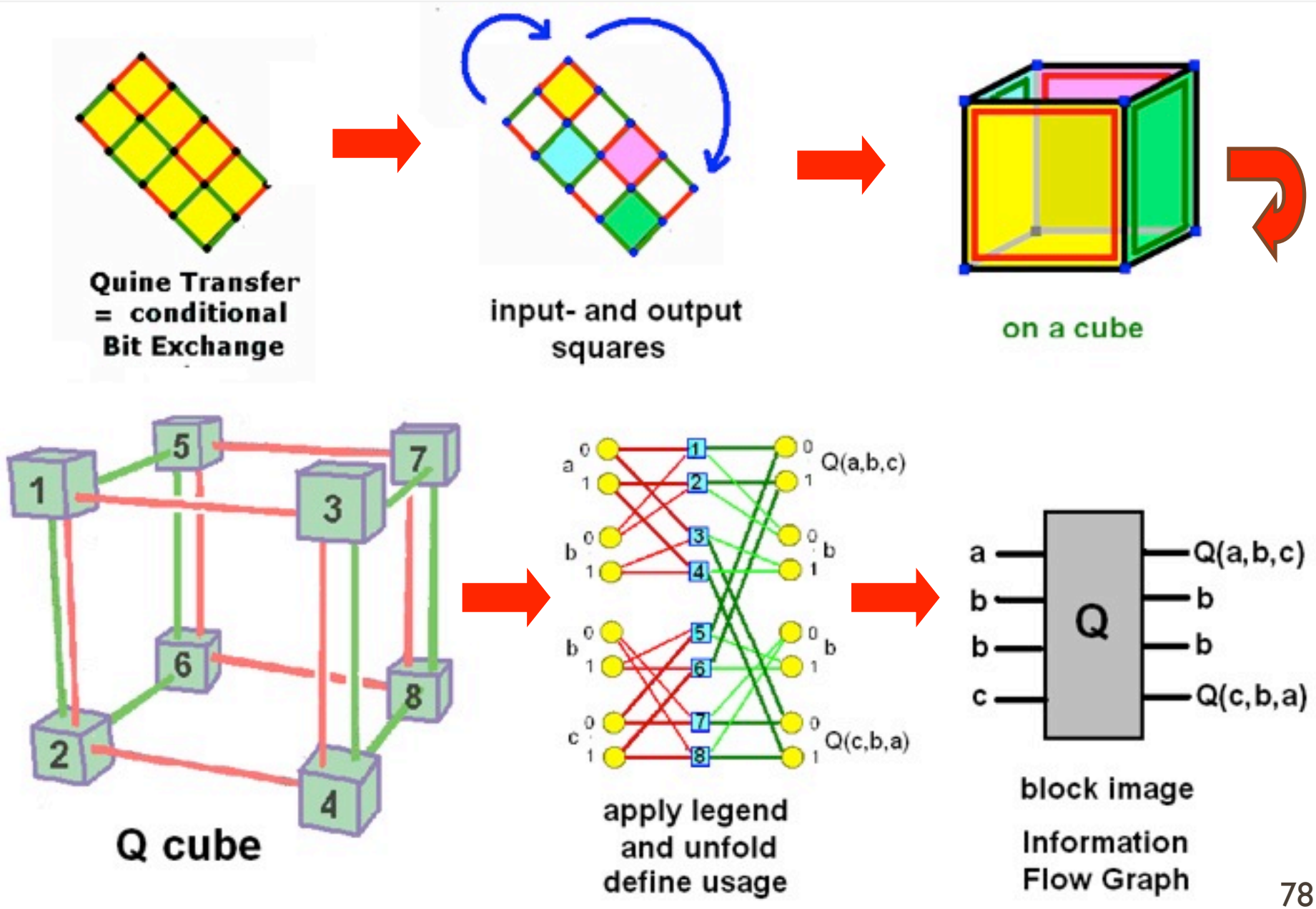
Quine Transfer = conditional Bit Exchange

Legend:





How the Net constructs can be generated from the rough images



Let be given a cycloid $(\alpha, \beta, \gamma, \delta)$ with $min(\alpha, \beta, \gamma, \delta) > 0$ and:

1. $c_0 := |M_0|$ the number of tokens in the initial marking, as defined by C.A. Petri and formalized by O. Kummer $\quad c_0 = \alpha + \beta$
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Then we claim

$\alpha = \frac{1}{2}(c_0 + n)$ and $\beta = \frac{1}{2}(c_0 - n)$ if $\alpha \geq \beta$ *or*

$\beta = \frac{1}{2}(c_0 + n)$ and $\alpha = \frac{1}{2}(c_0 - n)$ if $\alpha \leq \beta$.

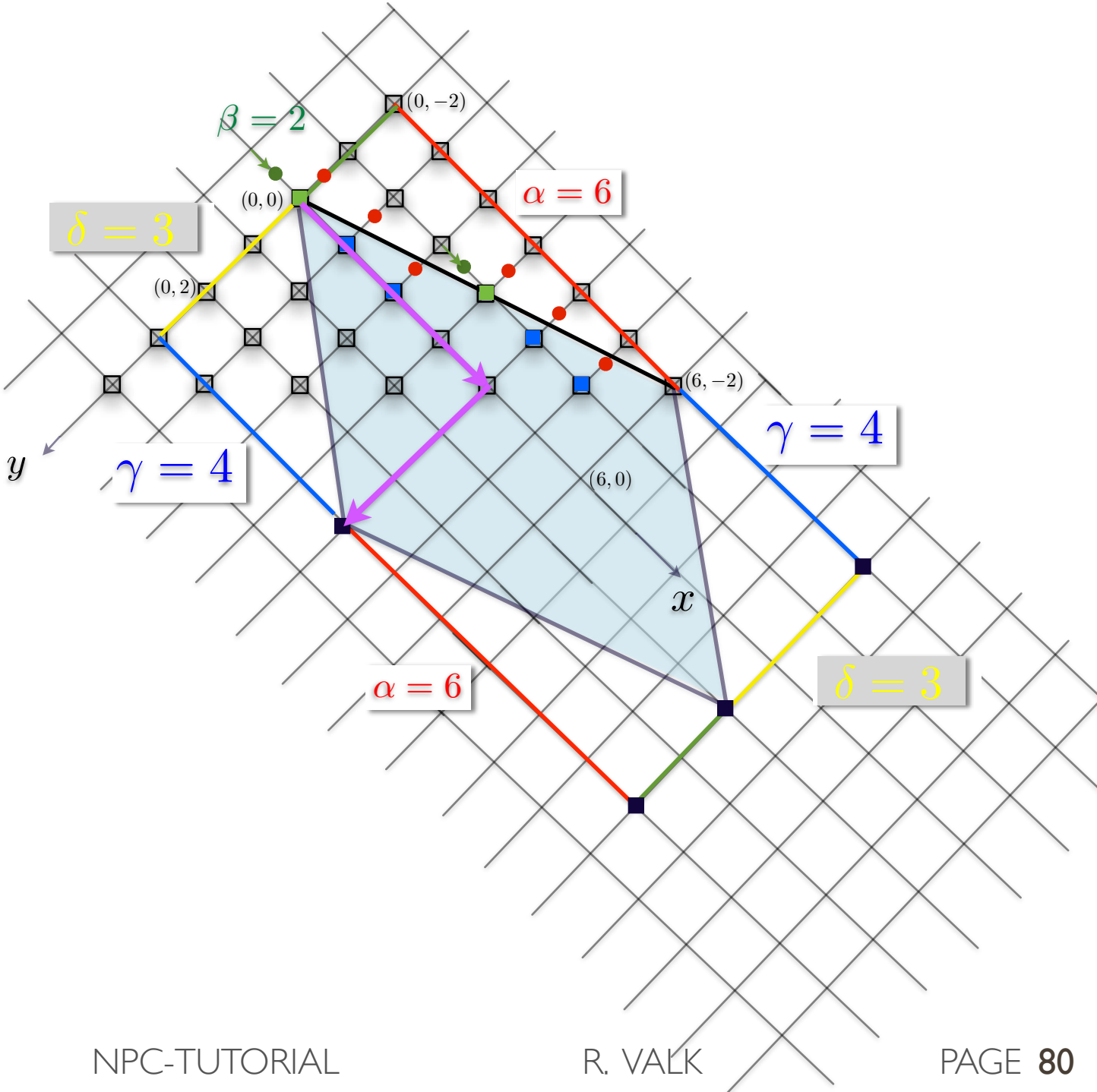
$\gamma = ?$

$\delta = ?$

$$A = \alpha \cdot \delta + \beta \cdot \gamma$$

$\gamma + \delta$

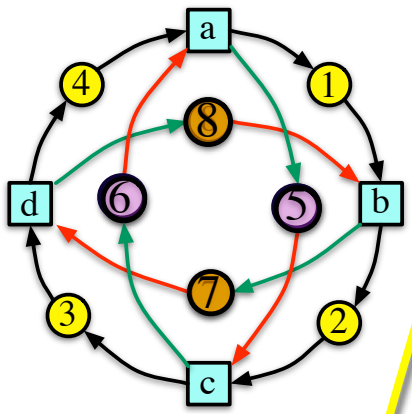
steps per
local basic circuits

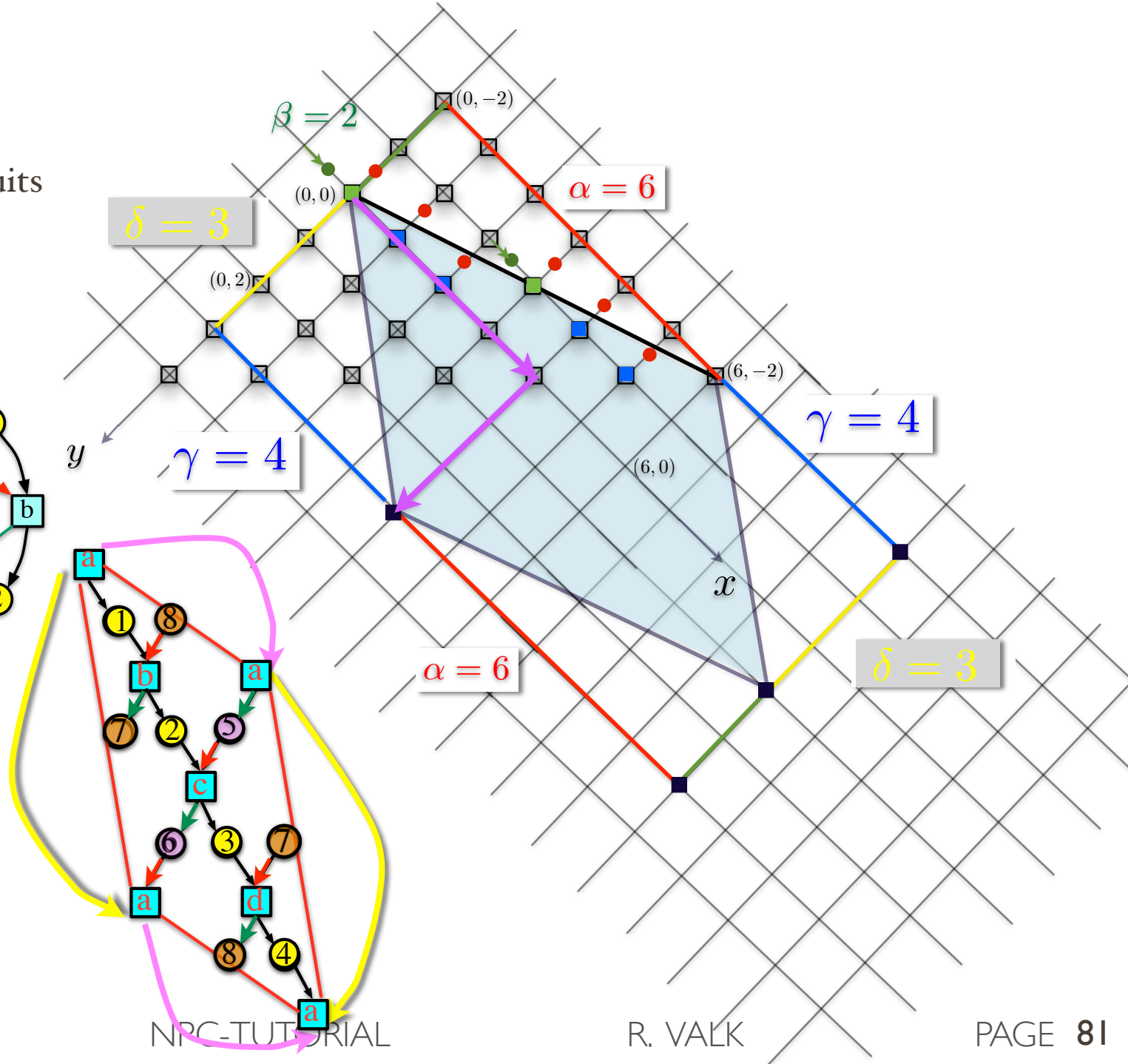


$\gamma + \delta$

steps per
local basic circuits

(2,1,2,1)





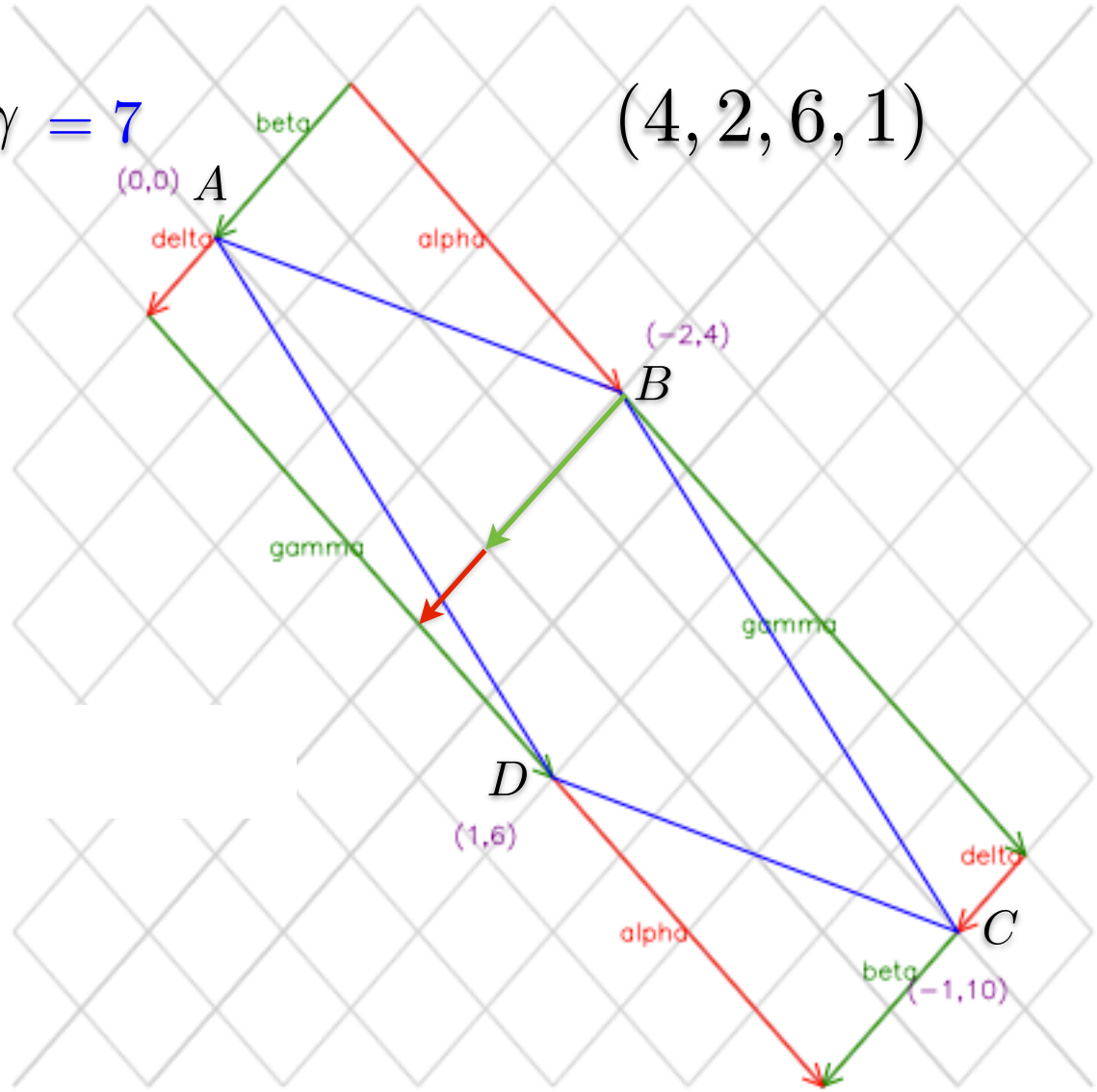
$$p = [A \to D] = \delta + \gamma = 7$$

$$q = [B \to D] = \beta + \delta + \gamma - \alpha = 5$$

if $\alpha \leq \gamma$

$$p \leq q \Leftrightarrow$$

$$(4, 2, 6, 1)$$



Let be given a cycloid $(\alpha, \beta, \gamma, \delta)$ with $min(\alpha, \beta, \gamma, \delta) > 0$ and:

1. $c_0 := |M_0|$ the number of tokens in the initial marking, as defined by C.A. Petri and formalized by O. Kummer $\quad c_0 = \alpha + \beta$
2. n the number tokens not involved in a maximal (initial) step $\quad n = |\alpha - \beta|$
3. $A := |T|$ the number of transitions $\quad A = \alpha \cdot \delta + \beta \cdot \gamma$
4. p or q the length of a minimal circuit $\quad p = \delta + \gamma$, $q = \beta + \delta + \gamma - \alpha$ if $\alpha \leq \gamma$

Then we claim

$\alpha = \frac{1}{2}(c_0 + n)$ and $\beta = \frac{1}{2}(c_0 - n)$ if $\alpha \geq \beta$ *or*

$\beta = \frac{1}{2}(c_0 + n)$ and $\alpha = \frac{1}{2}(c_0 - n)$ if $\alpha \leq \beta$.

$\alpha < \beta$

$$\gamma = \frac{A - \alpha \cdot p}{\beta - \alpha}$$

$$\delta = \frac{\beta \cdot p - A}{\beta - \alpha}$$

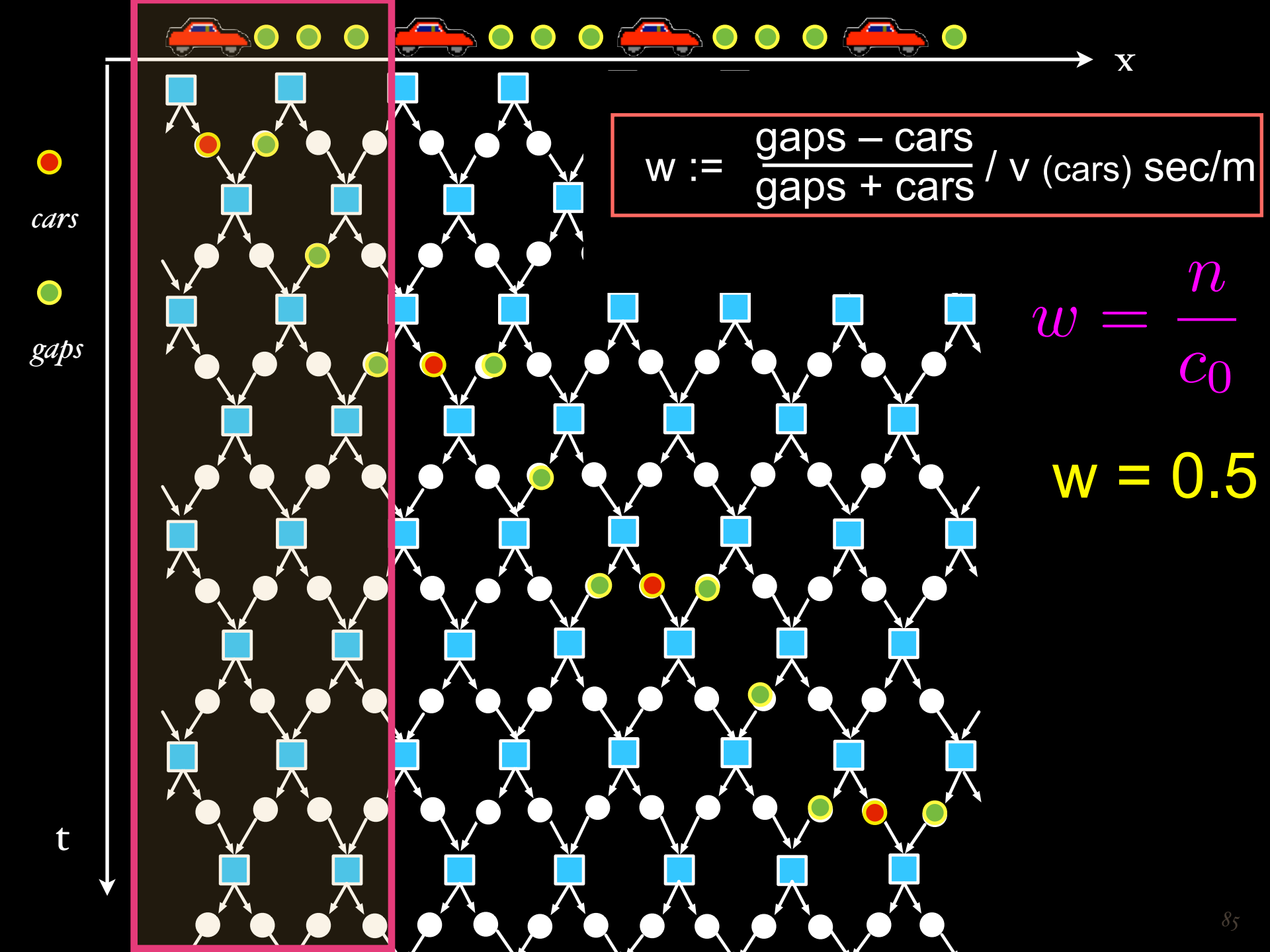
$$\gamma = \frac{\alpha(q + \alpha - \beta) - A}{\alpha - \beta}$$

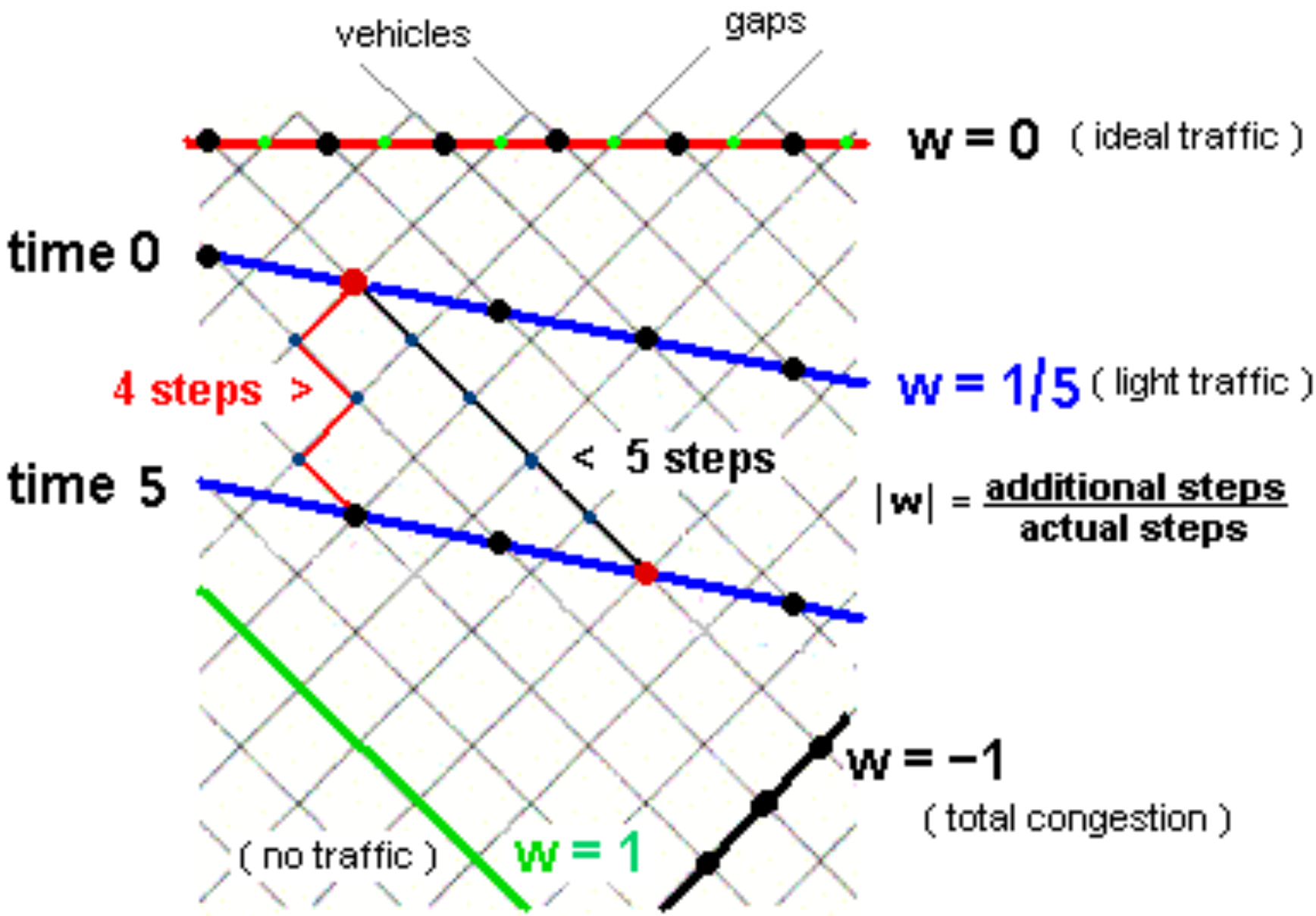
$$\delta = \frac{A - \beta(q + \alpha - \beta)}{\alpha - \beta}$$

$\beta < \alpha \leq \gamma$

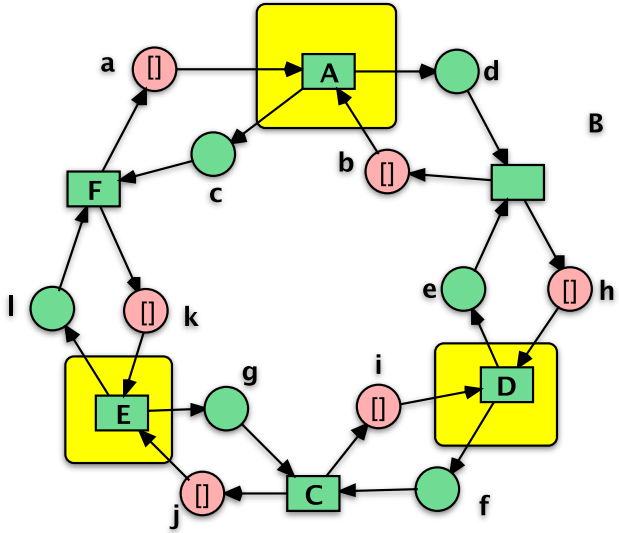
Contents:

1. Slowness: introduction and examples
2. From Physics to cycloids
3. Lorentz-transformation and slowness
4. Characterisation of cycloids
5. **Slowness: looking back**

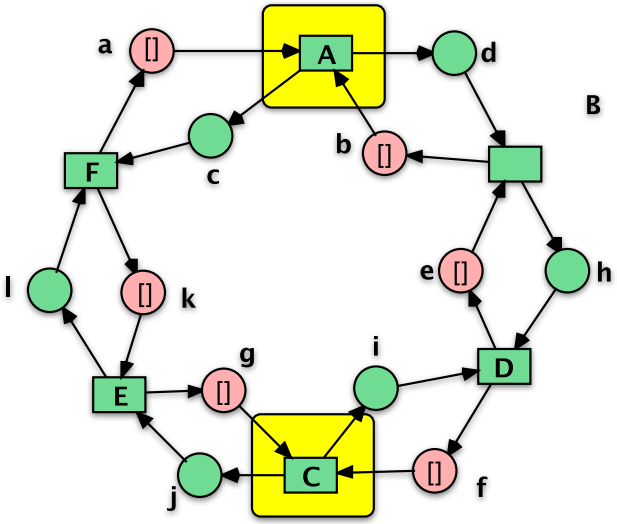

x
cars
gaps
t
w := (gaps – cars) / (gaps + cars) / v (cars) sec/m
$w = \frac{n}{c_0}$
w = 0.5



The Slowness w is measured in "time-units lost per space unit", e.g. in seconds per meter. Its combinatorial value ranges from -1 to +1.



$$w = \frac{n}{c_0}$$



(3,3,1,1) - slowness = 0

(4,2,1,1) - slowness = 1/3

$$w = \frac{0}{6} = 0$$

$$w = \frac{2}{6} = \frac{1}{3}$$

$$w = \frac{|m-n|}{k}$$

in general:

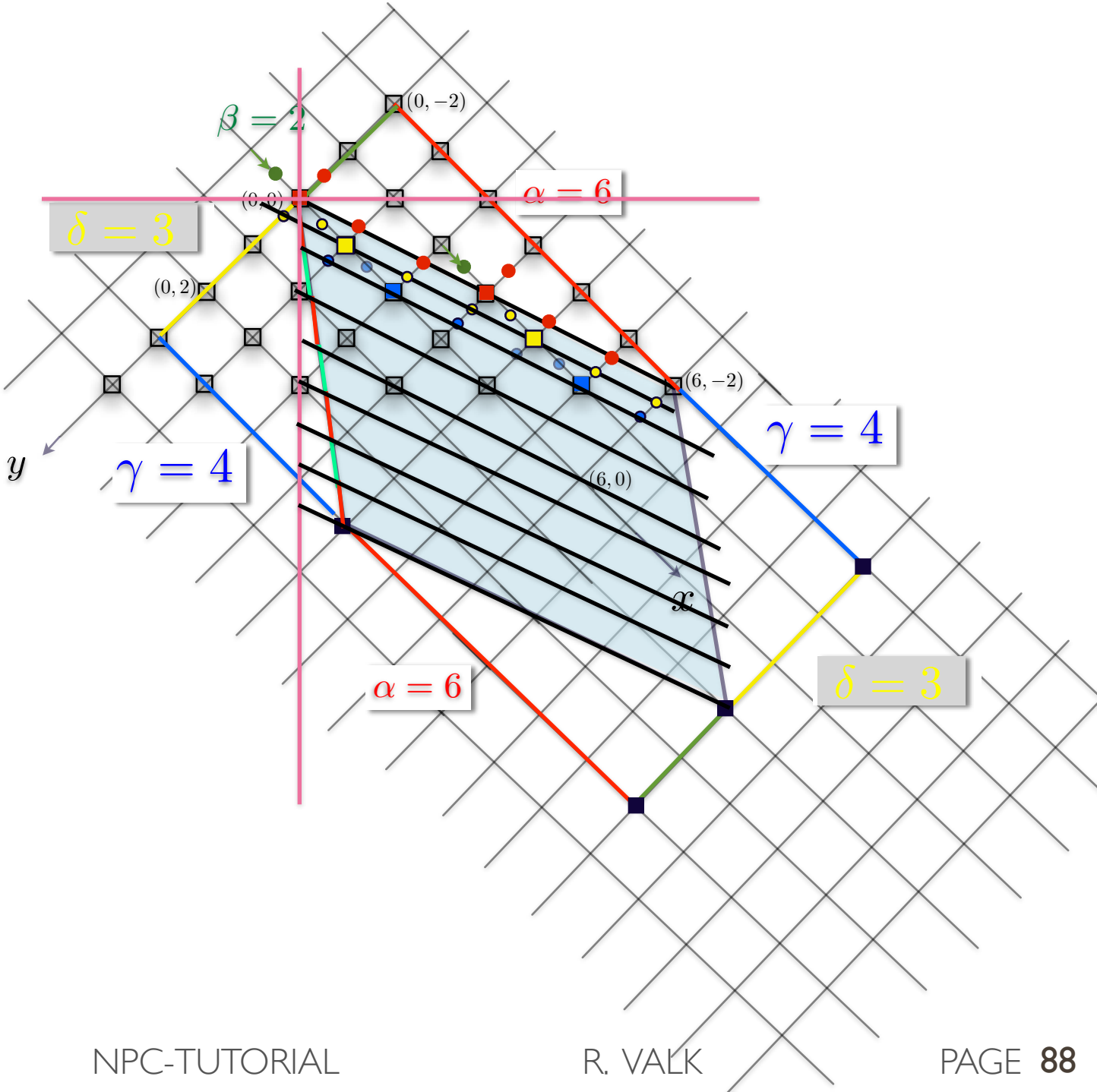
cycloids **(m,n,1,1)** *with* **m+n=k=const** *have the same structure*

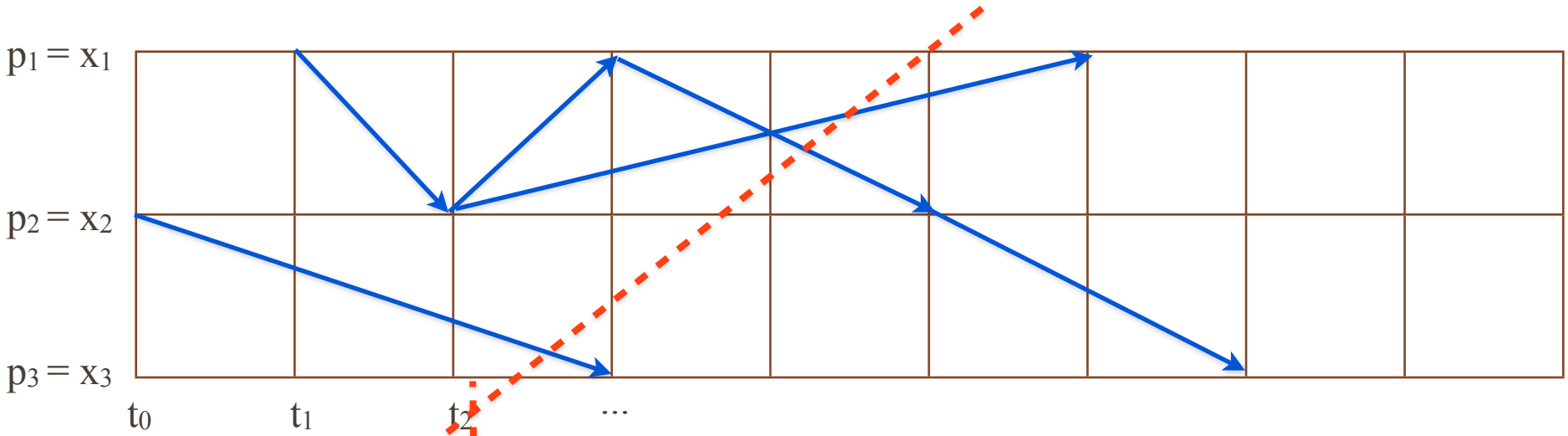
(6,2,6,2) *and* **(2,6,2,6)** *have the same structure*

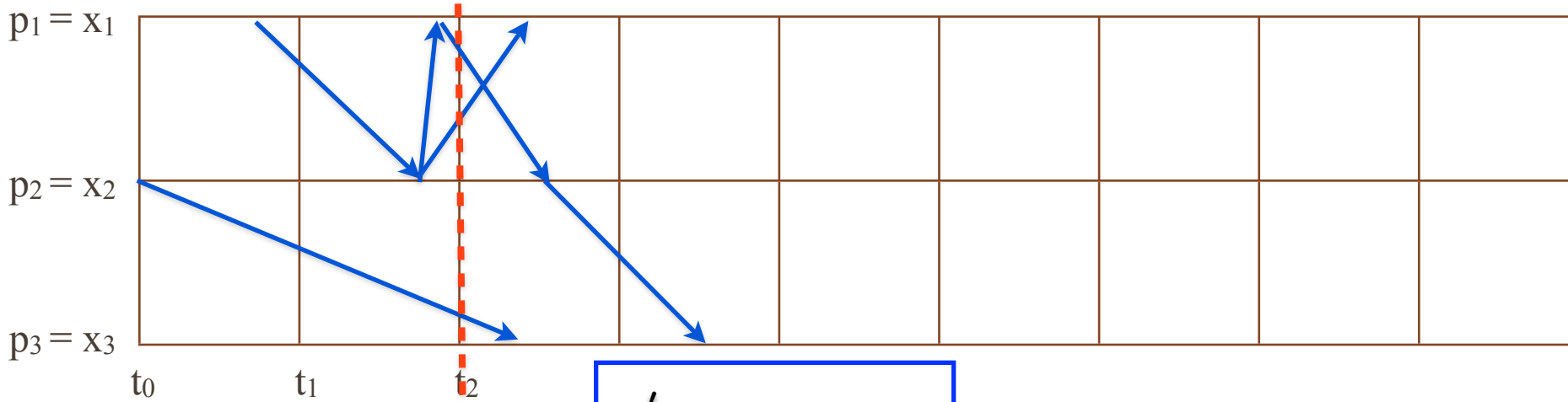
$$w = \frac{|6-2|}{8} = \frac{1}{2}$$

$\gamma + \delta$

steps per
local basic circuits







affine mapping

$$x' = x$$

$$t' = \frac{2}{5}(x + \frac{4}{5} \cdot t)$$

Summary and Conclusions:

1. Slowness: introduction and examples

- *example of car flow*
- *Little's Law*

2. From Petri-coordinates to cycloids

- *Petris „natural" coordinates represent movement in space <u>and</u> time*
- *interpretation of finite systems by folding in space <u>and</u> time*
- *tile parameters of cycloids*

3. Lorentz-transformation and slowness

- *Petri's view of physical laws*
- *from Galilei- to Lorentz-transformation*
- *the notions of velocity and slowness*
- *the travel pradoxon*
- *from* x-t-*coordinates to* ξ-η-*coordinates*

4. Characterisation of cycloids

- *computation of slowness from* α *and* β
- *slowness by restricting concurrency*
- *computation of* $\alpha,\beta,\gamma,\delta$ *from net properties*

5. Slowness: looking back

- *relating slowness of car-flow to net properties*
- *Lorentz-transformation and simultaneity*
- *general observation:*

  Petri's interpretation of phsical laws

  by ordinary coordination principles

THANK YOU !